School of Mathematics & Applied Statistics

MATH151: Mathematics 1A
Assignment Week 4
Autumn 2010

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked.
Not all the questions on the assignment will be marked.
You will receive a mark out of ten for this assignment.
This assignment is to be handed in during your tutorial in Week 6

The assignment that you hand in *must* include a cover page. You should use the first-page of this assignment as the cover sheet. As part of your assignment you should briefly answer the following questions

(a) What topic do you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) Did you have any problems with the assignment? What action did you take regarding any questions that you found difficult?

(d) Did you use any of the resources on the Summertime Mathematics project (`www.math.uow.edu.au/subjects/summer/subjects/m151.html`), or elsewhere on the net (including `www.uow.edu.au/~mnelson/teaching.dir/math151-resources.html`), to help in this assignment?

If you found a resource that was particularly helpful, please indicate where it is to be found — so that it can be added to the list of resources and other students can use it.

You should answer each question with a complete sentence. High quality feedback is important to continue improving this course. Therefore a bonus mark of one is available for answers to these questions. Thus the maximum mark on this assignment is 11/10.

These questions have various purposes. These include

- In answering the first three questions you are reflecting on the content of the course.
- Not all the questions on the assignment are marked. Your tutor will look at your answers to any questions that you found difficult (question c) which are not marked.
- In answering the last two questions you are providing feedback to the course co-ordinator. Depending upon the feedback received the co-ordinator may add appropriate review material in a subsequent lecture, or may revise the content of the course (helping students taking this course next year) or may develop additional resources for this subject (helping students taking this course next year).

You may choose to answer the questions on this assignment sheet by working in a group of upto four individuals. If you choose this option then you should: state which question(s) you answered as a group, list the members of your group and indicate what you learnt by working as a group.

School of Mathematics & Applied Statistics **MATH151: Mathematics 1A**
Assignment Week 4
Autumn 2010 Submission Receipt

Student Name: _____ *Student Number*: _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

Maths questions

The objectives of this section are to develop your mathematical skills as follows:

- To use basic mathematical skills;
- Be capable of applying logical, analytic and creative thinking to a range of problems.

You're holding a basketball. You're facing the basket. There's no-one else on the court. You're practising shooting hoops. That's how to think about these questions.

1. Is the following calculation correct?

$$\frac{1}{4}+\frac{2}{3}=\frac{1+2}{3+4}=\frac{3}{7}$$

   If not, why not?

2. If $g(t)=t^2-3t$ and $h(z)=2z-1$ find
   (a) $g(h(z))$
   (b) $h(g(t))$
   (c) $h(h(t))$
   (d) $g(g(z))$

3. The functions f, g and h given by the following rules are not valid for all real numbers. Write down the values of x where the rules do not apply.

$$f(x)=\frac{x}{x^2-9} \qquad g(x)=\frac{1}{\sqrt{x}} \qquad h(x)=\frac{\sqrt{x-5}}{x}$$

4. Draw graphs which
   (a) represents a function
   (b) does not represent a function — explain why your graph does not represent a function
   (c) represents a function having zeroes at $x=-3$ and $x=2$

5. Is $(x-1)^2$ the same quantity as $(1-x)^2$? Justify your answer.

6. Factorise completely the expression y^4-3y^2-4. Hint. Let $z=y^2$.

7. Without using your calculator, solve each of the following equations for x
   (a) $2\log(x)+3=5\log(x)$.
   (b) $\log(x)=4\log(2)-2\log(x)$.
   (c) $\log(x)-\log(x-1)=1$.

8. Is the following reasoning correct? If not, identify *all* errors.

   Question. Solve the equation

$$\log x-\log 3=\log 5-\log(x-2).$$

   Solution

$$\begin{aligned}
\log(x) - \log(3) &= \log(5) - \log(x-2), \\
\log(x) - \log(3) &= \log(5) - \log(x) - \log(-2), \\
2\log(x) - \log(3) &= \log(5) - \log(-2), \\
2\log(x) - \log(3) &= \log(5) + \log 2, \\
2\log(x) - \log(3) &= \log(7), \\
2\log(x) &= \log 10, \\
x &= 5.
\end{aligned}$$

The College Mathematics Journal, **39**(3), 229, 2008. Supplied by Dale Buske.

9. Are the following calculations correct? For each incorrect answer explain where the error is.

   Q. The definition of pH is

   $$\mathrm{pH} = -\log\left[\mathrm{H}^+\right],$$

   where $\left[\mathrm{H}^+\right]$ is the concentration of hydrogen ions measured in units of $\mathrm{mol\,dm^{-3}}$. Without using a calculator, what is the pH of a solution containing a concentration of hydrogen ions of $10^{-5}\,\mathrm{mol\,dm^{-3}}$? Simplify as far as possible.

