Discrete Population Models for a Single Species: *Quiz 2001*

Question 3. (15 marks)

1. Consider the difference equation

$$x_{n+1} = rx_n^2\,(1 - x_n)\,.$$

   (a) Show that the fixed points of this equation are given by $x^* = 0$ and $x^*_\pm = \frac{r \pm \sqrt{r(r-4)}}{2r}$.

   [3] marks

   (b) Determine the values for $x_\pm$ when $r = 5$ (correct to 5 decimal places). Calculate the corresponding eigenvalues and hence determine the stability of the two fixed points $x_\pm$.

   [3] marks

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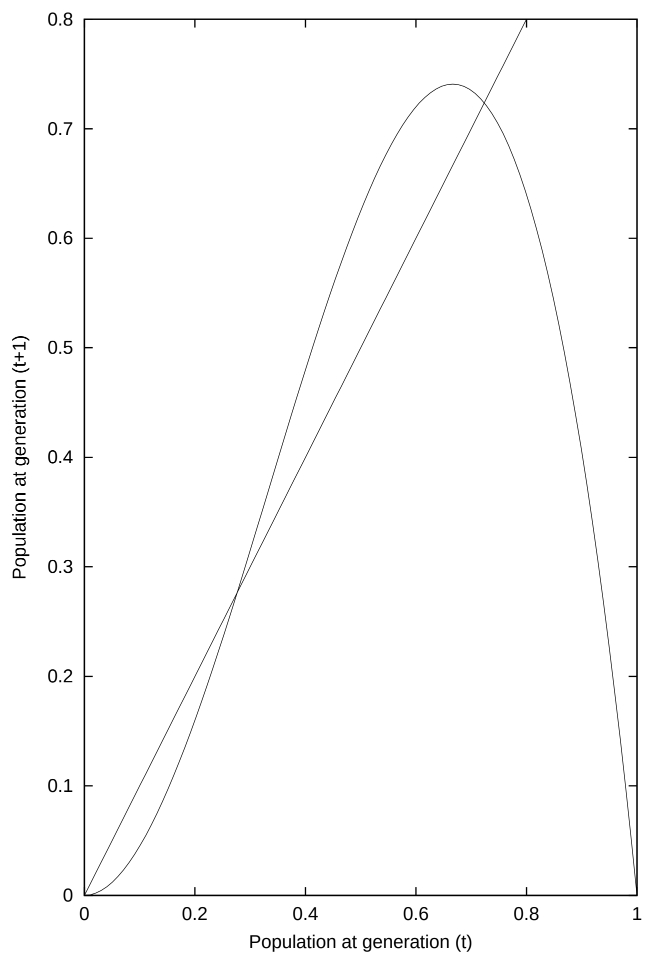
(c) The 3 figures in parts (i)–(iii) show the graph $x_{n+1} = 5x_n^2(1 - x_n)$ and the straight line $x_{n+1} = x_n$. Explain why the fixed points of the system are located at the intersection between $x_{t+1} = f(x_t)$ and $x_{t+1} = x_t$.

