Discrete Population Models for a Single Species: *Quiz 2002*

Question 4. (15 marks)

It has been suggested that a means of controlling insect numbers is to introduce and maintain a number of sterile insects in the population. One such model for the resulting population dynamics is

$$x_{n+1} = \frac{RMx_n^2}{(R-1)\,Mx_n^2 + Mx_n + S},$$

where $R > 1$ and $M > 0$ are constant parameters, and S is the constant sterile insect population.

1.

(a) Show that when $S = 0$ the difference equation can be simplified to

$$x_{n+1} = \frac{Rx_n}{(R-1)\,x_n + 1}.$$

(1 *mark*)

(b) Show that when $S = 0$ the fixed points are given by $x = 0$ and $x = 1$.

(2 *marks*)

(c) Calculate the eigenvalues associated with the two fixed points and hence determine their stability. 3 *marks*

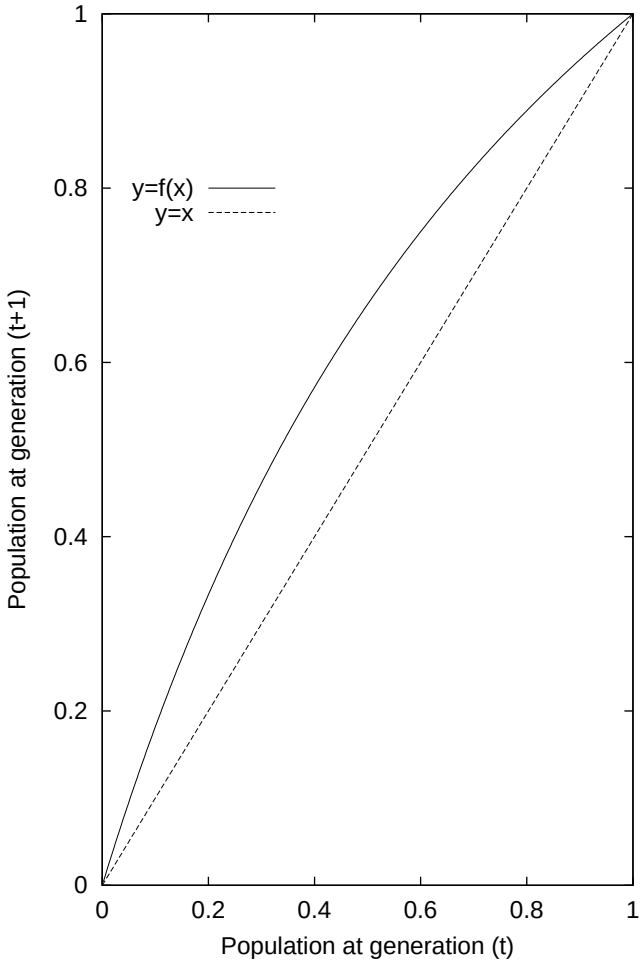
2. We now fix $R = 2$ and $M = 1$. The 3 figures in parts (b)–(d) show the graph

$$\begin{aligned} x_{n+1} &= f\left(x_n\right), \\ &= \frac{2x_n^2}{x_n^2 + x_n + S}, \end{aligned}$$

for various values of S, and the straight line $x_{n+1} = x_n$.

(a) The figure on this page shows the graphs $y = f(x)$ and $y = x$ when $S = 0$.

i. Suppose that the initial population (x_0) is $x_0 = 0.5$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population. (1 *mark*)

