Fundamentals Lecture Five

Geometry, Graphs of Functions

- The Straight Line
- Absolute Value
- Common functions (parabola, circle, hyperbola, exponential, logarithmic)

Trig Ratios

- Definitions: $\sin, \cos, \tan$.
- Complement of an angle.

Radian Measure

Angles of any Magnitude

Graphing trig functions Periodicity.

The straight line

$y = mx + c$

m

c

Line through the point (x_1, y_1) with gradient m.

$$\frac{y - y_1}{x - x_1} = m \qquad *$$

Line through the points (x_1, y_1) and (x_2, y_2).

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \qquad **$$

Question. Explain how equation $(**)$ follows from equation $(*)$.

gradient

intercept on the y-axis ($x = 0$)

The absolute-value function

$$y = |x|$$

$$|x| = \begin{cases} -x, & \text{when } x < 0, \\ x, & \text{when } x \geq 0. \end{cases}$$

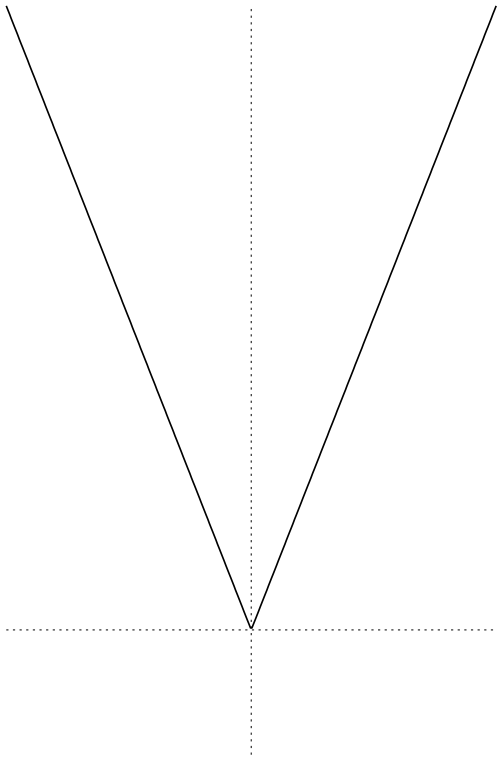
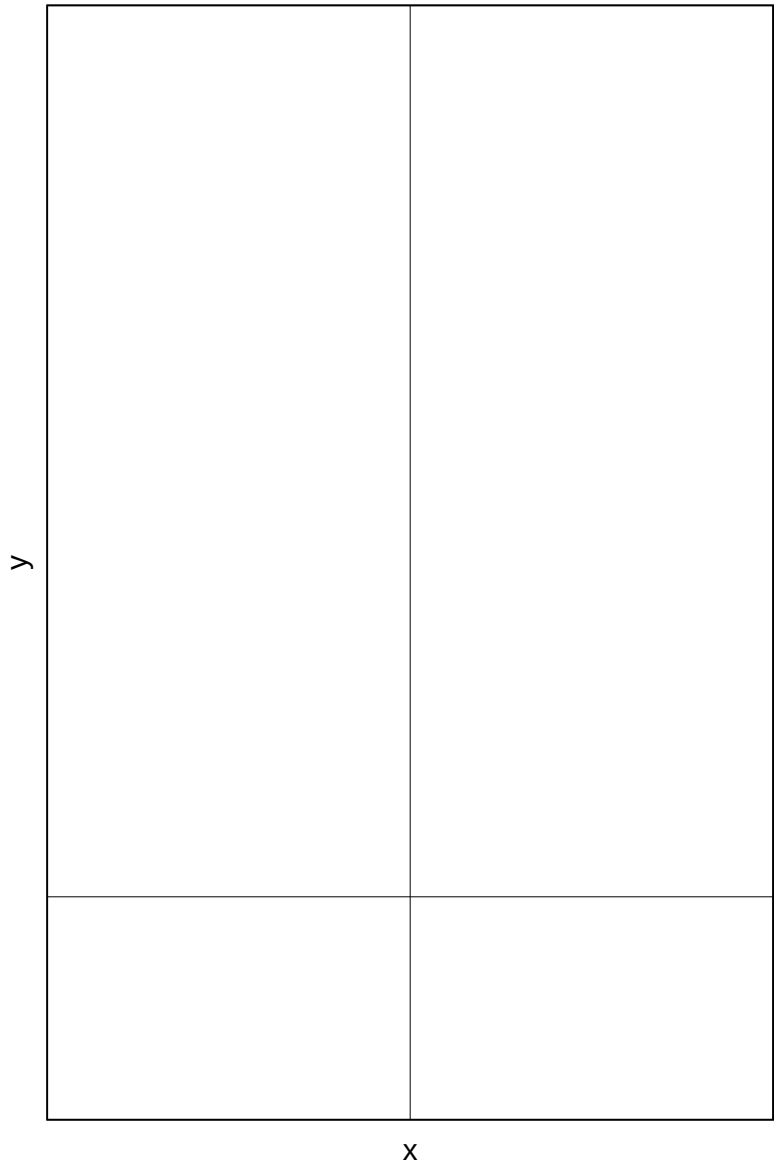


