School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 6

Spring 2004

Student Name: ______________________________ *Student Number:* ______________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 7

1. Consider the map
$$x_{n+1} = rx_n\left(1 - x_n^2\right).$$
   (a) Show that if $0 \leq x_n \leq 1$ then $0 \leq x_{n+1} \leq 1$ provided that $0 \leq r \leq \frac{3\sqrt{3}}{2}$.
   (b) Solve the fixed point equation, and show that there is only one fixed point ($x^* = 0$) for $0 \leq r \leq 1$ and three fixed points when $1 < r$. Give a formulae for the new pair $x^*_\pm$.
   (c) Only one of the new pair of solutions is biologically meaningful: which one is it?

2. Suppose that the number of deers (in thousands) in a forest can be modelled by the difference equation
$$p_{n+1} = 1.5p_n - 0.5p_n^2.$$
   (a) Determine the fixed points of this model.
   (b) Determine the stability of the fixed point(s).

3. **ADFA Test 2001** Consider the difference equation
$$x_{n+1} = rx_n^2\left(1 - x_n\right).$$
   (a) Show that the fixed points of this equation are $x^* = 0$ and $x^*_\pm = \frac{r \pm \sqrt{r(r-4)}}{2r}$.
   (b) Determine the values for $x_\pm$ when $r = 5$ (correct to 5 decimal places). Calculate the corresponding eigenvalues and hence determine the stability of the two fixed points $x_\pm$.

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Tutorial Class: __________ *Date Submitted:* ______________ *Tutor Initials:* ______________

4. The following discrete time population model has been used in the ecological literature.

$$x_{n+1} = \frac{rx_n}{1 + x_n^b}, \quad r > 0, \quad b > 1.$$

(a) Show that this equation has at most two real fixed points: $x_1^* = 0$ and the solution of the equation $x_2^{*b} = r - 1$.

(b) Let $f = \frac{rx}{1+x^b}$. Show that

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{r\left(1 + x^b\left[1 - b\right]\right)}{\left(1 + x^b\right)^2}$$

(c) Determine the stability of the fixed point x_1^* as a function of r and b.

(d) Show that the eigenvalue of the fixed point x_2^* is given by

$$f'\left(x_2^*\right) = (1 - b) + \frac{b}{r}$$

Hence determine the values of r, as a function of b, for which the fixed point x_2^* is stable.