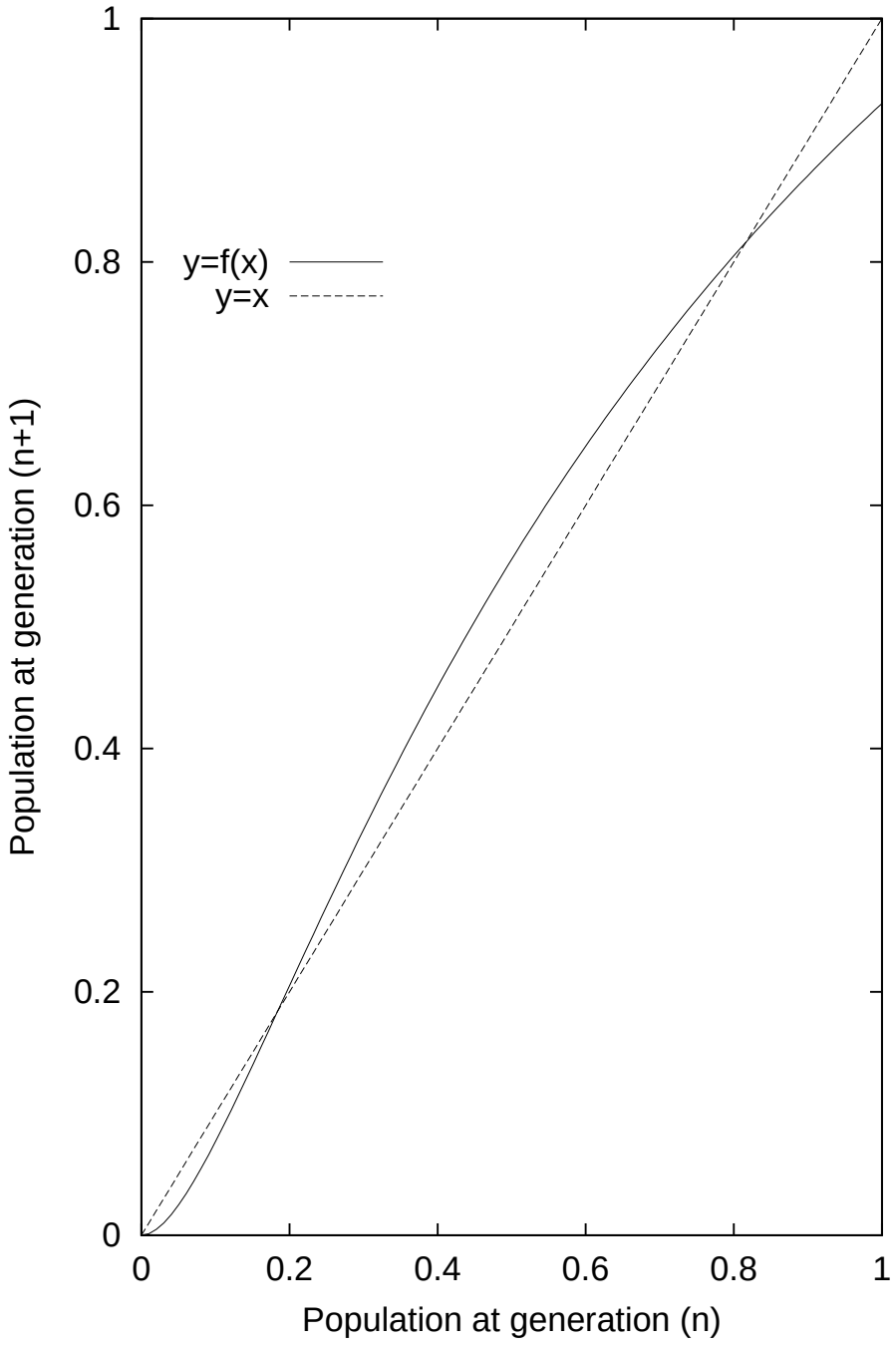


ii. Explain what your cobweb plot shows. (1 *mark*)

iii. How would your answer to part (ii) change if you were to chose a different value for x_0 with $0 < x_0 < 1$? (1 *mark*)

(b) The figure on this page shows the graphs $y = f(x)$ and $y = x$ when $S = 0.15$.

i. Suppose that the initial population (x_0) is $x_0 = 0.5$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population. 1 (*mark*)

