Integration Worksheet: Solutions

1.

$$\text{Use the formula } \frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x)\cdot g'(x)$$

Here we have

$$\begin{aligned} f(t) &= \sqrt{\tan^2 t + e^t}, \\ g(x) &= x^2, \\ g'(x) &= 2x. \end{aligned}$$

Thus

$$\frac{d}{dx}\int_1^{x^2} \sqrt{\tan^2 t + e^t}dt = 2x\sqrt{\tan^2(x^2) + e^{x^2}}.$$

2. Evalute the following integals.

(i) Use table 37 with $m = n = 1$.

$$\begin{aligned} \int \sin^m x \cos^n x dx &= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x dx + c \\ \int \sin^1 x \cos^1 x dx &= \frac{\sin^2 x \cos^0 x}{2} + 0 \times \int \sin^1 x \cos^{-1} x dx + c \\ &= \frac{\sin^2 x}{2} + c \end{aligned}$$

(ii) Use table 16 with $a = 2, b = 1$.

$$\begin{aligned} \int \frac{x}{\sqrt{ax+b}}dx &= \frac{2ax-4b}{3a^2}\sqrt{ax+b} + c \\ \int \frac{x}{\sqrt{ax+b}}dx &= \left(\frac{4x+4}{12}\right)\sqrt{2x-1} + c \\ &= \left(\frac{x+1}{3}\right)\sqrt{2x-1} + c \end{aligned}$$

(iii)

$$\begin{aligned} \text{Make the substitution } u &= 4 - x^2 \\ \text{Then } du &= -2x dx \end{aligned}$$

The limit $x = 0$ becomes $u = 4$. The limit $x = 2$ becomes $u = 0$.

$$\begin{aligned} \int_0^2 x\sqrt{4-x^2}dx &= -\frac{1}{2}\int_0^2 -2x\sqrt{4-x^2}dx \\ &= -\frac{1}{2}\int_4^0 u^{1/2}du \\ &= -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^0 \\ &= -\frac{1}{3}\left[u^{3/2}\right]_4^0 \\ &= -\frac{1}{3}\left[0 - (4)^{3/2}\right] \\ &= \frac{8}{3} \end{aligned}$$

(iv)

$$\begin{aligned} \text{Make the substitution } u &= 5x - 1 \\ \text{Then } du &= 5dx \end{aligned}$$

$$\begin{aligned} \int \frac{3}{5x-1}dx &= \frac{3}{5}\int \frac{5}{5x-1}dx \\ &= \frac{3}{5}\int \frac{du}{u} \\ &= \frac{3}{5}\ln|u| + c \\ &= \frac{3}{5}\ln|5x-1| + c \end{aligned}$$

(v)

$$\begin{aligned} \text{Make the substitution } u &= \tan x \\ \text{Then } du &= \sec^2 x dx \end{aligned}$$

The limit $x = 0$ becomes $u = 0$. The limit $x = \frac{\pi}{4}$ becomes $u = 1$.

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \tan x \sec^2 x dx &= \int_0^1 u du \\ &= \left[\frac{u^2}{2}\right]_0^1 \\ &= \frac{1}{2} \end{aligned}$$