Figure 1: The graph $y = |x|$

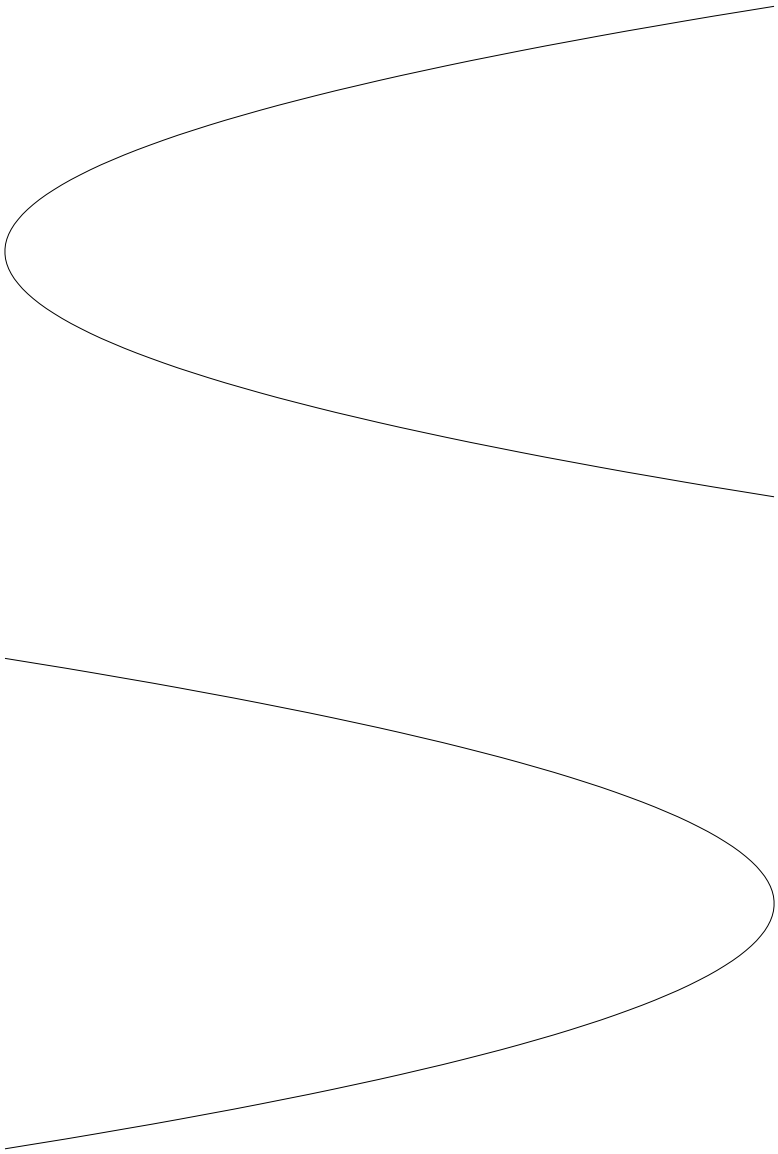
Example Plot the function $y = |x - 2|$

$$y = |x - 2| = \begin{cases} \quad , \\ \quad , \end{cases}$$



$$\begin{aligned} &2 - x \quad && \text{when } x < 2, \\ &x - 2 \quad && \text{when } x \geq 2. \end{aligned}$$

