

School of Mathematics & Applied Statistics
**MATH111: Mathematics Applied Mathematical
 Modelling 1**
Assignment Week 10
Spring 2004

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during your tutorial in Week 11

1. The population density of fish is modelled by the differential equation

$$\frac{du}{dt} = f(u), \quad u(t=0) = u_0,$$

where the function $f(u)$ has the following properties:

- $f(0) = f(K_0) = f(K) = 0$ where $0 < K_0 < K$.
- If $u \in (0, K_0)$ then $f(u) < 0$.
- If $u \in (K_0, K)$ then $f(u) > 0$.
- If $u > K$ then $f(u) < 0$.

- (a) Sketch the growth curve $f(u)$ as a function of u .
 - (b) Using your sketch determine the stability of the steady state solutions $u = 0$, $u = K_0$ and $u = K$ carefully explaining your reasoning.
 - (c) How does the long-term evolution of the differential equation depend upon the choice of the initial condition u_0 ?
 - (d) A disease spreads through the population reducing the population to a density $K_0/2$. What happens to the population? Justify your answer.
2. For some organisms finding a suitable mate may cause difficulties at low population densities, and a more realistic equation for population growth under these conditions may be

$$\frac{dN}{dt} = rN^2, \quad r > 0, N(0) = N_0.$$

- (a) Solve this problem and show that the solution becomes infinite in finite time.
- (b) The model above may be improved to

$$\frac{dN}{dt} = rN^2 \left(1 - \frac{N}{K}\right).$$

Without solving this equation find the steady-state solutions and say whether they are stable or unstable. (Do not calculate eigenvalues).

- (c) Derive the model of part (b) from that of part (a). (Hint... reread Chapter 11).

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3. Suppose a population satisfies a logistic model with carrying capacity 100 and that the population size is 10 when $t = 0$ and 20 when $t = 1$. Find the intrinsic growth rate.

Use the solution to the logistic equation

$$x(t) = \frac{Kx_0}{x_0 + (K - x_0)e^{-rt}}.$$

4. Nisbet & Gurney (1983) suggested the following form for the per-capita growth rate

$$r(x) = r \exp\left[1 - \frac{x}{K}\right] - d$$

Consider the associated population model

$$\frac{dx}{dt} = \left(r \exp\left[1 - \frac{x}{K}\right] - d\right)x, \quad x(0) = x_0, \quad r > de^{-1}.$$

- Find the steady-state(s) of the model. How do the number of steady-state solutions ($x^* \geq 0$) and their biological feasibility depend upon the values of r and d ?
- Explain why it is reasonable to assume that $r > de^{-1}$.
- Sketch $\frac{dx}{dt}$ as a function of x . Hence determine how the long-term dynamics of the model depends upon the initial value x_0 .