Discrete Population Models for a Single Species

Quiz 2002

Question 1. Consider the difference equation

$$x_{t+1} = f(x_n),$$

where f is a function.

(i) Explain what it means for the triple (x_0^*, x_1^*, x_2^*) to be a period-3 solution.

[1 mark]

(ii) The triple $x_0^* = 0.96069$, $x_1^* = 0.145380$, $x_2^* = 0.47834$ is a period-3 solution of the equation

$$x_{t+1} = 3.85x_t(1 - x_t).$$

   (a) Calculate the *eigenvalue* of the period-3 solution.

[2 marks]

(b) Is the period-3 solution stable or unstable? Explain your answer.

[2 marks]

Question 2. Consider the logistic equation with fixed harvesting

$$x_{n+1} = rx_n \left(1 - x_n\right) - h.$$

Suppose that $r = 2$ and $h = 0.05$.

(i) Show that the fixed points of this model are $x_2^* = 0.44365$ and $x_1^* = 0.05635$ (both correct to five decimal places).

[2 marks]

(ii) Calculate the stability of the two fixed points.

[2 marks]

(iii)

(a) Sketch the functions $y = x$ and $y = 2x(1 - x) - 0.05$, indicating the location of the fixed points.

[2 marks]

(b) Using your diagram and your answers to the previous parts of this question explain how the long-term population density depends upon the initial value x_0 ($0 \leq x_0 \leq 1$).

[3 marks]

(c) A company harvests deer from a wilderness area. Due to a drought the population density changes from 0.44365 to 0.02. Should the company change its harvesting strategy? Give reasons.

[1 mark]