   A1.

   $$\begin{aligned}
   \mathrm{pH} &= -\log[\mathrm{H}]^+, \\
   \Rightarrow \mathrm{pH} &= -e^{10^{-5}}
   \end{aligned}$$

   A2.

   $$\begin{aligned}
   \mathrm{pH} &= -\log[\mathrm{H}]^+, \\
   \Rightarrow \mathrm{pH} &= -\log 10^{-5}.
   \end{aligned}$$

Maths 'in action'

The objectives of this section are to develop your mathematical skills as follows:

- To use basic mathematical skills to solve a range of problems relevant to the scientific disciplines;
- Be capable of applying logical, analytic and creative thinking to a range of problems.
- To understand the applications of mathematics to other fields
- To apply mathematical principles to the formulation and solutions of problems.

In this section mathematical techniques are now placed within the context of a scientific. The situation is no longer as simple as shooting hoops on an empty court. The court is no longer empty. You have to worry about the opposition. You have to be able to shoot and score without thinking about the technique that you are using.

1. Assume that an atom of neon is spherical with a volume V of $4.85 \times 10^{-24}\,\mathrm{m^3}$. If the equation relating radius r and volume V of a sphere is:

   $$V = \frac{4}{3}\pi r^3$$

   determine the atomic radius of a neon atom.

2. Let $f(t)$ be the number of cm of rainfall that has fallen since mid-night, where t is the time in hours. Interpret the following in practical terms, giving units:

   $$f(1) = 3.1.$$

3. According to Sean Milo (2008) by the end of June 2008 the price of Brent crude had reached \$140/bbl, 40% higher than at the beginning of 2008, when it was already 50% higher than in June 2007.

   What was the price of Brent crude in June 2007?

   Milmo, S. (2008). Turbulence ahead. *the chemical engineer*, **806**, 30–31.

4. Ethanoic acid ionizes in water to form one anion and one solvated proton

$$C_2H_4O_2 \rightleftharpoons C_2H_3O_2^- + H^+$$

   The ionization does not proceed to completion; rather a proportion x ionizes, while the remainder persists as a neutral, covalent molecule, according to the equilibrium-following constant:

$$K = \frac{[\mathrm{H}^+]\,[\mathrm{anion}^-]}{[\text{un-ionized acid}]}.$$

   At equilibrium, the concentration of $[\mathrm{H}^+]$ and $[\mathrm{anion}^-]$ are both x, and the concentration of un-ionized acid will be $(0.1 - x)$. [1]

   (a) Explain why the equilibrium-following constant can be written in the form

$$K = \frac{x^2}{0.1 - x}.$$

   (b) Re-arrange the equation

$$K = \frac{x^2}{0.1 - x}$$

   to obtain a quadratic equation for x

   (c) The dissociation constant K of ethanoic acid is 1.75×10^{-5}. What is the proportion of the acid that is dissociated (x)? Hint. The value of x can not be negative.

5. (This question is harder)

   The Arrhenius equation is given by

$$k = A \exp\left[-\frac{E}{RT}\right],$$

   where k is the rate constant, A is called the pre-exponential factor, E is called the activation energy, R is the ideal gas constant and T is the temperature, measured in Kelvins.

   (a) By taking logs and simplifying show that this equation can be written in the form

$$\ln k = \ln A - \frac{E}{R} \cdot \frac{1}{T}.$$

   (b) Solve the set of simultaneous equations

$$y = 4x + 3,$$
$$y = 7x + 2.$$

   (c) Understanding the high-temperature behaviour of nitrogen oxides is essential for controlling pollution generated in vehicle engines. The decomposition of nitric oxide (NO) to nitrogen (N_2) and oxygen (O_2) has a rate constant of 0.0796 $\mathrm{mol}^{-1}\mathrm{s}^{-1}$ at 737 C and 0.0815 $\mathrm{mol}^{-1}\mathrm{s}^{-1}$ at 947 C. Using the equation you derived in the previous part of this question calculate the activation energy for the reaction.
   Notes.

   i. You need to first convert the temperature from centigrade to kelvin.
   ii. The value of the parameter R is $8.314\,\mathrm{J\,mol^{-1}K^{-1}}$.
   iii. Hint 1. Consider how you answered the second part of this question.
   iv. Hint 2. On the e-learning page you go to the folder
   `Useful Resources > Maths in Action videos`.
   In the Chemistry section find the `activation energy` video. This video shows you how to get started on this question.

[1] For chemistry students. The $(0.1 - x)$ assumes that the initial concentration (activity) of the acid was $0.1\,\mathrm{mol\,l^{-1}}$.

Miscellaneous

Goto `www.wolframalpha.com` which is a a computational knowledge engine. It's fun to play with, and very powerful.

Can you use it to check your answer to one of the questions in either this assignment or the assignment from week one? If so, give the command that you used to check your solution.