Fundamentals Lecture Four

Function Notation What is a function?

Zeros of Functions x is a zero of the function f if $f(x) = 0$.

Quadratic Equations $ax^2 + bx + c$

- Finding roots of quadratic equations.
- Equations that are reducible to quadratics. $4^z - 9\left(2^z\right) + 8 = 0$.
- Sketching quadratic equations.

Function Notation

A simple definition

A *function* is a mathematical rule that assigns to each and every input a *unique* output.

Examples of functions are

$$f(x) = \sin^2 x + 5x + 2$$

$$h(t) = e^{t^2} - 47$$

$$H(z) = \ln z + \frac{3}{1+z}$$

Note The 'name' of the variable is irrelevant. The rule for $f(x)$ could be written as $f\,(w) = \sin^2 w + 5w + 2$ or $f\,(t) = \sin^2 t + 5t + 2$. It is the rule that is important.

Example

If $g\left(t\right)=\frac{3t+2}{4t-1}$ find

$g(1)$

$g(0.5)$

$g(-t)$

$g\left(a+1\right)$.

Solution

$$\begin{aligned} g(1) &= && = \\ g\left(\frac{1}{2}\right) &= && = \\ g\left(-t\right) &= && = \\ g\left(a+1\right) &= \\ &= \\ &= \end{aligned}$$

$$\left(\frac{5}{3}\right)$$
$$\left(\frac{7}{2}\right)$$
$$\left(\frac{-3t+2}{-4t-1}\right)$$
$$\left(\frac{3a+5}{4a+3}\right)$$

Example

If $g(x) = 5$ find

$$
\begin{aligned}
g(1) &= & &= \\
g(-3) &= & &= \\
g(-x) &= & &= \\
g(h+t) &= \\
&= \\
&=
\end{aligned}
$$

Exercises on Function Notation

Exercise 1.6.2 (page 12)

Zeros of Functions
Definition

A *zero* of the function $f(x)$ is a *solution* to the equation $f(x) = 0$. □

Note : A zero of the function $f(x)$ can also be referred to as a *root* of the equation $f(x) = 0$.

For example, the zeroes (or roots) of $f(x) = x^2 + 5x - 14$ can be found by solving the equation $f(x) = 0$

$$\text{i.e.}\, x^2 + 5x - 14 = 0$$

$$\text{Factorising} \qquad\qquad = 0$$

$$\text{i.e.}\, x = \_\_\_ \,\text{and}\, x = \_\_\_$$

So, $x = \_\_\_$ and $x = \_\_\_$ are the zeroes (or roots) of $f(x) = x^2 + 5x - 14$.

$$(x - 2)(x + 7)$$

Exercises on Functions

Exercise 1.7.2 (page 13)

Quadratic Equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

1. Solution by factors

Example

Find the roots of the equation

$$3x^2 - 13x - 10 = 0$$

Solution

$$0 = 3x^2 - 13x - 10$$

$$0 = (\qquad\qquad)(\qquad\qquad)$$

$$\therefore \qquad\qquad = 0 \text{ or } \qquad\qquad = 0$$

$$\therefore x = \qquad\qquad \text{or } x = $$

Thus the roots are and

2. Quadratic Formula

The roots of the quadratic equations $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta = b^2 - 4ac$ is called the *discriminant* of the quadratic equation.

If $\Delta > 0$: roots are real and unequal
If $\Delta = 0$: roots are real and equal
If $\Delta < 0$: no real roots

3. Completing the Square

Not needed.

Example

Solve the equation

$$2x^2 - 3x - 4 = 0$$

Solution

$a = \qquad\qquad , \; b = \qquad\qquad , \; c =$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore\; x =$$

$$=$$

$$=$$

Equations Reducible to Quadratics

Many harder equations can be reduced to quadratic equations by use of a simple substitution.

Examples

Solve

1. $4x^4 + 7x^2 - 15 = 0$
2. $4^x - 9\left(2^x\right) + 8 = 0$

Solve $4x^4 + 7x^2 - 15 = 0$. Let $u = x^2$ and equation becomes

$$0 =$$

$$\therefore 0 =$$

$$\therefore u = \quad \text{or } u =$$

But $u = x^2$, therefore

$$x^2 = \qquad \text{OR} \qquad x^2 =$$

$$\therefore x = \qquad\qquad \text{No real sols}$$

$$\frac{5}{4} \qquad -3$$

$$\frac{5}{4} \qquad -3$$

$$\pm\frac{\sqrt{5}}{2}$$

Solve $4^x - 9\left(2^x\right) + 8 = 0$. Let $u = 2^x$ and equation becomes

$$0 =$$

$$=$$

$$\therefore \quad u = \quad \text{OR} \quad u =$$

But $u = 2^x$

$$\therefore 2^x = \quad \text{OR} \quad 2^x =$$

$$= \qquad\qquad =$$

$$\therefore x = \quad \text{OR} \quad x =$$

Exercise 1.8.3
(pages 15 & 16)

