

School of Mathematics & Applied Statistics
**MATH111: Mathematics Applied Mathematical
 Modelling 1**
Assignment Week 4
Spring 2004

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during your tutorial in Week 5

1. How long will it take \$1000
 - (a) to earn \$100 at 15% simple interest?
 - (b) to accumulate to at least \$1200 at 13.5% simple interest?
2. A cash discount of 4% is given if a bill is paid 30 days in advance of its due date. What is the highest simple interest rate at which you can afford to borrow money in order to take advantage of the cash discount?
3. What is the
 - (a) interest rate compounded monthly that is equivalent to 10.08% compounded yearly?
 - (b) interest rate compounded every two months that is equivalent to 12% compounded quarterly?
 - (c) interest rate compounded monthly that is equivalent to 5% compounded every half-year?

(See question 6 in chapter 3.7.2 for the definition of *equivalent* interest rates).
4. To prepare for early retirement, a self-employed consultant desposits \$5500 into a retirement saving plan each year, starting on her 31st birthday. When she is 51, she wishes to draw out 30 equal annual payments. What is the size of each withdrawal, if interest was compounded annually at 12% for the first ten years, compounded annually at 10% for the next ten-year period, and compounded annually at 11% for the 30-year retirement period?
5. Dr. Susan Calvin buys a piece of land worth \$40,000 by paying down \$10,000 down and then taking out a loan for \$30,000. The loan will be retired with quarterly payments over 15 years with a quarterly compounded interest rate of 8%. Find her equity at the end of nine years. See questions 2 & 3 in section 3.7.5.
6. Consider the following map

$$x_{n+1} = \frac{27rx_n^2(1-x_n)}{16}$$

- (a) Show that if $0 \leq r \leq 4$ and $0 \leq x_n \leq 1$ then $0 \leq x_{n+1} \leq 1$.
- (b) Show that there is only one fixed point ($x^* = 0$) for $0 \leq r < \frac{64}{27}$, two fixed points when $r = \frac{64}{27}$ and three fixed points for $\frac{64}{27} \leq r \leq 4$. Give a formulae for the new pair x_{\pm}^* .

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Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

7. In a remote region in Canada, the dynamics of a fly population has been studied and found to satisfy the difference equation

$$x_{n+1} = 11 - 0.01x_n^2,$$

where $x_n (> 0)$ is the fly population density at generation n ,

- (a) Determine the fixed point(s) of this model.
- (b) Assume that the initial fly population density is 30. By drawing a cobweb diagram determines what happens to the fly population after a very long time (i.e. as $n \rightarrow \infty$).