

Essential Mathematics Skills - Question Database - Chapter 4

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4 DIFFERENTIATION

1. Find $\frac{dy}{dx}$ if $y = \frac{2}{x^3}$.

Solution:

$$\begin{aligned} y' &= 2 \frac{d}{dx} x^{-3} \\ &= -6x^{-4} \\ &= \frac{-6}{x^4} \end{aligned}$$

2. Find $\frac{dy}{dx}$ if $y = \frac{4}{x^2}$.

Solution:

$$\begin{aligned} y &= 4x^{-2} \\ \implies y' &= 4 \cdot (-2) \cdot x^{-3} \\ &= -\frac{8}{x^3} \end{aligned}$$

3. Find $\frac{dy}{dx}$ if $y = x - \frac{1}{x^2}$.

Solution:

$$\begin{aligned} y &= x - x^{-2} \\ \implies y' &= 1 - 1 \cdot (-2)x^{-3} \\ &= 1 + \frac{2}{x^3} \end{aligned}$$

4. If $f(x) = 1 - \frac{1}{x^2}$ then $f'(x)$ is

Solution:

$$\begin{aligned} f(x) &= 1 - x^{-2} \\ \implies f'(x) &= 0 - 1 \cdot (-2) \cdot x^{-3} \\ &= \frac{2}{x^3} \end{aligned}$$

5. If $f(x) = \frac{1}{2x}$ then $f'(x)$ is

Solution:

$$\begin{aligned} f(x) &= \frac{1}{2}x^{-1} \\ \implies f'(x) &= \frac{-1}{2}x^{-2} \\ &= \frac{-1}{2x^2} \end{aligned}$$

6. If $f(x) = \frac{1}{3}x^{-3}$ then $f'(x)$ is

Solution:

$$\begin{aligned} f(x) &= \frac{1}{3}x^{-3} \\ \implies f'(x) &= -3 \cdot \frac{1}{3}x^{-4} \\ &= -x^{-4} \end{aligned}$$

7. Find $\frac{dy}{dx}$ if $y = \sin 2x$.

Solution:

Since

$$\begin{aligned} \frac{d}{dx} \sin x &= \cos x \\ \implies y' &= \frac{d2x}{dx} \cos 2x \\ &= 2 \cos 2x \end{aligned}$$

8. Find $\frac{dy}{dx}$ if $y = \frac{1}{3} \cos 3x$.

Solution:

Since

$$\begin{aligned} \frac{d}{dx} \cos x &= -\sin x \\ \implies y' &= -\frac{1}{3} \frac{d3x}{dx} \sin 3x \\ &= -\sin 3x \end{aligned}$$

9. Find dy/dx if $y = \sin(3x^2)$.

Solution:

Since

$$\begin{aligned}\frac{d}{dx} \sin x &= \cos x \\ \text{and } \frac{d}{dx} x^2 &= 2x \\ \implies y' &= 3 \cdot 2x \cdot \cos(3x^2) \\ &= 6x \cos(3x^2)\end{aligned}$$

10. Find y' if $y = 2 \cos(x^2)$.**Solution:**

Since

$$\begin{aligned}\frac{d}{dx} \cos x &= -\sin x \\ \text{and } \frac{d}{dx} x^2 &= 2x \\ \implies y' &= -2 \cdot 2x \cdot \sin(x^2) \\ &= -4x \sin(x^2)\end{aligned}$$

11. Find $f'(x)$ if $f(x) = \frac{1}{(1-3x)^2}$.**Solution:**

$$\begin{aligned}f(x) &= (1-3x)^{-2} \\ \implies f'(x) &= (-2)(-3)(1-3x)^{-3} \\ &= \frac{6}{(1-3x)^3}\end{aligned}$$

12. Find $\frac{dy}{dx}$ if $y = x^2 e^x$.**Solution:**Since $\frac{d}{dx} x^2 = 2x$ and $\frac{de^x}{dx} = e^x$ then

$$\begin{aligned}y' &= \frac{dx^2}{dx} e^x + x^2 \frac{de^x}{dx} \\ &= 2x e^x + x^2 e^x \\ &= x e^x (2 + x)\end{aligned}$$

13. Find $f'(x)$ if $f(x) = \frac{x}{x^2 - 3}$.

Solution:

$$\begin{aligned}
 f(x) &= \frac{x}{x^2 - 3} \\
 \Rightarrow f'(x) &= \frac{\frac{dx}{dx}(x^2 - 3) - x \frac{d}{dx}(x^2 - 3)}{(x^2 - 3)^2} \\
 &= \frac{(x^2 - 3) - x \cdot 2x}{(x^2 - 3)^2} \\
 &= \frac{-x^2 - 3}{(x^2 - 3)^2}
 \end{aligned}$$

14. Find $x'(t)$ if $x(t) = e^{-t} \sin(2t)$.**Solution:**

$$\begin{aligned}
 x(t) &= e^{-t} \sin(2t) \\
 \Rightarrow x'(t) &= \sin 2t \frac{d}{dt}[e^{-t}] + e^{-t} \frac{d}{dt}[\sin 2t] \\
 &= \sin 2t(-e^{-t}) + e^{-t}(2 \cos 2t)
 \end{aligned}$$

