School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 6

Spring 2007

Student Name: ____________________ *Student Number:* ____________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 7

The assignment that you had in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

If you fail to provide a cover-page your assignment will automatically be marked 'unsatisfactory'.

You may choose to answer one of the questions on this assignment sheet by working in a group of upto four individuals. If you choose this option then at the end of your group answer you must list the members of your group.

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
Assignment Week 6
Spring 2007 Submission Receipt

Student Name: ____________________ *Student Number:* ____________

Tutorial Class: ________ *Date Submitted:* ____________ *Tutor Initials:* ____________

1. Consider the population model

$$N_{t+1} = rN_t \left(1 + \alpha N_t\right)^{-\beta},$$

where N_{t+1} and N_t are the populations in successive generations, r is the finite net rate of increase and α and β are constants defining the density dependent feedback term [1].

   (a) By defining $x_t = \alpha N_t$ obtain the scaled equation

   $$x_{t+1} = rx_t \left(1 + x_t\right)^{-\beta},$$

   showing all necessary working. What is the importance of the scaled equation?

   (b) Show that the, non-negative, fixed points of the scaled equation are given by

   $$x_1^* = 0,$$
   $$x_2^* = (r)^{\frac{1}{\beta}} - 1.$$

   What condition must be imposed upon the value of r for the steady-state solution x_2^* to be non-negative?

   (c) Determine the stability of the trivial fixed-point solution as a function of r.

   (d) Assuming that $\beta = 2$ determine the stability of the trivial fixed-point solution as a function of r.

   (e) For the case $\beta = 2$ draw a steady-state diagram for the non-negative fixed points as a function of r.

[1] M.P. Hassell, J.H. Lawton and R.M. May. (1976). Patterns of Dynamical Behaviour in Single-Species Populations. *Journal of Animal Ecology* , **45**(2): 471–86.

You should use maple for the following questions where-ever possible!

2. (a) Given the graph $y = f(x)$ explain how to obtain the graph $y = f(x) - h$, with $h > 0$.

   (b) Consider the scaled Ricker model

   $$x_{n+1} = x_n \exp\left[r\left(1 - x_n\right)\right].$$

   In order to draw a cobwebbing diagram for this population model a figure containing the straight line $y = x$ and the curve $y = x\exp\left[r\left(1 - x\right)\right]$ is required.

      (i) Why is the straight line $y = x$ required in a cobwebbing diagram?

      (ii) Write appropriate maple code to plot the straight line $y = x$ and the curve $y = x\exp\left[r\left(1 - x\right)\right]$ on the same figure with $0 \le x \le 3$ and $r = 1.8$. Choose an appropriate scale for the y-axis.

      (iii) Mark the fixed-points of the scaled Ricker model on your output. You may do this by hand.

   (c) Consider the scaled Ricker model subject to fixed harvesting with $r = 1.8$

   $$x_{n+1} = x_n \exp\left[1.8\left(1 - x_n\right)\right] - h.$$

   By providing appropriate graphs show that

      (i) Harvesting is sustainable when $h = 0.25$.

      (ii) Harvesting is unsustainable when $h = 1$.

   (d) Estimate the critical value of the fixed harvesting parameter, h_{cr}, to two decimal places such that if $h = h_{\mathrm{cr}} + 0.01$ harvesting is unsustainable whilst if $h = h_{\mathrm{cr}} - 0.01$ harvesting is sustainable.
   HINT. Use maple to plot appropriate graphs. In your answer you must make it clear how you used maple to answer this question.

3. Plot the following curves given in parametric form [2].

   (i) $(x, y) = \left(\frac{2}{3}\cos(t) + \frac{5}{9}\cos(2t), \frac{2}{3}\sin(t) - \frac{5}{9}\sin(2t)\right).$
   This curve is an example of an hypocycloid. What is an hypocycloid?

   (ii) $(x, y) = (\cos(3t), \sin(2t)).$
   This curve is an example of the Lissajous curve. What is the Lissajous curve?

[2]. L. Berman. (2006). Folding Beauties. *The College Mathematics Journal* **37**(3), 176–186.