MATH111 – Spring 2007
Tutorial Sheet – Week 2

This tutorial sheet principally covers chapters 1 & 2 of the notes.

Part One: Revision of Key Ideas

'Hands-on' questions

The first question is 'hands-on', to get you used to the idea of finding a solution to a discrete equation by using the process of *iteration*. The second question is designed to help you get used to the idea of translating a word program into a corresponding mathematical problem. You should discuss these questions with your neighbours if you are stuck.

1. Consider the difference equation
$$x_{n+1} - 2x_n + x_{n-1} = \frac{2}{n^2}x_n$$
with initial conditions $x_1 = 1$ and $x_2 = 4$.
   (a) What is the order of this equation? Is it a linear or non-linear equation? Is it an autonomous or non-autonomous equation?
   (b) By putting $n = 2$ into the equation calculate the value for x_3.
   (c) By putting $n = 3$ into the equation calculate the value for x_4.
   (d) By putting $n = 4$ into the equation calculate the value for x_5.
   (e) Hence, or otherwise, guess the "closed-form" solution of x_k.
   (f) Verify that your formula satisfies the difference equation and the initial conditions.
2. Consider the problem of modelling patient flow in a department of geriatric medicine. Each day the following activities occur:
   - A number of new patients are admitted to the department for acute care.
   - A fraction, α, of the current patients are treated and discharged.
   - A fraction, β, of the current patients, unfortunately, die.
   - A fraction of the current patients, γ, are transferred to another section.

   (a) Write down a **word** equation that defines this problem.
   (b) Write down, formally, the difference equation that describes the above scenario. Define **all** variables and explain your terms.

Book work questions

The following questions test your understanding of some of the basic ideas introduced in the first two chapters.

1. Given an example of a fifth order difference equation.
2. Give an example of a 'linear difference equation' and a 'non-linear difference equation'. Clearly explain why your example is a linear/non-linear difference equation.
3. Give an example of an 'autonomous difference equation'? and a 'non-autonomous difference equation'. Clearly identify what makes it an autonomous/non-autonomous difference equation.

4. Give an example of a quantity that might be measured in discrete intervals and represented by a difference equation.
5. What does the word 'dynamics' mean?
6. How many initial conditions are required to solve an nth order difference equation?

Part Two

1. Give the orders of of the following difference equations and state whether they are linear, nonlinear, autonomous or non-autonomous.
   (a) $nx_{n+2} + 3n^2 x_n = x_{n-1} + 2$
   (b) $x_{n-1} + \cosh(x_n) = 2$
2. Consider the difference equation
   $$y_k = k y_{k-1}, \quad k = 1, 2, 3 \ldots$$
   with initial condition $y_0 = 1$.
   (a) Calculate y_1, y_2, y_3, y_4 and make a guess at the "closed-form" solution of y_k.
   (b) Verify that your formula satisfies the difference equation and the initial condition.
3. Solve the following difference equations to obtain solutions in "closed form".
   (a) $x_n - 2x_{n-1} = 0$
   (b) $x_n = x_{n-1} + 3$
   (c) $x_n + x_{n-1} = n$
   (Hint: Arithmetic-Geometric Series $\displaystyle\sum_{k=1}^{n} (-1)^{n-k} k = \frac{1}{4}(2n+1) - \frac{1}{4}(-1)^n$)
4. Imagine this scenario, if you will. Economic rationalism has taken hold of your workplace and it's time to renegotiate your contract. Knowing a thing or two about maths, you make the following proposal. "Boss, I've been far too greedy. But I've come to my senses, after reading *Animal Farm*, and propose a new pay scale. Starting tomorrow, I would like you to pay me two cents..." "*It's a deal*" "...raised to the power of the number of days..." "*Sign here!*" "...the commencement of my new..." "*Next!* "...contract."

   Day one, you are paid 2c (2 raised to the power of one). Day two, 4c (2 squared). Day three, 8c (2^3). Day four, 16c (2^4). Day five, 32c. For week one, you take home 62c.
   (a) How much do you take home in week two?
   (b) How much to you take home in week three?
   (c) How much do you take home in week four?

   Based on an article by Jeremy Chunn that appeared in Mens Style *Summer 2003*