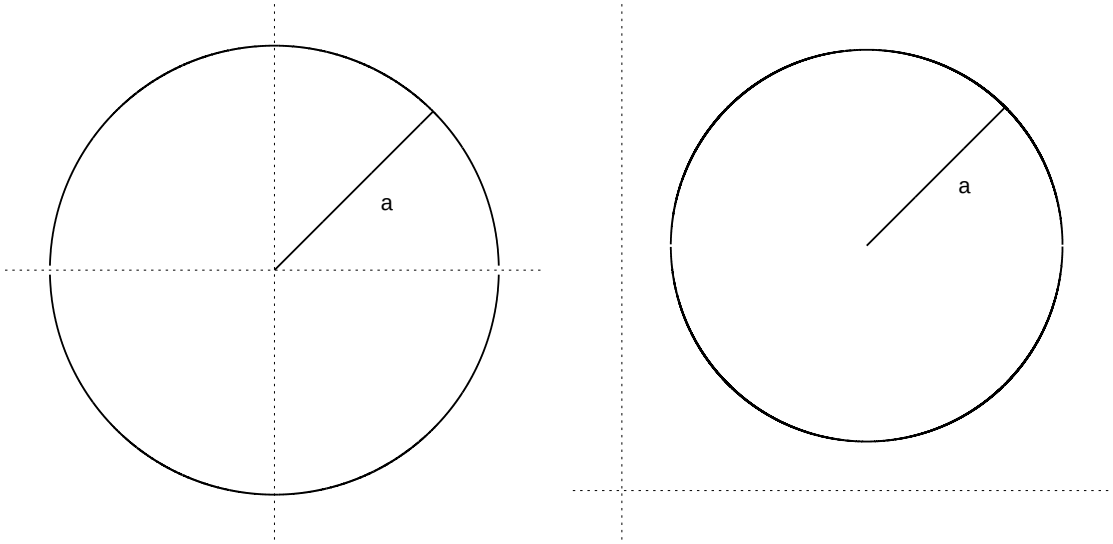
The parabola $x = ay^2 + by + c$



depending upon if $a > 0$ or $a < 0$.

Question Which figure has $a > 0$ and which has $a < 0$?

