2.9 LOGARITHMIC DIFFERENTIATION

We use **Logarithmic Differentiation** when we have a function of the form $y = g(x)^{h(x)}$. (Note that $g(x)$ may be a constant.) For example
$y = 2^x,\ y = x^x,\ y = x^{\tan x}$.

Recall

1. $e^{\ln a} = a,\ a > 0$
2. $\ln a^b = b \ln a$
3. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[e^{f(x)}\right] = e^{f(x)} f'(x)$

Example

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left[b^x\right] &= \underline{\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{\ln b^x}\right]} && \text{(using (i))}\\
&= \underline{\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{x \ln b}\right]} && \text{(using (ii))}\\
&= \underline{e^{x \ln b}\frac{\mathrm{d}}{\mathrm{d}x}(x \ln b)} && \text{(using (iii))}\\
&= \underline{e^{x \ln b} \cdot \ln b}\\
&= \underline{b^x \ln b}
\end{aligned}$$

Example

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{\sin x}\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{ln x^{\sin x}}\right] \text{ (using (i))}$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{\sin x \ln x}\right] \text{ (using (ii))}$$

$$= e^{\sin x \ln x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(\sin x \ln x) \text{ (using (iii))}$$

$$= e^{\sin x \ln x}\left(\cos x \ln x + \sin x \frac{1}{x}\right)$$

$$= x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

2.9.1 Questions

Do them!