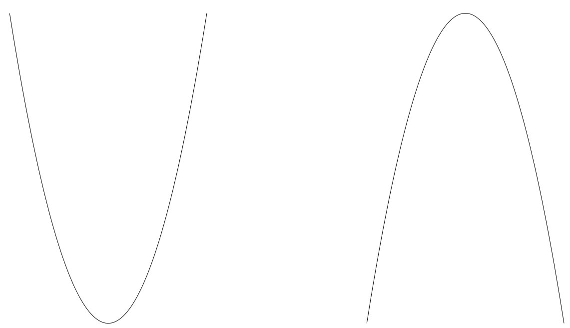
$$(u-8)(u-1)$$

3 0

Mark's Guide to: *Sketching Quadratic Functions*

$$y = ax^2 + bx + c$$

1. Ignoring the placement of the axis the graph looks like one of these



depending upon if $a > 0$ or $a < 0$.

2. Note that if $a > 0$ the curve has a *minimum* whilst if $a < 0$ it has a *maximum*.

$$\boxed{y = ax^2 + bx + c}$$

3. The position of the maximum/minimum is at $x = \frac{-b}{2a}$. The y value is $\frac{4ac-b^2}{4a}$.

4. Notice that the curve is *symmetric* around the maximum/minimum.

5. Find the values of x at which $y = 0$, i.e.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   If $b^2 - 4ac < 0$ then there are no (real) roots and the graph does not cross the y-axis.

6. Calculate the value of y when $x = 0$.

Example Sketch $y = x^2 + 2x + 3$.

Solution

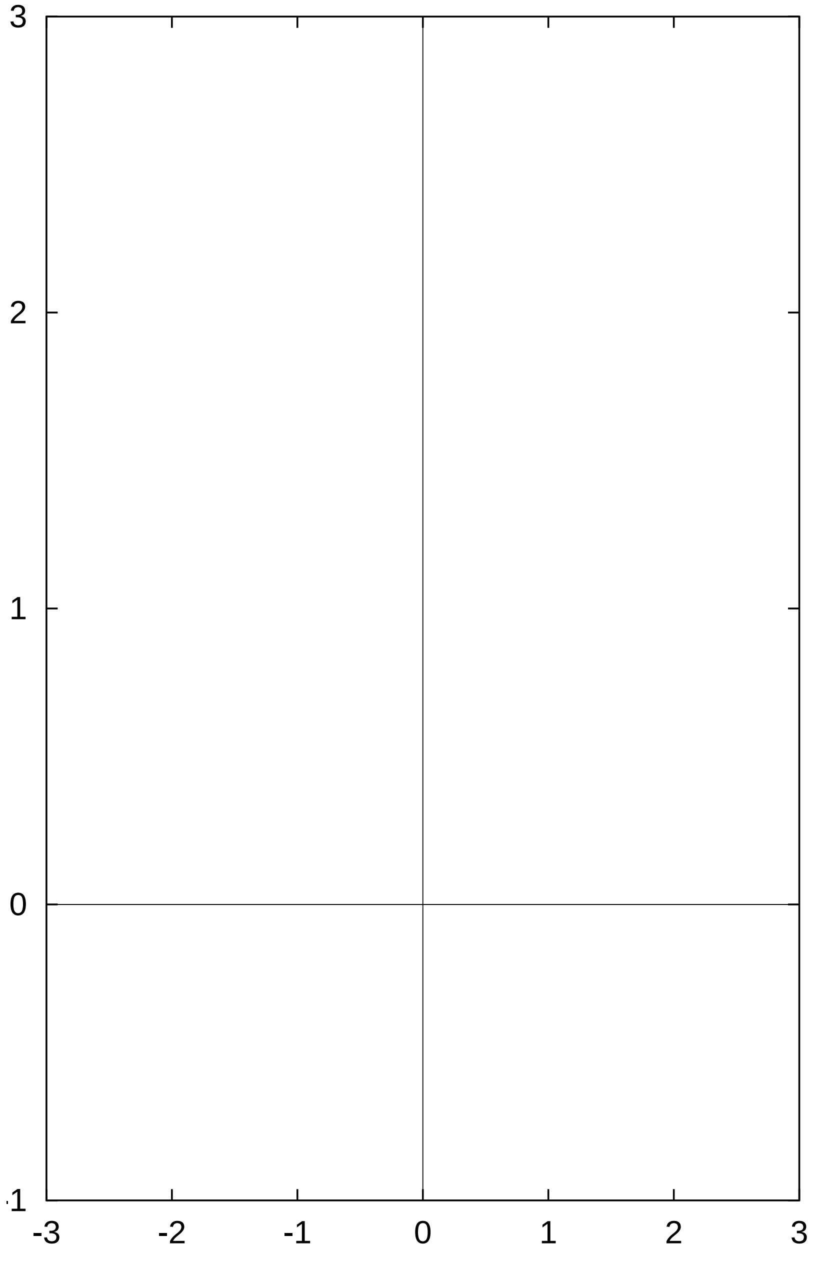
1. a is positive/negative therefore y has a maximum/minimum.
2. The location of the max/min is