15. Find $\frac{dy}{dt}$ if $y(t) = e^t(\cos t + \sin t)$.**Solution:**

$$\begin{aligned}
 \Rightarrow y'(t) &= (\cos t + \sin t) \frac{d}{dt}[e^t] \\
 &\quad + e^t \frac{d}{dt}[\cos t + \sin t] \\
 &= e^t(\cos t + \sin t - \sin t + \cos t) \\
 &= 2e^t \cos t
 \end{aligned}$$

16. Find $\frac{dy}{dx}$ if $y = \sin^3(2x)$.**Solution:**

$$\begin{aligned}
 y &= \sin^3(2x) \\
 \Rightarrow y' &= 3 \sin^2(2x) \frac{d}{dx}[\sin 2x] \\
 &= 3 \sin^2(2x) \frac{d2x}{dx} \cos 2x \\
 &= 3 \sin^2(2x) 2 \cos 2x \\
 &= 6 \sin^2(2x) \cos 2x
 \end{aligned}$$

17. Find $\frac{dy}{dx}$ if $y = \ln(\ln x)$.

Solution:

since $\frac{d}{dx} \ln x = \frac{1}{x}$ then

$$\begin{aligned} y' &= \frac{1}{\ln x} \frac{d}{dx} [\ln x] \\ &= \frac{1}{\ln x} \frac{1}{x} \\ &= \frac{1}{x \ln x} \end{aligned}$$

18. Find $\frac{dy}{dx}$ if $y = \sin(x + x^3)$.

Solution:

$$\begin{aligned} y &= \sin(x + x^3) \\ \implies y' &= \cos(x + x^3) \frac{d}{dx} [x + x^3] \\ &= \cos(x + x^3)(1 + 3x^2) \end{aligned}$$

19. Find dy/dx if $y = x \cos x$.

Solution:

Since $\frac{d}{dx} \cos x = -\sin x$ then by the product rule

$$\begin{aligned} y' &= \cos x \frac{d}{dx} x + x \frac{d}{dx} \cos x \\ &= \cos x - x \sin x \end{aligned}$$

20. Find $\frac{dy}{dx}$ if $y = x \sin x$.

Solution:

$$\begin{aligned} y &= x \sin x \\ \implies y' &= \frac{d}{dx} [x] \sin x + x \frac{d}{dx} [\sin x] \\ &= \sin x + x \cos x \end{aligned}$$

21. Find $\frac{dy}{dx}$ if $y = x^2 \sin x$.

Solution:

$$\begin{aligned}
 y &= x^2 \sin x \\
 \Rightarrow y' &= \frac{d}{dx}[x^2] \sin x + x^2 \frac{d}{dx}[\sin x] \\
 &= 2x \sin x + x^2 \cos x
 \end{aligned}$$

22. Find $\frac{dy}{dx}$ if $y = x(1-x)^4$.

Solution:

$$\begin{aligned}
 y &= x(1-x)^4 \\
 \Rightarrow y' &= \frac{d}{dx}[x](1-x)^4 + x \frac{d}{dx}[(1-x)^4] \\
 &= (1-x)^4 + x(4(1-x)^3) \frac{d}{dx}[1-x] \\
 &= (1-x)^4 + 4x(1-x)^3(-1) \\
 &= (1-x)^4 - 4x(1-x)^3
 \end{aligned}$$

23. Find $\frac{dy}{dx}$ if $y = x(x+2)^6$.

Solution:

$$\begin{aligned}
 y &= x(x+2)^6 \\
 \Rightarrow y' &= \frac{d}{dx}[x](x+2)^6 \\
 &\quad + x \frac{d}{dx}[(x+2)^6] \\
 &= (x+2)^6 + x6(x+2)^5
 \end{aligned}$$

24. Find $\frac{dy}{dx}$ if $y = xe^x$.

Solution:

$$\begin{aligned}
 y &= xe^x \\
 \Rightarrow y' &= \frac{d}{dx}[x]e^x + x \frac{d}{dx}[e^x] \\
 &= e^x + xe^x
 \end{aligned}$$

25. Find $\frac{d}{dx} \left(\frac{\sin x}{x} \right)$.

Solution:

$$\begin{aligned}\frac{d}{dx} \left(\frac{\sin x}{x} \right) &= \frac{\frac{d}{dx} [\sin x]x - \sin x \frac{d}{dx} [x]}{x^2} \\ &= \frac{\cos(x)x - \sin x}{x^2}\end{aligned}$$

26. Find $\frac{d}{dx} (\cos^2(3x))$.

Solution:

$$\begin{aligned}d dx (\cos^2(3x)) &= 2 \cos(3x) \frac{d}{dx} [\cos 3x] \\ &= 2 \cos(3x)(-\sin 3x) \frac{d}{dx} [3x] \\ &= 2 \cos(3x)(-\sin 3x)3 \\ &= -6 \cos 3x \sin 3x\end{aligned}$$