[1] mark

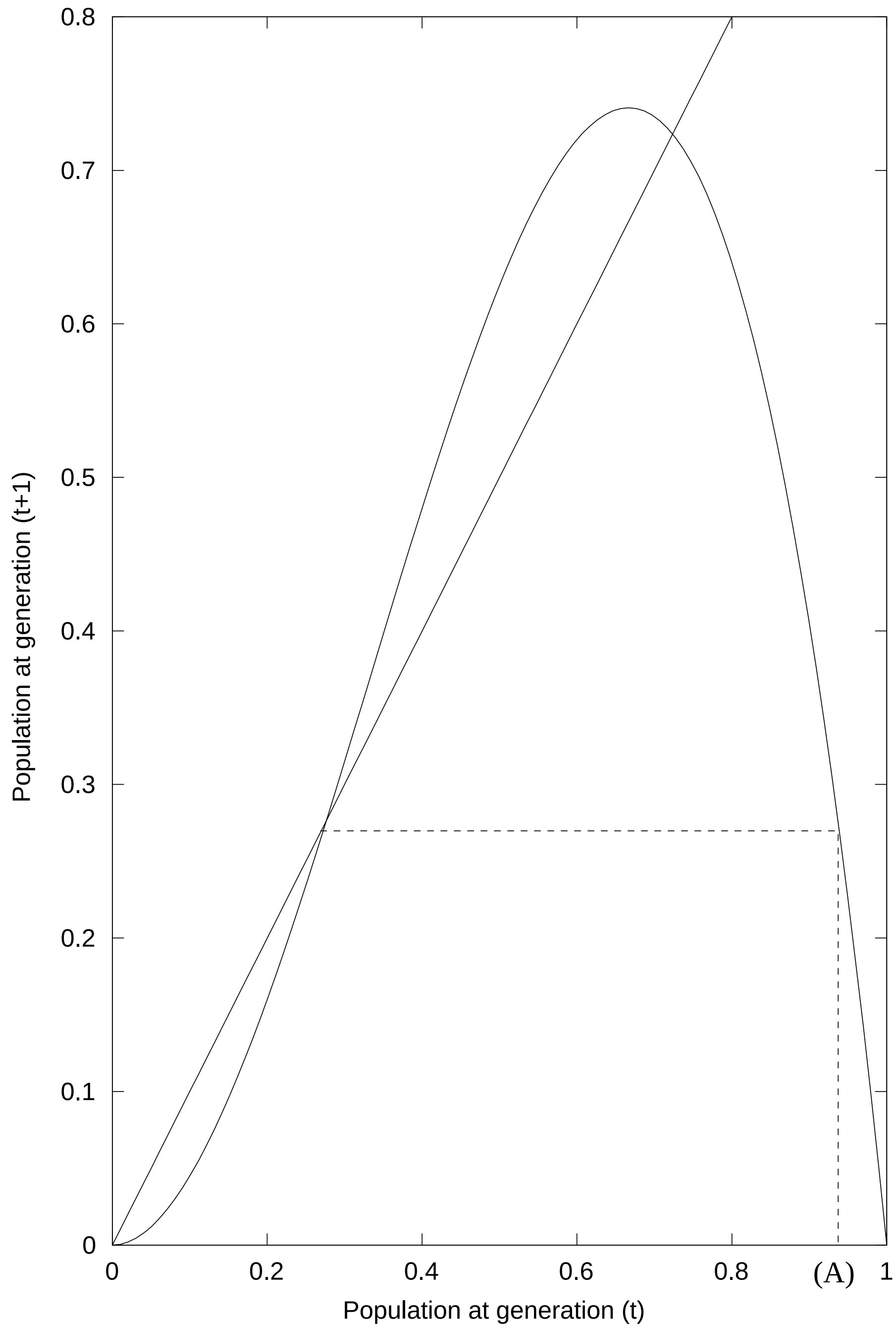
i. Suppose that the initial population (x_0) is in the range $0 < x_0 < x_-^*$, where x_-^* is the smaller of the two fixed points $x_\pm^*$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population.

[2]marks



Explain what your cobweb plot shows.

