

1 Limits Worksheet

1.1 Exercises from Handout

1. Find, if possible, the following limits:-

$$\begin{array}{llll} \text{(a)} \quad \lim_{x \rightarrow 0} x^2 - 4x^2 + x + 6 & \text{(b)} \quad \lim_{x \rightarrow 0} 2^x & \text{(c)} \quad \lim_{x \rightarrow 1} \frac{x}{\sqrt{x^2 + 1} - 1} & \text{(d)} \quad \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1} \\ \text{(e)} \quad \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2} & \text{(f)} \quad \lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2} & \text{(g)} \quad \lim_{x \rightarrow 0} \frac{\frac{1}{x} - 1}{x - 1} & \text{(h)} \quad \lim_{x \rightarrow 2} \frac{2 - x}{x} \\ \text{(i)} \quad \lim_{x \rightarrow 0} \frac{x^2 - x}{x} & \text{(j)} \quad \lim_{x \rightarrow 4} \frac{x - 4}{x^2 - 16} & \text{(k)} \quad \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} & \text{(l)} \quad \lim_{x \rightarrow a} \frac{x^2 - ax}{x - a} \end{array}$$

(a) 6

(b) 1

(c) $\sqrt{2} + 1$

(d) 0

(e) 1

(f) 12

(g) -1

(h) 0

(i) -1

(j) $\frac{1}{8}$

(k) $\frac{5}{4}$

(l) a

2. Given that $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$. Find, if possible, the following limits:-

$$\begin{array}{lll} \text{(a)} & \lim_{x \rightarrow \infty} \frac{1}{2x} & \text{(b)} \quad \lim_{x \rightarrow -\infty} \frac{1}{2x+1} & \text{(c)} \quad \lim_{x \rightarrow \infty} \frac{x}{x^2+1} \\ \text{(d)} & \lim_{x \rightarrow \infty} \frac{2x}{x+2} & \text{(e)} \quad \lim_{x \rightarrow -\infty} \frac{2x^2}{5x^2+1} & \text{(f)} \quad \lim_{x \rightarrow \infty} \frac{5x-4}{2x+1} \\ \text{(g)} & \lim_{x \rightarrow \infty} \frac{4x^2+x}{x^2+2x+1} & \text{(h)} \quad \lim_{x \rightarrow -\infty} \frac{x^2+2x-3}{3x^2-x+2} \end{array}$$

(a) 0.

(b) 0.

(c) 0.

(d) 2.

(e) $\frac{2}{5}$.

(f) $\frac{5}{2}$.

(g) 4.

(h) $\frac{1}{3}$.

3. Evaluate the following limits (if possible).

$$\text{(a)} \quad \lim_{x \rightarrow 1} 3(2x-1)(x+1) \quad \text{(b)} \quad \lim_{x \rightarrow 5} \frac{x^2-25}{x-5} \quad \text{(c)} \quad \lim_{x \rightarrow 0} \frac{x^2}{x-1} \quad \text{(d)} \quad \lim_{x \rightarrow 3} \frac{x^3-27}{x-3}$$

(a) 3.

(b) 10.

(c) 0.

(d) 27.

4. If $\lim_{x \rightarrow c} f(x) = 2$ and $\lim_{x \rightarrow c} g(x) = 5$, find the following limits.

$$\text{(a)} \quad \lim_{x \rightarrow c} \left(\frac{f}{g} \right) (x) \quad \text{(b)} \quad \lim_{x \rightarrow c} (fg)(x) \quad \text{(c)} \quad \lim_{x \rightarrow c} \frac{f(x)}{3g(x)}$$

(a) $\frac{2}{5}$.

(b) 10.

(c) $\frac{2}{15}$.

5. Give an example of functions f and g such that the following hold.

(a) $\lim_{x \rightarrow 0} f(x) = 0$, $\lim_{x \rightarrow 0} g(x) = 0$, and $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 4$.

(b) $\lim_{x \rightarrow 0} f(x) = 0$, $\lim_{x \rightarrow 0} g(x) = 0$, and $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ does not exist.

(a) $f(x) = x$, $g(x) = x^2 + \frac{1}{4}x$.

(b) $f(x) = x$, $g(x) = x^2$.