

School of Mathematics & Applied Statistics
MATH111: Applied Mathematical Modelling
Assignment Week 7 Spring 2007

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked.

This assignment is to be handed in at the end of your during the Wednesday lecture of week 9.

Assignment Guidelines

You are expected to structure your assignments as if writing a report for presentation to people unfamiliar with the work covered by the assignment.

Your report should be structured as follows:

1. introductory remarks
 - state purpose of assignment; and
 - state proposed tasks.
2. discussion of theory (if appropriate)
3. discussion of results
 - present outputs of each task;
 - analyse outputs of each task, including
 - comment on what your mathematical results mean in terms of the underlying physical problem.
 - comment on unexpected results;
4. concluding remarks: state whether purpose of assignment was achieved. You should answer the following questions regarding this assignment.
 - What was the most important thing that you learnt? Why was it ‘important’?
 - What was the most puzzling thing that you did? (If nothing was puzzling, say so!)
5. Bibliography (if required)
6. Appendices
 - MAPLE program(s), containing comment lines explaining the purpose of your code.
 - MAPLE outputs where you think they are required to further amplify comments you have made in your report. Do *not* include every output you generated.
7. If you are not certain what is required in your report you should speak to the lecturer before you hand it in. If you don’t ask, don’t whinge if you lose marks because you didn’t do what you should have done.

Continued on next page

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling**
Assignment Week 5 Spring 2007 Submission Receipt

Student Name: _____ *Student Number:* _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

Graphs and tables should be included at appropriate locations in the body of the report or as appendices at the end of the report. Please ensure all handwritten work is tidy and legible and that every page is present and in the intended order.

The grade a student receives will be the lab demonstrator's subjective assessment of how much effort that student seems to have put into creating their report. Please note that missing or incomplete outputs, inadequate discussions, and/or poor presentation will result in a low grade even if you have successfully completed all the assigned tasks. Note that marks to questions/tasks (if provided) is only indicative.

Here are some good ways to *lose* marks (5% for each one):

- No title.
- No introduction.
- No theory.
- No sections/section headings.
- Not including the model equations.
- Not discussing the model equations.
- No conclusions or summary.
- No figures.
- Inadequate referencing of sources.
- No appendices (if required).
- Repeating the questions in your report and answering them. You're supposed to write a report!
- Using the question/task numbers in your report. These don't make sense to a reader who hasn't read the assignment sheet.
- Including every graph you generated during your investigation. Summarise your findings where appropriate!
- Poor graphs: no title, no labels, too small, too large, not numbering figures etc.
- Stating that your graph uses colour, such as 'blue' and 'black' lines, but only providing a black and white graphic.
- Poor quality output: difficult to read; pages out of order.
- Not showing signs of having carried out further reading when you have been asked to read specific article(s). (-10%)

This list is *not* exhaustive.

Instructions

You should work your way through this assignment, answering questions and making notes where appropriate. Where appropriate you should adapt Maple programs that you have used in previous lab sessions.

You will find it very *useful* to save any programs that you write onto a disk which you bring to subsequent labs.

1. Use a text editor such as NotePad (Programs/Accessories/NotePad) to write your program.
2. Save your program onto a disk (or alternatively onto the C drive) as a *text* file.
3. To load your program into Maple enter `read "A:/file";` where `file` is the name of your program.
4. If your program generates an error message:
 - (a) Enter the command `restart;` into Maple.
 - (b) Look at your code for syntax errors. Correct the code and reload it.
 - (c) If you can't find your error, ask for assistance.

Fixed harvesting in the Ricker model

1 Background

1.1 The mode

The Ricker model with fixed harvesting is given by

$$x_{n+1} = x_n \exp \left[r \left(1 - \frac{x_n}{K} \right) \right] - H, \quad n = 0, 1, 2, \dots \quad (1)$$

In this equation x_n the size of the population after n breeding seasons, r is the intrinsic growth rate, K is the carrying capacity of the environment and H is the amount harvested.

