

Geometric Progression

C.1 What is a Geometric Progression?

A Geometric Progression (GP) is a sequence taking the form

$$k, kr, kr^2, kr^3, \dots, kr^{n-1}, \dots$$

where k is the first term and r is called the common ratio.

The following are examples of GPs

- (i) 3, 6, 12, 24, 48, ...
(common ratio = 2)
- (ii) 2, -4, 8, -16, 32, ...
(common ratio = -2)
- (iii) 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...
(common ratio = $\frac{1}{2}$)

Question C.1 *Can you write down a difference equation that describes the terms of a G.P.?*

The sum of a G.P. is

$$\begin{aligned} S_n &= k + kr + kr^2 + \dots + kr^{n-2} + kr^{n-1}, \\ &= \frac{k(r^n - 1)}{r - 1} \quad \text{where } r \neq 1 \end{aligned}$$

If $-1 < r < 1$, then

$$S_\infty = \frac{k}{1-r}$$

C.2 Questions

1. Evaluate

$$1 + 3 + 3^2 + 3^3 + \dots + 3^{n-2} + 3^{n-1}$$

2. Show that the expression

$$\sum_{p=1}^n a^{n-p}$$

is a geometric expression and hence evaluate it.

3. Imagine this scenario, if you will. Economic rationalism has taken hold of your workplace and it's time to renegotiate your contract.

Knowing a thing or two about maths, you make the following proposal. "Boss, I've been far too greedy. But I've come to my senses, after reading *Animal Farm*, and propose a new pay scale. Starting tomorrow, I would like you to pay me two cents..." "*It's a deal*" "...raised to the power of the number of days..." "*Sign here!*" "...the commencement of my new..." "*Next!*" "...contract."

3. (*continued*)

Day one, you are paid $2c$ (2 raised to the power of one). Day two, $4c$ (2 squared). Day three, $8c$ (2^3). Day four, $16c$ (2^4). Day five, $32c$. For week one, you take home $62c$.

- (a) How much do you take home in week two?
- (b) How much do you take home in week three?
- (c) How much do you take home in week four?

Based on an article by Jeremy Chunn that appeared in *Mens Style Summer 2003*