School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 8

Spring 2007

Student Name: ______________________________ *Student Number:*________________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 9

The assignment that you had in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

If you fail to provide a cover-page your assignment will automatically be marked 'unsatisfactory'.

You may choose to answer one of the questions on this assignment sheet by working in a group of upto four individuals. If you choose this option then at the end of your group answer you must list the members of your group.

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
Assignment Week 6
Spring 2006 Submission Receipt

Student Name: ______________________________ *Student Number*: ________________

Tutorial Class: __________ *Date Submitted:* ________________ *Tutor Initials:* ________________

1. The model discussed in this section is analysed in Kubota and Maibach (1992).

   A drug delivery vehicle, such as a skin patch, is attached to a patient. The amount of drug per unit area (X) in the skin is described by the following differential equation

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\left(k_{12} + k_{1B}\right) X + k_{21} A_0, \quad X\left(0\right) = 0. \tag{1}$$

   In this equation A_0 is the concentration of drug per unit area in the delivery device, which is maintained at a constant level. The parameters k_{12} and k_{21} are the rate at which drug enters the vehicle from the skin and the skin from the vehicle respectively. The parameter k_{1B} is the rate at which drug leaves the skin and enters the blood stream.

   (a) Solve equation (1) to find the amount of drug per unit area in the skin as a function of time.

   (b) The cumulative amount of drug per unit area excreted from the skin (Ae_s) is expressed by

$$Ae_s = \int k_{1B} X \,\mathrm{d}t, \quad Ae_s\left(0\right) = 0.$$

   Determine Ae_s.

   K. Kubota and H.I. Maibach. A compartment model for percutaneous absorption: Compatibility of lag time and steady-state flux with diffusion model. *Journal of Pharmaceutical Sciences*, **81**(9):863–865, 1992.

2. After irradiation, the number of unavailable binding sites in the spleen for injected CFU decreases according to the differential equation

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\lambda_1 U, \qquad U\left(0\right) = U_0. \tag{2}$$

   The number F of available binding sites in the spleen changes according to

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \lambda_1 U - \lambda_2 F, \quad F\left(0\right) = 0, \tag{3}$$

   where the term $\lambda_2 F$ represents the decay of available sites due to radiation damage.

   (a) Solve equation (2) to find the number of unavailable sites as a function of time, $U\left(t\right)$.

   (b) Substitute your expression for $U\left(t\right)$ into equation (3). Solve the resulting differential equation to find the number of available sites as a function of time, $F\left(t\right)$.

   (c) Find the value of t, $t_{\max}$, where the number of available sites (F) is maximised (F_{max}).

   (d) Determine the maximum number of available sites available. Simplify your expression as far as possible.

   This question is based upon Matioli (1976).

   G. Matioli. (1976). Consideration of some functional properties of the hemopoietic sources. *Mathematical Biosciences*, **28**, 373–376.

In the mid-session test and/or the final exam you may be asked a question about Maple.

Your answer should include all maple code that you used to obtain the answer.

3. The Shannon function is defined as

$$H\left(p\right) = -\left[p \log_{10}\left(p\right) + \left(1-p\right)\log_{10}\left(1-p\right)\right],$$

   where $0 < p < 1$.

   (a) Plot the Shannon function.

   (b) What value of p maximises the value of the Shannon function?

   (c) What is the maximum value of the Shannon function?

   The importance of the Shannon function is described in Bruen & Forcinito (section 9.2, pages 162-164).

   A.A. Bruen and M.A. Forcinito. Cryptography, Information Theory, and Error-Correction. Wiley-Interscience, Hoboken, NJ, 2005.