The circle



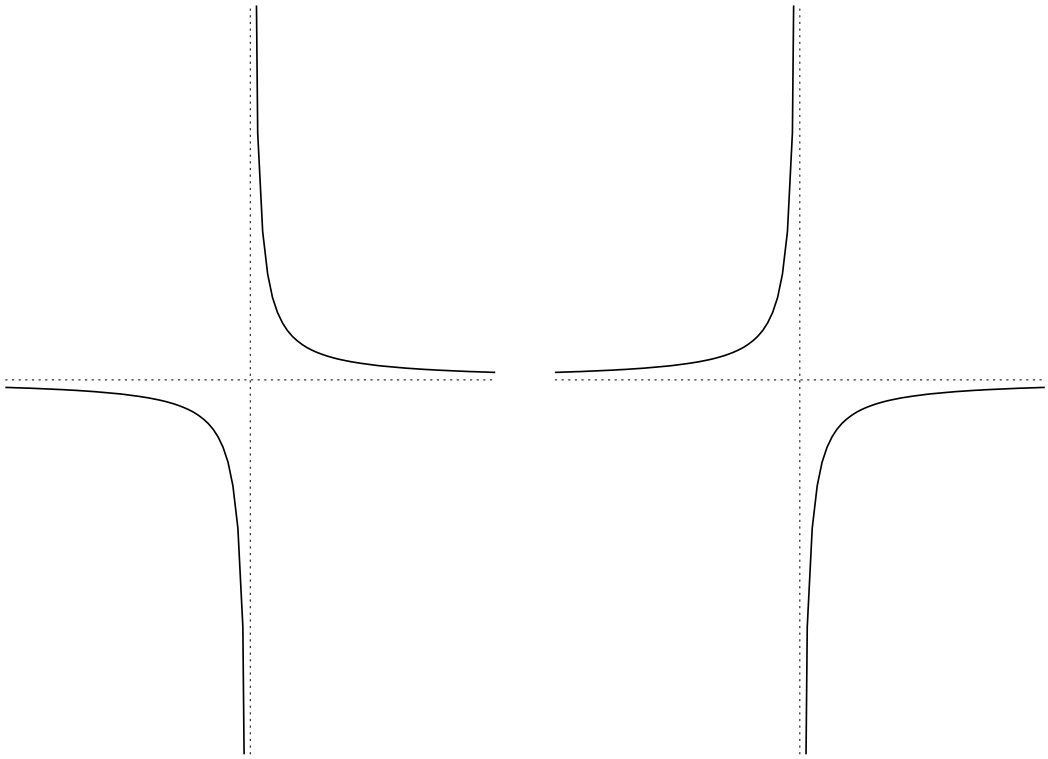
Circle Origin Radius

$x^2 + y^2 = a^2$

$(x - x_1)^2 + (y - y_1)^2 = a^2$

$$
\begin{array}{ll}
(0,0) & a \\
(x_1, y_1) & a
\end{array}
$$

Rectangular Hyperbola



The graphs $xy = c$ $(c > 0)$ and $xy = c$ $(c < 0)$.

Questions

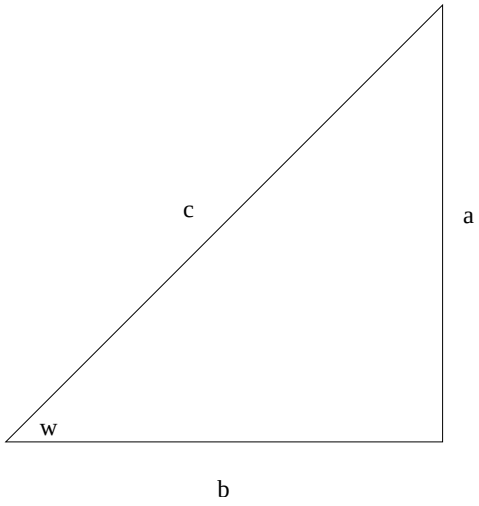
1. Which graph is which? *Why*
2. In each graph what is the value of y as $x \to \pm\infty$ and $x \to \pm 0$?

Exercises on Geometry, Graphs of Functions

Exercise 1.9.2
(pages 22&23)

Do as many of the questions as you require in order to gain mastery of the technique.

Trigonometric ratios for right-angled triangles



Direct Ratios

$$\sin w =$$

$$\cos w =$$

$$\tan w =$$

Figure 2:

Reciprocal Ratios

$$\operatorname{cosec} w =$$

$$\sec w =$$

$$\cot w =$$

$$\frac{a}{c}$$
$$\frac{b}{c}$$
$$\frac{a}{b}$$

$$\frac{1}{\sin w}$$
$$\frac{1}{\cos w}$$
$$\frac{1}{\tan w}$$

Complement of an angle

If w is measured in radians, then the **complement** of w is $\frac{1}{2}\pi - w$.

$\sin w = \cos\left(\frac{1}{2}\pi - w\right)$
$\sec w = \operatorname{cosec}\left(\frac{1}{2}\pi - w\right)$
$\tan w = \cot\left(\frac{1}{2}\pi - w\right)$

Question. Justify each of these relationships.

