School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 2

Spring 2007

Student Name: ______________________________ *Student Number:* ______________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 3

The assignment that you had in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

If you fail to provide a cover-page your assignment will automatically be marked 'unsatisfactory'.

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
Assignment Week 2
Spring 2007 Submission Receipt

Student Name: ______________________________ *Student Number*: ______________

Tutorial Class: __________ *Date Submitted:* ______________ *Tutor Initials:* ______________

You may choose to answer one of the questions on this assignment sheet by working in a group of upto four individuals. If you choose this option then at the end of your group answer you must list the members of your group.

1. Give an example of
   (a) A third order nonlinear autonomous difference equation.
   (b) A fifth order linear nonautonomous difference equation.

   For each example, explain why your equations satisfies the stated criteria.

2. The number of chickens in Mr & Mrs Tweedy's farm is modelled by the difference equation,

$$c_n = (1 + g - \alpha)\, c_{n-1} - P, \quad n = 1, 2, 3 \ldots$$

   where c_i is the number of chickens in the ith week, g is the fractional growth rate of chickens each week, α is the fraction of current chickens who are killed by foxes each week, and P is the constant number of chickens that are converted into pies each week. Assume that g and α are constant. For convenience let

$$\beta = 1 + g - \alpha,$$

   and suppose that in week 0 there are c_0 chickens present.

   (a) Find the general solution of the chicken model, simplifying as far as possible.
   (b) Suppose that $c_0 = 200$, $g = 0.25$ and $\alpha = 0.05$.
      (i) Suppose that 20 chickens a week are converted into pies. What is the number of chickens on the farm in the limit $n \to \infty$?
      (ii) Suppose that 60 chickens a week are converted into pies. What is the number of chickens on the farm in the limit $n \to \infty$?
      (iii) What number of chickens (P) should be converted into pies each week if the number of chickens on the farm is to remain constant?
      (iv) At the beginning of the first week the pie machine breaks down before any chickens are converted into pies. It will take Mr. Tweedy eight weeks to fix the pie machine. When it is fixed Mrs Tweedy will convert all the chickens into pies. If one chicken produces four pies how many pies will Mrs Tweedy have?
      (v) How do the chickens feel about being converted into pies? What should they do?

3. Use induction to prove that

$$\sum_{k=1}^{n} a^{n-k} k = \frac{a^{n+1} - (n+1)\, a + n}{(a-1)^2}.$$

   Simplify your expression as far as possible for the case when $a = -1$.

4. Consider the early-stages of a disease spreading through a population. Those individuals who have caught the disease are known as the 'infectives'. Each week the following activities occur:

   - Each infective infects a certain number of uninfected people.
   - A fraction of the infectives recover naturally from the disease.
   - A fraction of the infectives die as a result of the disease.
   - A certain number of the infectives are treated and recover from the disease.

   (a) Write down a **word** equation that defines this problem.
   (b) Write down, formally, the difference equation that describes the above scenario. Define **all** variables and explain your terms.

5. In the mid-session test and/or the final exam you may be asked a question in which you need to write simple Maple code to solve a problem.

   The following expression was derived by Golay (1958) for capillary chromatography with a retentive layer.

$$\Lambda = \frac{1 + 6\delta + 11\delta^2}{48\,(1+\delta)^3}.$$

   In this equation Λ is the normalised dispersion, δ is a function of the capacitance ratio and the void fraction of the chromatography bed.

   What is the maximum value of the normalised dispersion, Λ_{max}, and what is the corresponding value of δ?

M.J.E. Golay (1958). Theory of chromatography in open and coated tubular columns with round and rectangular cross-sections, in Gas Chromatography, D.H. Desty, ed., Butterworth, London, pp 36-49.