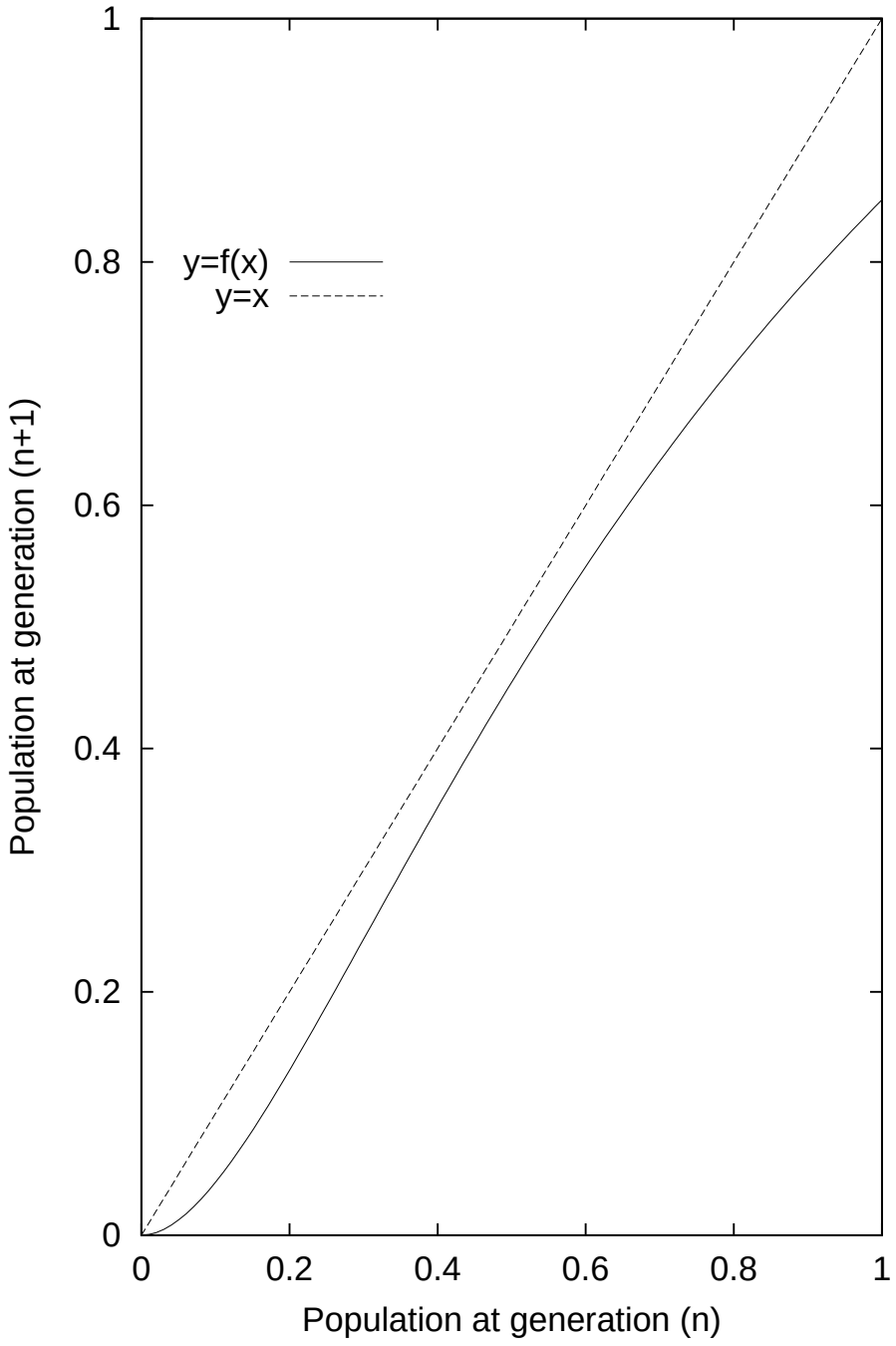


ii. Explain what your cobweb plot shows. 1 *mark*

iii. How would your answer to part (b)(ii) change if you were to choose a different value for x_0 with $0 < x_0 < 1$? (1 *mark*)

(c) The figure on this page shows the graphs $y = f(x)$ and $y = x$ when $S = 0.35$.

i. Suppose that the initial population (x_0) is $x_0 = 0.5$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population. (1 *mark*)



(d) Comment on the biological implications of your answers to parts (a-c) of this question. (2 *marks*)