

# MATH111 – Spring 2007

## Tutorial Sheet – Week 3

This tutorial sheet principally covers chapter 3 of the notes. Note that the first part of this tutorial sheet is to be detached from the tutorial sheet and handed in at the end of the tutorial.

### Part One: Revision of Key Ideas

This question is designed to help you get used to the idea of translating a word problem into a corresponding mathematical problem. You should try to solve it without looking at your lecture notes. You should discuss this question with your neighbours if you are stuck

1. You take out a loan and make equal-sized payments at regular intervals to reduce the principal and to pay interest on the amount still owing. Compound interest accumulates on your loan.
  - (a) Write a *word* equation for the debt after  $n$  payments.
  - (b) Translate your word equation into a *mathematical equation*, carefully defining your notation.

### Book work questions

The following questions test your understanding of some of the basic ideas introduced in chapter three.

1. How many days are there in a year when calculating ordinary simple interest?
2. The solution of the difference equation

$$x_n - ax_{n-1} = b(n),$$

is given by

$$x_n = a^n x_0 + \sum_{p=1}^n a^{n-p} b(p).$$

Hence, or otherwise, Show that the solution of the difference equation

$$y_n = \left(1 + \frac{\alpha p}{100}\right) y_{n-1} - D$$

is given by

$$y_n = \left(1 + \frac{\alpha p}{100}\right)^n \left(y_0 - \frac{100D}{\alpha p}\right) + \frac{100D}{\alpha p}.$$

3. Re-arrange the loan repayment formula

$$y_n = \left(1 + \frac{\alpha p}{100}\right)^n \left(y_0 - \frac{100D}{\alpha p}\right) + \frac{100D}{\alpha p}.$$

to find a formula for the repayments needed to clear a debt of size  $y_0$  after  $n$  payments when interest is compounded at  $p\%$  p.a. with a conversion period  $\alpha$ .

You should make a note of your solution for  $D$  as it is very helpful in answering questions of this nature.

## Part Three

You work on the first question in a group. Note that the second question in this set is a ‘real’ financial mathematics problem, one that many Australians have considered in the last decade.

1. Suppose that at time  $t = 0$  you borrow an amount  $y_0$  from a bank and thereafter you borrow an amount  $B$  at regular intervals. You pay compound interest on your total borrowings.
  - (a) Write a word equation to describe this scenario.
  - (b) Convert your word equation to a difference equation;
  - (c) Solve your difference equation to find the amount of money you owe at time  $n$ .
  - (d) Suppose that at the start of January 1994 Belinda and Peter borrow \$2,000 and at the start of each subsequent month they borrow an additional \$500. At the start of January 2004 they have borrowed a total of \$62,000. What is their accumulated debt if they are being charged 8.5% interest compounded monthly?
  - (e) The equation that you have derived is the same as the equation for annuities. Explain why this makes sense.
  
2. Suppose that in January 1994 Belinda and Peter had \$1000 to invest. They invested it in an Australian equity fund and contributed \$250 per month for the next 10 years.
  - (a) What is their total investment after 10 years?
  - (b) If over the period of investment the average annual return on the fund is 10.6% how much would money would they have in their equity fund?
  - (c) Borrowing money to invest it called gearing. It is a popular investment strategy.  
Suppose that in January 1994 Belinda and Peter borrowed \$2000 to add to their initial investment of \$1000 and that thereafter they borrowed an additional \$500 a month to add to the \$250 they invested. At the end of 10 years how much would they have in their equity fund?
  - (d) At the end of ten years Belinda and Peter close their equity fund. How much money do they have after they have paid off the accumulated debt on their loan? (Use your answer to the final part of question 1.

(Based on an example in. “We can open your eyes to a world of investment options”. Bridges. 2006.)

3. You have been given \$1000 to invest for one year. You have a choice of three bank accounts.
  - ‘You Beaut’ bank offers you 10% p.a. compounded annually. At the end of the year you will pay \$20 in fees.
  - ‘Fair Go’ bank offers you 10% p.a. compounded quarterly. At the end of the year you will pay \$30 in fees.
  - ‘Ocker’ bank offers you 11% p.a. compounded every four months. At the end of the year you will pay \$25 in fees.

Which bank should you put your money into (justify your answer)?

4. A cash discount of 4% is given if a bill is paid 30 days in advance of its due date. What is the highest simple interest rate at which you can afford to borrow money in order to take advantage of the cash discount? (50%)
  
5. To prepare for early retirement, a self-employed consultant deposits \$5500 into a retirement saving plan each year, starting on her 31st birthday. When she is 51, she wishes to draw out 30 equal annual payments. What is the size of each withdrawal, if interest was compounded annually at 12% for the first ten years, compounded annually at 10% for the next ten-year period, and compounded annually at 11% for the 30-year retirement period? (38, 878.17)