School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 8

Spring 2006

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 9

1. Give an example of a *linear* differential equation and a *non-linear* differential equation explaining why your equation is linear/non-linear.
2. Identify if the following differential equations are autonomous or non-autonomous. You *must* justify your answer.
   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}t} = y^2$
   (b) $\dfrac{\mathrm{d}^2t}{\mathrm{d}y^2} = t\dfrac{\mathrm{d}t}{\mathrm{d}y} + \dfrac{1}{t}$
   (c) $\dfrac{\mathrm{d}t}{\mathrm{d}y} = \cos t$
3. For two consecutive reactions
$$\mathcal{A} \xrightarrow{k_1} \mathcal{B} \xrightarrow{k_2} \mathcal{C}$$
occurring in a batch reactor the concentrations of the species $\mathcal{A}$ and $\mathcal{B}$ satisfy the differential equations
$$\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}t} = -k_1\mathcal{A}, \qquad \mathcal{A}(0) = C_A, \tag{1}$$
$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}t} = k_1\mathcal{A} - k_2\mathcal{B}, \quad \mathcal{B}(0) = 0. \tag{2}$$
The chemical species $\mathcal{A}$ is known as the reactant, the chemical species $\mathcal{B}$ is known as the intermediate product and the chemical species $\mathcal{C}$ is known as the final product.
   (a) Solve the system of differential equations to find the concentrations of the reactant and the intermediate product as a function of time.
   (b) In many cases the intermediate product is more valuable than the final product and hence we want to maximise its production. At what time, t_m, should we stop the batch reactor from operating to achieve this aim?
   (c) Define the fractional yield of the intermediate product by
$$\mathcal{Y}_B = \frac{\mathcal{B}}{C_A}.$$
   What is the maximum fractional yield?

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
Assignment Week 6
Spring 2006 Submission Receipt

Student Name: _____ *Student Number*: _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

In the mid-session test and/or the final exam you may be asked a question about Maple.

Your answer should include all maple code that you used to obtain the answer.

4. In section 10.2.1 of the notes we considered the problem of pollutant being dumped at time $t = 0$ into a clean lake into which only fresh water flows. We found that the time taken for the pollutant to reach 5% of its initial value is given by

$$t_{0.05} = \frac{V}{q} \ln 20.$$

In section 10.2.2 we consider the same problem but with a seasonal flowrate. The value of $t_{0.05}$ was found to satisfy the equation

$$\ln(0.05) + \frac{q_0}{V}\left[t_{0.05} + \frac{365\epsilon}{2\pi}\sin\left(\frac{2\pi t_{0.05}}{365}\right)\right] = 0. \tag{3}$$

In the following we take $V = 10^5 \mathrm{m}^3$ and $q = 5 \times 10^3 \mathrm{m}^3 \mathrm{hr}^{-1}$.

(a) Find $t_{0.05}$ when $\epsilon = 0$.

(b) Find $t_{0.05}$ when $\epsilon = 0.05$.

(c) Draw a graph showing how $t_{0.05}$ depends upon the value for ϵ. Label your axis.

You may find it useful to use the following maple commands: `fsolve` and `implicitplot`.