Find an exact value for $\sin \frac{\pi}{12}$

**Solution**

$$\frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$$

$$\text{Now, } \tfrac{\pi}{12} = \tfrac{\pi}{3} - \tfrac{\pi}{4}$$

$$\begin{aligned}
\therefore \sin\frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\
&= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\
&= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\
&= \frac{1}{2\sqrt{2}}\left(\sqrt{3} - 1\right) \\
&= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)
\end{aligned}$$

Use the sum of angles formulae and the double angle formulae to show that

$$\sin 3\omega = 3\sin\omega - 4\sin^3\omega$$

$\sin 3\omega = \sin(\omega + 2\omega)$

$$= \sin w \cos 2w + \cos w \sin 2w$$

using the sum of angles formulae for $\sin A + B$.

$$= \sin w\left(1 - 2\sin^2 w\right) + \cos w\,(2\sin w \cos w)$$

using the double angle formulae for $\cos 2w$ and $\sin 2w$.

$$= \sin w - 2\sin^3 w + 2\sin w \cos^2 w$$

$$= \sin w - 2\sin^3 w + 2\sin w\left(1 - \sin^2 w\right)$$

using Pythaoreas formula

$$= 3\sin w - 4\sin^3 w$$