$$x =$$

$$y =$$

3. What is the value of y when $x = 0$?
4. Does the quadratic have real roots? If yes, where are they? What are the corresponding valuers for y?

Graph of $y = x^2 + 2x + 3$



Example Sketch $y = -2x^2 + 5x - 2$

Solution

1. a is positive/negative therefore y has a maximum/minimum.

2. The location of the max/min is

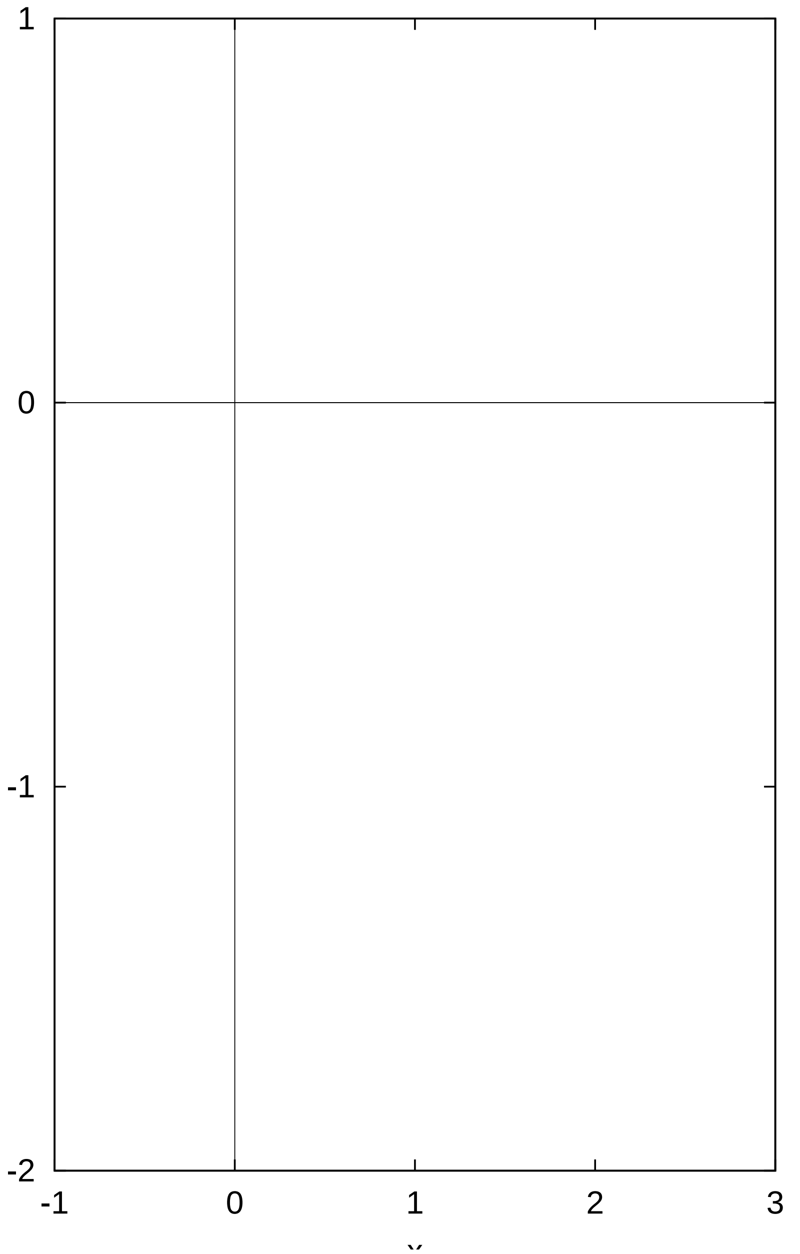
$$x =$$

$$y =$$

3. What is the value of y when $x = 0$?

4. Does the quadratic have real roots? If yes, where are they? What are the corresponding valuers for y?

Graph of $y = -2x^2 + 5x - 2$



Exercises on Quadratic functions

Exercise 1I (page 1-19)
Q1. parts (b) and (c) & Q2.

Do as many of the questions as you require in order to gain mastery of the technique.