

1 Limits Worksheet

1.1 Exercises from Handout

1. Find, if possible, the following limits:-

$$\begin{array}{llll}
 \text{(a)} \quad \lim_{x \rightarrow 0} x^2 - 4x^2 + x + 6 & \text{(b)} \quad \lim_{x \rightarrow 0} 2^x & \text{(c)} \quad \lim_{x \rightarrow 1} \frac{x}{\sqrt{x^2 + 1} - 1} & \text{(d)} \quad \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1} \\
 \text{(e)} \quad \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2} & \text{(f)} \quad \lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} & \text{(g)} \quad \lim_{x \rightarrow 0} \frac{\frac{1}{x} - 1}{x - 1} & \text{(h)} \quad \lim_{x \rightarrow 2} \frac{2 - x}{x} \\
 \text{(i)} \quad \lim_{x \rightarrow 0} \frac{x^2 - x}{x} & \text{(j)} \quad \lim_{x \rightarrow 4} \frac{x - 4}{x^2 - 16} & \text{(k)} \quad \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} & \text{(l)} \quad \lim_{x \rightarrow a} \frac{x^2 - ax}{x - a}
 \end{array}$$

2. Given that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$. Find, if possible, the following limits:-

$$\begin{array}{llll}
 \text{(a)} \quad \lim_{x \rightarrow \infty} \frac{1}{2x} & \text{(b)} \quad \lim_{x \rightarrow -\infty} \frac{1}{2x + 1} & \text{(c)} \quad \lim_{x \rightarrow \infty} \frac{x}{x^2 + 1} \\
 \text{(d)} \quad \lim_{x \rightarrow \infty} \frac{2x}{x + 2} & \text{(e)} \quad \lim_{x \rightarrow -\infty} \frac{2x^2}{5x^2 + 1} & \text{(f)} \quad \lim_{x \rightarrow \infty} \frac{5x - 4}{2x + 1} \\
 \text{(g)} \quad \lim_{x \rightarrow \infty} \frac{4x^2 + x}{x^2 + 2x + 1} & \text{(h)} \quad \lim_{x \rightarrow -\infty} \frac{x^2 + 2x - 3}{3x^2 - x + 2}
 \end{array}$$

3. Evaluate the following limits (if possible).

$$\text{(a)} \quad \lim_{x \rightarrow 1} 3(2x - 1)(x + 1) \quad \text{(b)} \quad \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \quad \text{(c)} \quad \lim_{x \rightarrow 0} \frac{x^2}{x - 1} \quad \text{(d)} \quad \lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$$

4. If $\lim_{x \rightarrow c} f(x) = 2$ and $\lim_{x \rightarrow c} g(x) = 5$, find the following limits.

$$\text{(a)} \quad \lim_{x \rightarrow c} \left(\frac{f}{g} \right) (x) \quad \text{(b)} \quad \lim_{x \rightarrow c} (fg)(x) \quad \text{(c)} \quad \lim_{x \rightarrow c} \frac{f(x)}{3g(x)}$$

5. Give an example of functions f and g such that the following hold.

$$\text{(a)} \quad \lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = 0, \text{ and } \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 4.$$

$$\text{(b)} \quad \lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = 0, \text{ and } \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} \text{ does not exist.}$$