B Sigma notation

B.1 What is Sigma notation?

The symbol "$\sum$" (Sigma) is a notation used to denote the summation of terms and is defined as

$$\sum_{i=1}^{n} f(i) = f(1) + f(2) + f(3) + \cdots + f(n-1) + f(n)$$

For example

(i) $\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

(ii) $\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$

(iii) $\sum_{p=1}^{4} p(p+3) = [1 \cdot (1+3)] + [2 \cdot (2+3)] + [3 \cdot (3+3)] + [4 \cdot (4+3)]$

(iv) $\sum_{i=3}^{5} i^{\frac{1}{2}} = 3^{\frac{1}{2}} + 4^{\frac{1}{2}} + 5^{\frac{1}{2}}$

(v) $\sum_{n=1}^{7} 3 =$ _____

Some common properties:

(a) $\sum_{k=1}^{n} c = cn$ (where c is a constant)

e.g. $\sum_{k=1}^{20} 3 = 3 \times 20 = 60$

(b) $\sum_{k=1}^{n} cF(k) = c\sum_{k=1}^{n} F(k)$

e.g. $\sum_{k=1}^{n} 4n = 4\sum_{k=1}^{n} n$

(c) $\sum_{k=1}^{n}[F(k) \pm G(k)] = \sum_{k=1}^{n} F(k) \pm \sum_{k=1}^{n} G(k)$

e.g. $\sum_{k=1}^{n}(3n^2 + 4n) = \sum_{k=1}^{n} 3n^2 + \sum_{k=1}^{n} 4n = 3\sum_{k=1}^{n} n^2 + 4\sum_{k=1}^{n} n$

Here are some **very** useful results which, if needed in quizzes and exams, would be provided by the examiner.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$$

B.2 Questions

1. Find the following [Barry & Davies]
   (a) $\sum_{i=1}^{4} i^2$
   (b) $\sum_{i=1}^{6} (i+1)$
   (c) $\sum_{i=1}^{4} (i+2)$
   (d) $\sum_{i=1}^{4} (2i+1)$

2. Use induction to prove the following results

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$$