

Fundamentals

Lecture Three

Logarithms Simplify

$$\log_a 18 - 2 \log_a 3$$

Factorisation Simplify

$$2x^2 - 5x - 12$$

Algebraic Fractions Simplify

$$\frac{m^2 + m - 2}{m^2 - m}$$

Logarithms

Consider a simple example,

$$16 = 2^4.$$

2 is called the *base* of the number, and 4 is called the *index* or the *logarithm to base 2 of 16*.

$$16 = 2^4,$$

$$\text{number} = \text{base}^{\text{Index}}.$$

$$\text{OR } 4 = \log_2 16,$$

$$\text{logarithm} = \log_{\text{base}} \text{number}.$$

More formally. If

$$N = b^x \Leftrightarrow x = \log_b N,$$

where b and N are positive real numbers. then x is called the *logarithm to base b of N* .

Rules

Let x and y be positive real numbers.

$$1. \log_b xy = \log_b x + \log_b y$$

$$2. \log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

$$3. \log_b x^p = p \log_b x$$

$$4. \log_a a = 1 \text{ (because } a^1 = a)$$

$$5. \log_a 1 = 0 \text{ (because } a^0 = 1)$$

Note: There is *no* rule of logarithms for simplifying either $\log(x + y)$ or $\log(x - y)$

Common Logarithms are logarithms to the base 10, and are written either as $\log_{10} N$ or, simply, $\log N$.

Natural Logarithms are logarithms to the base e , and are written either as $\log_e N$ or, most frequently, as $\ln N$.

Note:

(i) $\ln e = 1.$

(ii) $\ln e^x = x,$

(iii) $e^{\ln x} = x,$

Simplify the following expressions

1. $\log_2 16$

2. $\log_5 50 + \log_5 10 - \log_5 4$

3. $\frac{\log 32}{\log 8}$

4. $4 \log 3 - \log 27$

Exercises on Logarithms

Exercise 1.3.2. (page 7)

For each of the questions do as many of the subquestions as you require in order to gain mastery of the basic technique.

(4)

(3)

$\left(\frac{5}{3}\right)$

(log 3)

Factorisation

Factorisation of an algebraic expression consists of rewriting the given algebraic expression as a *product*. The terms in the product are called *factors*.

Recall

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Common Factors

Factorise

$$ab + ac + 3a =$$

Common Factors by Grouping

Factorise the following expressions

1. $ax + bx + ay + by$

2. $x^2 - y^2 - 6x + 6y$

3. $x^2 - 5x + 6 - ax + 2a$

Difference of Two Squares

Factorise the following expressions

1. $x^2 - y^2$

2. $a^4 - b^4$

3. $(2x + 3y)^2 - (x - 4y)^2$

$$((a + b)(x + y))$$

$$((x - y)(x + y - 6))$$

$$((x - 2)(x - 3 - a))$$

$$((x - y)(x + y))$$

$$((a - b)(a + b)(a^2 + b^2))$$

$$((3x - y)(x + 7y))$$

Quadratic Factors

Factorise the following expressions

1. $x^2 + (a + b)x + ab$

2. $x^2 - 9x + 20$

3. $6x^2 + 11x + 3$

Exercises on Factorisation

Exercise 1.4.1 (page 9)

Do as many of the questions as you require in order to gain mastery of the technique.

$$(x + a)(x + b)$$

$$(x - 5)(x - 4)$$

$$(2x + 3)(3x + 1)$$

Algebraic Fractions

Rules

$$1. \frac{am}{bm} = \frac{a}{b}$$

$$2. \frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c} \quad (c \neq 0)$$

$$3. \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \quad (b, d \neq 0)$$

$$4. \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b, d \neq 0)$$

$$5. \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \quad (b, c, d \neq 0)$$

Note

1. In $\frac{a+m}{b+m}$ the 'm' *cannot* be cancelled.

$$2. \frac{a}{\frac{b}{c}} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$$

$$3. \frac{\frac{a}{b}}{c} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$$

Examples

Simplify the following expressions

$$1. \frac{(x-2)}{(x+2)} \times \frac{3x+6}{x^2+2x-8}$$

$$2. \frac{2}{a-3} \div \frac{6}{a^2-9}$$

$$3. \frac{3}{b^2-4} - \frac{5}{3b-6}$$

$$\left(\frac{3}{x+4}\right)$$

$$\left(\frac{a+3}{3}\right)$$

$$\left(\frac{-5b-1}{3(b-2)(b+2)}\right)$$

Exercises on Algebraic Expressions

Exercise 1.5.2 (page 11)

Do as many of the questions as you require in order to gain mastery of the technique.