Graphs of Trigonometric Functions

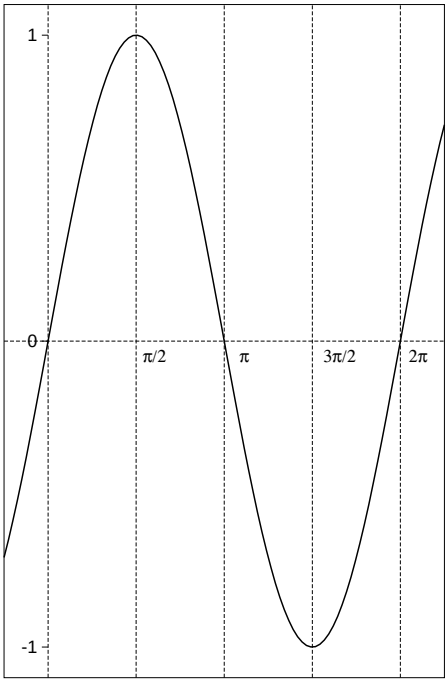


Figure 3: $y = \sin x$

When graphing a trigonometric function you need to

1. Show $x < 0$ and $x > 0$.
2. Show at least one complete period.

You *need* to be able to draw the following graphs.

- $y = \cos x$.
- $y = \tan x$.
- $y = \cot x$.
- $y = \sec x$.
- $y = \operatorname{cosec} x$.

I suggest you familiarise yourself with the graphs of these functions!

Radian Measure

In most theoretical mathematics and in *all* calculus applications of the trigonometric functions, angles are measured in radians. The relationship between radians and degrees is given by

$$\pi \text{ radians} = 180 \text{ degrees}$$

Important special cases are

$\frac{\pi}{6}$ radians =	$\frac{2\pi}{3}$ radians =
$\frac{\pi}{4}$ radians =	$\frac{5\pi}{6}$ radians =
$\frac{\pi}{3}$ radians =	π radians =
$\frac{\pi}{2}$ radians =	2π radians =

30^o 120^o

45^o 150^o

60^o 180^o

90^o 360^o

The following table gives the exact value of the trigonometric ratios for 'standard' values of $A, 0 \leq A \leq \frac{\pi}{2}$.

It is expected that students would be able to write down the exact values listed in this table.

A	$\sin A$	$\cos A$	$\tan A$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined

Angles of any magnitude

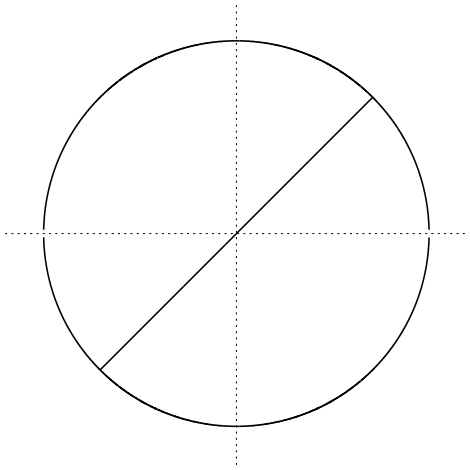


Figure 4:

Positive Angles area measured in an **anti-clockwise** direction.

Negative Angles area measured in an **clockwise** direction.

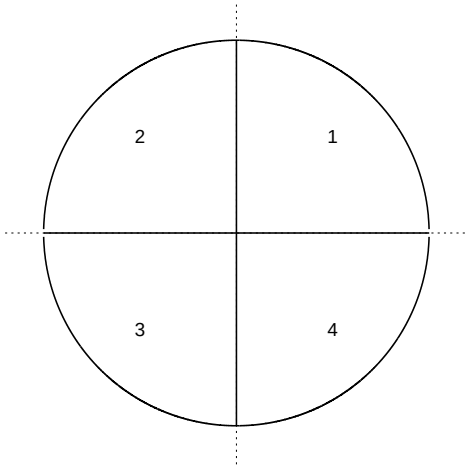


Figure 5:

1st quadrant angle: w.

2nd quadrant angle: .

3rd quadrant angle: .