Question 1 (3 marks) *The scaled Ricker model. is obtained by introducing a new variable, x_{n+1}^* , defined by*

$$x_{n+1}^* = \frac{x_{n+1}}{K}.$$

1. *Derive the scaled Ricker model*

$$x_{n+1}^* = x_n^* \exp [r (1 - x_n^*)] - h, \quad n = 0, 1, 2, \dots \quad (2)$$

2. *How is the scaled harvesting parameter h defined in terms of the parameters in equation (1)?*

1.2 Maple code

The maple code used in this assignment can be adapted from that used in previous assignments. In order to prevent the population becoming negative, when it is over-harvested, you should change a line such as

```
f := x -> x*exp(r*(1-x)) -h;  
to  
f := x -> max(x*exp(r*(1-x)) -h,0);
```

Question 2 (2 marks) *How does the indicated code ensure that the population can not become negative?*

Before answering the assignment questions check your code by running it using the following parameter values:

- (a) $r = 1.7$ and $h = 0.0$.
- (b) $r = 1.7$ and $h = 0.7$.
- (c) $r = 1.7$ and $h = 0.8$.

You should obtain:

- (a) a period-one solution with $x^* = 1.0$.
- (b) a period-one solution with $x^* = 0.439971044$.
- (c) a period-one solution with $x^* = 0$.

Question 3 (7 marks)

- (a) Compare your answer in (b) to your answer in (a). What does this mean biologically? [2]
- (b) Compare your answer in (c) to your answer in (a). What does this mean biologically? [2]
- (c) The answers to (a) and (b) differ in an important biological way to (c). Determine a critical value of h , h_{cr} , such that if $h = h_{cr} - 0.01$ the solution is biologically the same as (a) and (b) whereas if $h = h_{cr} + 0.01$ the solution is biologically the same as (c). [3]
Provide graphs showing what happens on either side of your critical value.

2 Tasks

In investigating the solution of equation (2) we are interested in determining the ‘long-time’ behaviour of the populations, i.e. the value of x_n when n is “sufficiently large”. How large does n have to be for it to be “sufficiently large”? You will have to discover that by trial and error, it will depend upon the values used for the parameters in the model (r & h).

1. Determine the critical value of h , h_{cr} , for $r = 0.1, r = 0.2, r = 0.3, \dots r = 2.0$. [20]
 - Determine h_{cr} to an accuracy of 0.01.
 - You do *not* need to show the graphs establishing your critical values.
 - Provide your data in the form of a table with the first column showing the value for r and the second column showing the corresponding value for h_{cr} .
2. Plot your data with the value of r on the x -axis and the value of h_{cr} on the y -axis. [5]
 - **Note.** Suppose you want to plot the datapoints (1,1), (2,2) and (3,3). The Maple command to use is:
`plot([[1,1],[2,2],[3,3]],style=POINT);`
Put a straight line between consecutive datapoints.
 - If your curve is not smooth determine more datapoints.
3. How does the value for h_{cr} vary as r is increased? [1]

Question 4 (5 marks) For a particular fishery appropriate parameter values are given by $r = 1.25$ and $K = 1000$. The value for K is given in “tonnes of fish”.

- (a) The fishery manager asks you to recommend a value for H . What recommendation do you give?
- (b) How does your recommendation change when the fishery manager tells you that from experience he knows that in practice the harvest is always 110% of the set value?

Question 5 (5 marks) For a particular fishery appropriate parameter values are given by $r = 1.05 \pm 0.05$ and $K = 1200 \pm 100$. The value for K is given in “tonnes of fish”.

The fishery manager asks you to recommend a value for H . What recommendation do you give?

In this assignment we investigated the dynamics of the scaled Ricker model with fixed harvesting

$$x_{t+1} = x_t \exp[r(1 - x_t)] - h.$$

Question 6 (2 marks) *From the perspective of harvesting we require $h \geq 0$. What might the situation $h < 0$ represent 'physically'?*

3 Marking

Every student starts with a mark of 100. The questions and tasks for this assignment are worth a certain number of marks. Every time your answer to a question or task is incomplete or wrong you lose marks. In addition to losing marks in this way you can also lose marks for a badly written report. There is no upper bound on the number of marks you can lose. If you make 17 bad mistakes (see the list on page two) then you will lose 85 marks. However, your mark will not be reduced below zero.