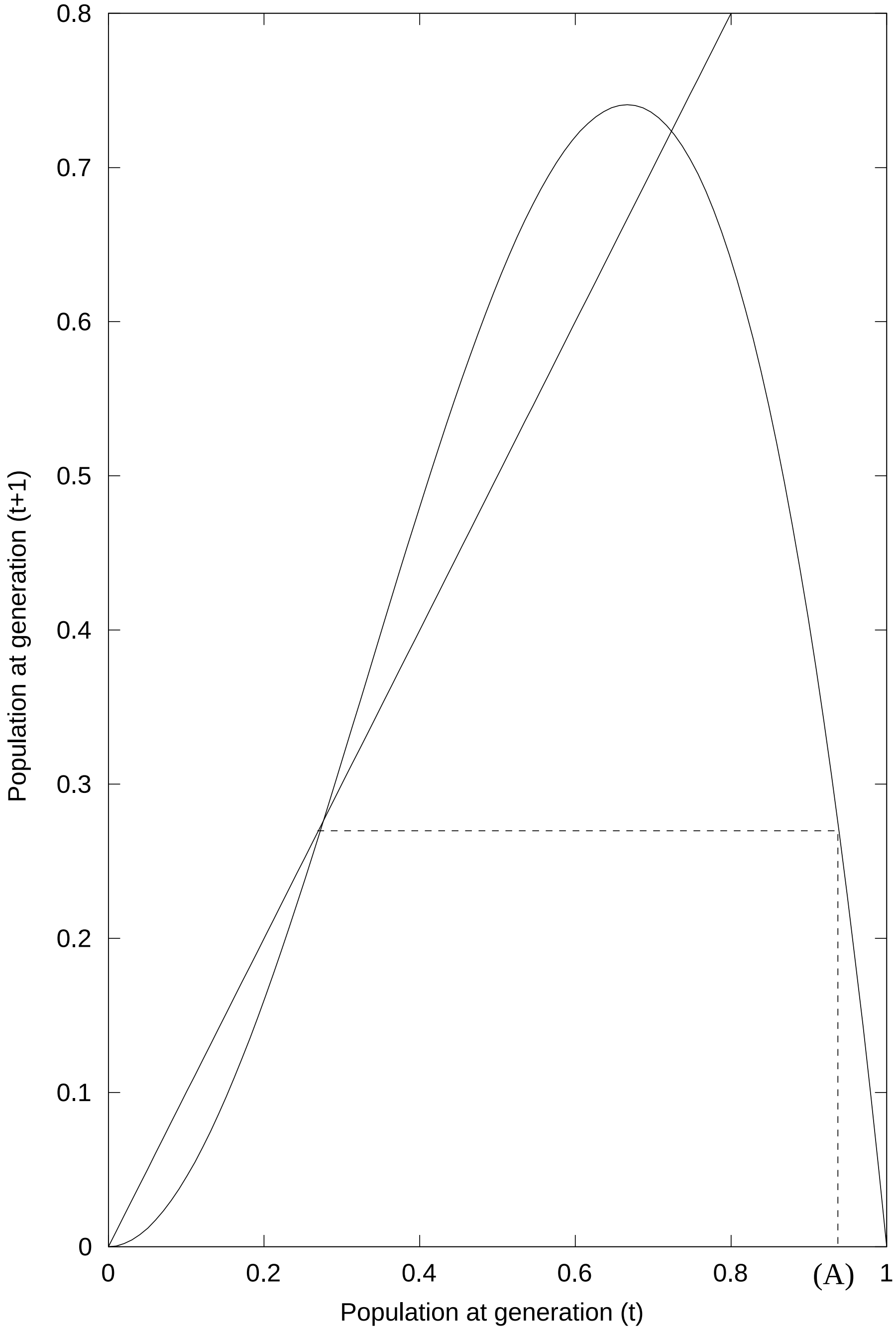
ii. Now suppose that the initial population (x_0) is in the range $x_-^* < x_0 < A$, where A is the point marked on the figure. By drawing successive iterations on the cobweb diagram determine the long-term evolution of the population. [2] marks



Explain what your cobweb plot shows.

iii. By using similar techniques as above, explain what happens if the initial population is now in the range $A < x_0 < 1$.

[2] marks



Explain what your cobweb plot shows.

(d) Suppose that x represents the population of vermin on an island and that the current population size is x^*_+, where x^*_+ is the larger of the two fixed points $x^*_\pm$. Is it possible to eradicate the species by killing a fraction of the current population size? Justify your answer by referring to your answers to part (c).

[2] marks