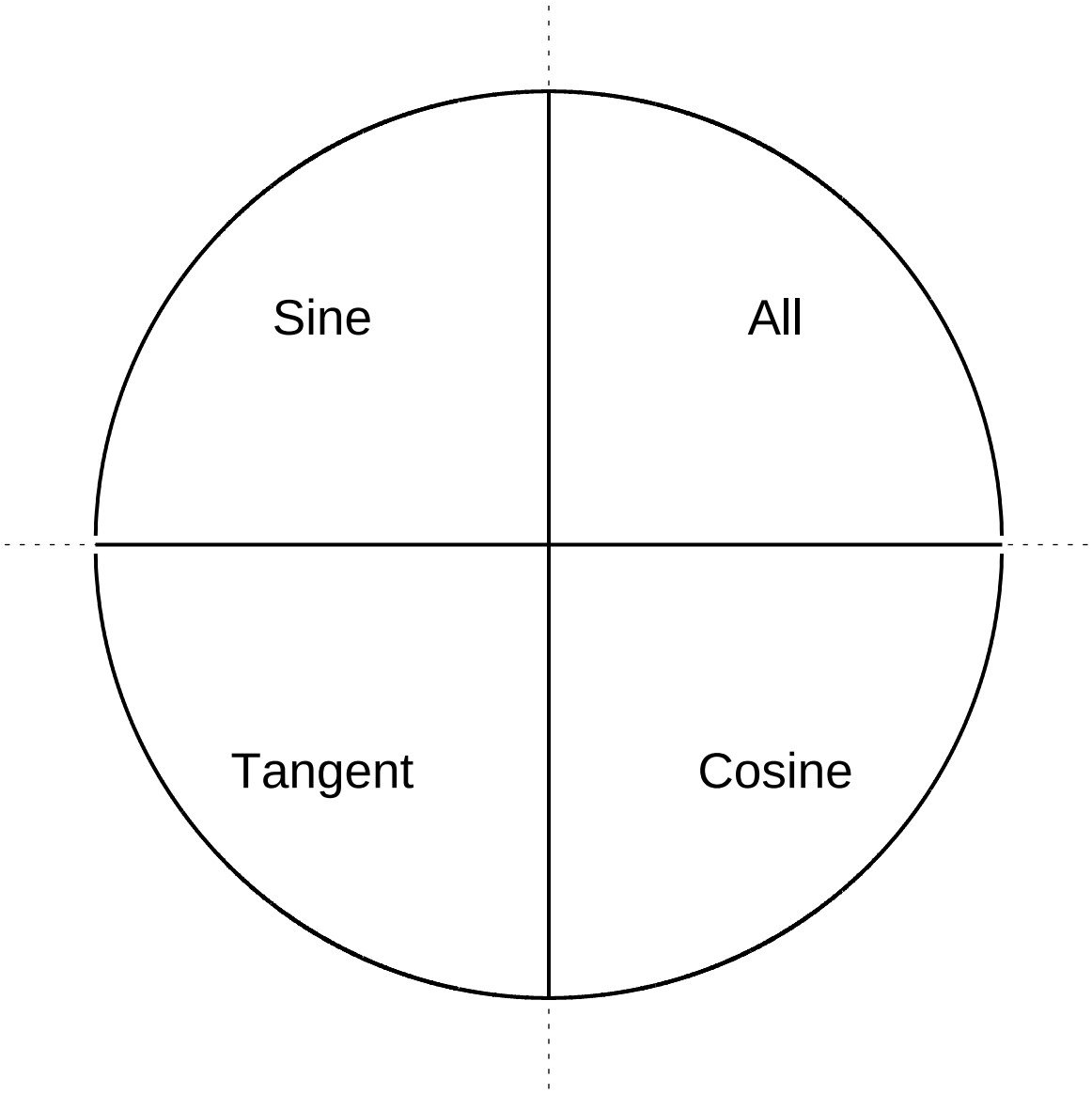
4th quadrant angle: .

$$\pi - w$$

$$\pi + w$$

$$2\pi - w$$

In which quadrants are the trigonometric ratios positive and negative?



Exercises on Trig Functions

Exercise 1.10.5
Pages 27&28

Do as many of the questions as you require in order to gain mastery of the technique.