

School of Mathematics & Applied Statistics  
**MATH111: Mathematics Applied Mathematical  
 Modelling 1**  
**Assignment Week 8**  
**Spring 2004**

*Student Name:* \_\_\_\_\_ *Student Number:* \_\_\_\_\_

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during your tutorial in Week 9

1. A *batch reactor* has neither inflow nor outflow of reactants or products whilst the reaction is being carried out. Suppose that the reaction



occurs in a batch reactor. For an  $n$ th-order reaction the rate of change of reactant concentration in the reactor is given by

$$\frac{dA}{dt} = -k_1 A^n, \quad A(0) = A_0,$$

where  $A_0$  is the initial concentration of the reactant.

- (a) By solving the appropriate differential equation determine how the concentration of reactant in the reactor depends upon the time since the reactor was started for
- (i) a first-order reaction,
  - (ii) a second-order reaction.
- (b) Hence obtain a formula for the time taken ( $t_R$ ) for the concentration of reactant to decrease to 10% of its initial value for
- (i) a first-order reaction,
  - (ii) a second-order reaction.
- (c) (i) For a given first-order reaction  $k_1 = 10^{-4} \text{ s}^{-1}$ . Determine  $t_R$ .
- (ii) For a given second-order reaction the product  $k_1 A_0 = 10^{-3} \text{ s}^{-1}$ . Determine  $t_R$ .

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*Tutorial Class:* \_\_\_\_\_ *Date Submitted:* \_\_\_\_\_ *Tutor Initials:* \_\_\_\_\_

2. A company is discharging a herbicide into a river that flows into a marsh, where it is degraded. The rate of degradation of the herbicide,  $A$ , is assumed irreversible and to follow first-order homogeneous kinetics.



This process is represented by the differential equation

$$V \frac{dA}{dt} = qA_0 - qA - V k_1 A. \quad (1)$$

Assume that the marsh is rectangular with width  $W = 100$  m, length  $L = 1000$  m and average depth  $D = 0.25$  m. The other parameter values are:  $A_0 = 10^{-2} \text{ mol m}^{-3}$ ,  $k_1 = 16 \times 10^{-5} \text{ h}^{-1}$ ,  $q = 2 \text{ m}^3 \text{ hr}^{-1}$ .

- (a) In equation (1) what do the symbols  $A, A_0, V, k_1, q$  &  $t$  mean?
- (b) Given that initially there is no herbicide present in the marsh: obtain the solution to equation (1).
- (c) (i) Let  $A(\infty)$  be the concentration of herbicide in the marsh at time  $t = \infty$ . What is  $A(\infty)$ ?
- (ii) How many days does it take for the level of herbicide in the marsh to reach half of its final value?
- (iii) Suppose that the legal maximum level of herbicide in the marsh is given by  $A_{\max} = \frac{1}{30} A(\infty)$ . On which day must the company stop pumping herbicide into the marsh?