2.8 THE DERIVATIVE OF AN INVERSE FUNCTION

Example To find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right)$

Let $y = \sin^{-1} x, \quad \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\therefore x = \sin y$$

$$\therefore \frac{\mathrm{d}x}{\mathrm{d}y} = \_\_\_\_$$

$$\text{i.e. } \frac{\mathrm{d}y}{\mathrm{d}x} = \_\_\_\_$$

But $\sin^2 y + \cos^2 y = 1$. Therefore
$\cos y = \pm\sqrt{1 - \sin^2 y}$

$\cos y$

$\frac{1}{\cos y}$

Now, if $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$ then $\cos y > 0$.

$$\text{Thus} \cos y = \sqrt{1 - \sin^2 y}$$

$$\text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$\therefore \frac{\mathrm{d}}{\mathrm{d}x}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

The derivatives of the other inverse trigonometric functions and the inverse hyperbolic functions are found in a similar manner.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
		$\lvert x\rvert < 1$	
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	$\coth^{-1} x$	$\frac{1}{1-x^2}$
		$\lvert x\rvert > 1$	

2.8.1 Exercises

Do them!