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Randomness and recurrence in dynamical systems. A real analysis approach. (English)

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The book presents an interesting and original selection of fundamental ideas, concepts and results from areas of the modern theory of dynamical systems closely related to probability. According to the author, the purpose of this book is "to connect the material with some recent research, with the aim of giving a more 'contemporary' feel to the material and contributing in a small way towards 'bridging the gap' between undergraduate teaching and the world of current mathematical ideas and research. In some ways the work should be regarded as 'lecture notes', in that questions and motivation of the ideas, and thinking about the results from different viewpoints, are considered perhaps more than usual".

The topics chosen by the author for this nice book can be classified as "real analysis with a decisive flavor of probability theory". The material is divided into five chapters. Chapter 1 has an introductory character; it provides several basic ideas and prepares the reader to the subject through a number of definitions and examples. The prerequisites include the knowledge of set theory, complex numbers, elementary calculus and some real analysis. In particular, a good understanding of the concept of a function and related notions is required. Suggestions for background references are provided for the reader who needs to refresh the material. Fundamental definitions and terminology are also introduced here along with the main principles of mathematical reasoning and proof.

Chapter 2 relates irrational numbers to dynamical systems; it is focused on two key results: Kronecker's and Weyl's theorems regarding distribution of irrational numbers over the interval $[0, 1)$. Chapter 3 pursues connections between elementary real analysis and probability theory. The main interest is in the case where the sample space is a bounded interval on the real axes, usually the unit interval $[0, 1)$, and a main technical tool for this is the relationship between $[0, 1)$ and the sample space consisting of all sequences of ones and zeros. The topics discussed in this chapter include sets and events, recurrence, randomness, binary expansions, law of averages, the Walsh functions, leading digits, etc. Some earlier results, namely, the Recurrence Theorem and Borel's Theorem, are interpreted here from the point of view of dynamical systems.

In Chapter 4, the question of recurrence is viewed from a mathematical point of view, emphasizing the case of recurrent phenomena in dynamical systems on a bounded interval of numbers. Length-preserving transformations, Poincaré Recurrence Theorem, Kac's result on average recurrence times are considered along with applications to the Kronecker and Borel dynamical systems. The final Chapter 5 has an expository nature and collects more advanced material that uses ideas and results from measure theory. It is centered around the idea of averaging in dynamical systems and Birkhoff's Individual Ergodic Theorem. One of the primary goals is to review many ideas and topics

introduced earlier in the book using powerful tools of measure theory. This chapter also presents several underlying ideas in measure theory and ergodic theory that may not be found in standard texts on the subject.
In the end of all chapters in the book, the reader can find exercises sections, investigation sections with more challenging problems and small projects related to practical situations, notes with additional information and related historical details, and a detailed bibliography. In the beginning of Chapters 2–5, flow charts explaining the organization of the material and relationship between different sections are provided.
The author made each and every effort to present in the book a selection of important topics related to dynamical systems and randomness that are usually encountered in graduate texts at the level accessible for undergraduates. To this end, auxiliary material from probability theory, measure theory or integration is introduced whenever needed. The clarity of exposition does not necessarily imply its simplicity. However, an interested and motivated reader will be enriched with a deeper understanding of remarkable, sometimes surprising connections between dynamical systems and randomness, both being subjects of the extensive research in modern mathematics.

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∗37-01 Instructional exposition (Dynamical systems and ergodic theory)