MATH111 – Spring 2007
Tutorial Sheet – Week 7

This tutorial sheet covers chapters 8 & 9 of the notes.

Chapter 8: Differential Equations: Introduction, Definitions and Basic Concepts

Book work questions

1. Give an example of a third order differential equation.
2. Give an example of 'a linear differential equation'? and a non-linear differential equation. Clearly explain why your example is a linear/non-linear differential equation.
3. Give an example of an 'autonomous differential equation'? and a 'non-autonomous differential equation'. Clearly identify of a non-autonomous differential equation.
4. How many initial conditions are required to solve an nth order differential equation?

Chapter 9: First-Order Differential Equations

Exercises

1. The population density of cells in an experiment is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \lambda - Dx, \quad x(0) = x_0.$$

   where λ is the rate at which new cells are born and D is the death-rate of cells.

   (a) Solve this equation.

   (b) What is the density of cells in the limit $t \to \infty$?

2. A company is discharging a herbicide into a river that flows into a marsh, where it is degraded. The rate of degradation of the herbicide, A, is assumed irreversible and to follow first-order homogeneous kinetics.

$$A \xrightarrow{k_1} \text{Products.}$$

   This process is represented by the differential equation

$$V\frac{\mathrm{d}A}{\mathrm{d}t} = qA_0 - qA - Vk_1A. \tag{1}$$

   (a) Given that initially there is no herbicide present in the marsh: obtain the solution to equation (1).

   (b) From now on we assume that the marsh is rectangular with width $W = 100\,\mathrm{m}$, length $L = 1000\,\mathrm{m}$ and average depth $D = 0.25\,\mathrm{m}$. The other parameter values are: $A_0 = 10^{-2}\,\mathrm{mol\,m^{-3}}$, $k_1 = 16 \times 10^{-5}\,\mathrm{h^{-1}}$, $q = 2\,\mathrm{m^3hr^{-1}}$.

      (i) Let $A(\infty)$ be the concentration of herbicide in the marsh at time $t = \infty$. What is $A(\infty)$?

      (ii) How many days does it take for the level of herbicide in the marsh to reach half of its final value?

      (iii) Suppose that the legal maximum level of herbicide in the marsh is given by $A_{\max} = \frac{1}{30}A(\infty)$. On which day must the company stop pumping herbicide into the marsh?

3. A *batch reactor* has neither inflow nor outflow of reactants or products whilst the reaction is being carried out. Suppose that the reaction

$$A \to B$$

occurs in a batch reactor. For an nth-order reaction the rate of change of reactant concentration in the reactor is given by

$$\frac{\mathrm{d}A}{\mathrm{d}t} = -k_1 A^n, \quad A(0) = A_0,$$

where A_0 is the initial concentration of the reactant.

(a) By solving the appropriate differential equation determine how the concentration of reactant in the reactor depends upon the time since the reactor was started for

(i) a first-order reaction ($n = 1$),

(ii) a second-order reaction ($n = 2$).

(b) Hence obtain a formula for the time taken (t_R) for the concentration of reactant to decrease to 10% of its initial value for

(i) a first-order reaction,

(ii) a second-order reaction.

(c) (i) For a given first-order reaction $k_1 = 10^{-4}\,\mathrm{s}^{-1}$. Determine t_R.

(ii) For a given second-order reaction the product $k_1 A_0 = 10^{-3}\,\mathrm{s}^{-1}$. Determine t_R.

A worked solution to question 1 is given in the lecture book. Questions 2 & 3 were on the week 8 assignment sheet (2004); worked solutions are available on the web page.