MID-YEAR EXAMINATION 2001

Course Number: AMAT.2110

Course Name: Discrete Dynamics

Question 4. (13 marks)

(a) Consider the difference equation

$$x_{n+1} = f(x_n),$$

(i) What do we mean when we say that 'x^* *is a fixed point of the function* f'? [1 *mark*]

(ii) Write down the conditions for x^* to be stable and unstable, carefully defining all the terms that appear in your solution. [3 *marks*]

(b) The logistic difference equation is

$$x_{n+1} = rx_n(1 - x_n).$$

(i) When $r = 3.4$ the pair $x_0^* = 0.84215$ and $x_1^* = 0.45196$ is a period-2 orbit. Calculate the eigenvalue corresponding to this solution and hence determine its stability. [2 *marks*]

(ii) When $r = 3.5$ the pair $x_0^* = 0.85714$ and $x_1^* = 0.42857$ is a period-2 orbit. The eigenvalue of this orbit is $\lambda \approx -1.25$. What is the stability of the period-2 orbit? [1 *mark*]

(iii) Hence, or otherwise, explain how you would expect the nature of the solution to the logistic difference equation to change as r is increased from 3.4 to 3.5? [2 *marks*]

(c) Let f be a function defined on the interval $[0, 1]$ with $0 \leq f(x) \leq 1$. Consider the difference equation

$$x_{n+1} = f(x_n).$$

Draw a graph of a function f having a period 263 orbit. Explain why your function f has an orbit of this period. [4 *marks*]

Question 5. (20 marks)

The population of a species is governed by the equation

$$x_{t+1} = 1.8x_t\,(1 - x_t) - bx_t,$$

where the harvesting parameter b, $0 \leq b \leq 1$, is the fraction of the population that is harvested.

(a) Show that the fixed points of the above difference equation are $x^* = 0$ and $x^* = \frac{0.8-b}{1.8}$.
[4 *marks*]

(b) Show that there is a critical value of the harvesting parameter, $b = b_{cr}$, such that if $b > b_{cr}$ one of the fixed points is negative. Comment upon the biological implications of this situation.
[3 *marks*]

(c) Determine the stability of the fixed points found in part (a).
[8 *marks*]

(d) The number of animals that are harvested at equilibrium ($\mathcal{H}$) is given by the equation

$$\mathcal{H} = bx^*,$$

where x^* is the non-trivial fixed-point.

(i) Show that there is a value of the harvesting parameter, $b = b_m$, such that $\mathcal{H}$ obtains its maximal value, $\mathcal{H}_m$, at $b = b_m$.
[2 *marks*]

(ii) Determine the values $\mathcal{H}_m$ and b_m.
[2 *marks*]

(iii) Explain the biological significance of these values of $\mathcal{H}_m$ and b_m.
[1 *mark*]