Modeling Dual Polarization Wireless Fading Channels using Quaternions

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Abstract

In the letter we describe signal transmitted between two dual-polarized antennas using a quaternion notation. The channel can then be modeled by a single quaternion gain, instead of a matrix of four complex gains as it is done in a classical approach. The model is very useful as it allows for a simple accommodation of any polarization twist between the transmit and receive antennas, as well as, modeling random fading in both polarizations with or without cross-polar scattering.

1. Introduction

Polarization diversity has been widely studied in the past, e.g. [1], and its diversity gain under Rayleigh fading considered in [2]. Recently, the demand for high rates in mobile communications has again raised the interest in applying polarization diversity, often together with other forms of diversity, like those considered in [3] or [4]. It has been shown there that polarization diversity can significantly add to the performance improvements offered by other diversity techniques, and be nearly as effective as spatial diversity for base station antennas, without a noticeable increase in their dimensions. In addition, the use of dual-polarized antennas for both transmitter and receiver combined with channel coding has been studied in [5].

The transmission channel for the case when dual-polarized antennas are used at both transmitter and receiver is usually described using four complex coefficients that leads to significant complexities when more than one antenna pair is considered. In this letter, we propose a simple mathematical model to describe such transmission channels using quaternions [6], where the channel is described by its gain coefficient being a single quaternion number.

The paper is organized as follows. In Section II, we introduce the description of the transmitted signal using quaternion numbers for dual-polarized antenna systems. The channel model is introduced in Section III. Simulation results are presented in Section IV, while Section V concludes the paper.

2. Signal Description

In [7], Isaeva and Sarytchev showed that the polarization state of an electromagnetic wave can be nicely modeled by means of quaternion representation, whereby two complex signal constellations

$$z_1 = x_1 + iy_1 \quad \text{and} \quad z_2 = x_2 + iy_2 \quad (1)$$

are orthogonal to each other on a polarization plane, just forming a quaternion number:

$$s = z_1 + z_2 j = x_1 + iy_1 + jx_2 + ky_2 \quad (2)$$

where i, j, and k are hypercomplex numbers [7] satisfying

$$i^2 = j^2 = k^2 = -1 \quad \text{and} \quad ijk = -1 \quad (3)$$

In general, a quaternion number a is defined as a linear combination of hypercomplex numbers i, j, and k, i.e.

$$a = a_1 + a_2 i + a_3 j + a_4 k$$

where a_m; $m = 1, 2, 3, 4$, are real numbers. From (3), the following equalities can be drawn

$$
\begin{aligned}
ij &= -ji = k \\
ki &= -ik = j \\
jk &= -kj = i
\end{aligned} \quad (4)
$$

which means that multiplication of quaternion numbers is not commutative. A product ab of two quaternion numbers a and b is thus given by:

$$
\begin{aligned}
ab = &\; a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 \\
&+ (a_1b_2 + a_2b_1 + a_3b_4 - a_4b_3)i \\
&+ (a_1b_3 - a_2b_4 + a_3b_1 + a_4b_2)j \\
&+ (a_1b_4 + a_2b_3 - a_3b_2 + a_4b_1)k
\end{aligned} \quad (5)
$$

and, of course, $ab \neq ba$.

A rotation s_φ of the signal s on a polarization plane by an angle of φ can be simply represented in the quaternion notation as:

$$
\begin{aligned}
s_\varphi &= se^{j\varphi} = (z_1 + z_2 j)e^{j\varphi} \\
&= (z_1 + z_2 j)[\cos(\varphi) + j\sin(\varphi)]
\end{aligned} \quad (6)
$$

Similarly, any change to the polarization bases, e.g. due to differences between transmit and receive antenna alignments, can be easily represented in the quaternion notation, as long as the polarization bases are orthogonal [7].

3. Channel Model

Conventionally, transmission between two dual-polarized antennas (e.g. having both vertical and horizontal polarizations) is considered in a similar way as if there were 2 transmit and 2 receive antennas, resulting in 4 sub-channels, each characterized by its own complex gain (see Figure 1.a) much the same as in case of a 2-input 2-output MIMO system [8]. Assuming that an input signal to transmit antenna is of the form given by (1), with z_1 transmitted in the horizontal polarization and z_2 through the vertical one, the received signals r_1 and r_2 in the horizontal and vertical polarization of the receive antenna are:

$$r_1 = z_1 h_{hh} + z_2 h_{vh}$$
$$r_2 = z_1 h_{hv} + z_2 h_{vv} \qquad (7)$$

where h_{hh}, h_{vh}, h_{hv}, and h_{vv} are complex channel gain coefficients for transmissions between horizontal and horizontal, vertical and horizontal, horizontal and vertical and vertical and vertical antennas, respectively.
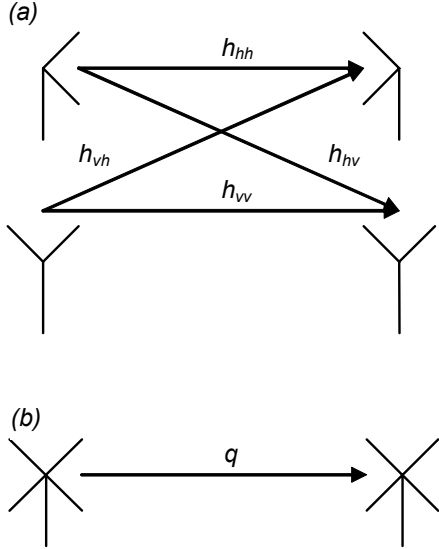
(a)

(b)

Figure 1: Schematic diagrams of channels utilizing dual-polarized antennas; (a) classical approach – the horizontally and vertically polarized antennas are considered separately and each of the channel gains, h_{hh}, h_{vh}, h_{hv}, and h_{vv} are complex numbers, (b) proposed model – the channel between two dual-polarized antennas is described by a single quaternion gain q.

Hence, the output r of the receive antenna system in the quaternion notation is

$$r = r_1 + r_2 j \qquad (8)$$

We propose an alternative channel model, where the channel between the dual-polarized transmit and receive antennas is described by a single quaternion gain, $q = q_1 + q_2 i + q_3 j + q_4 k$, as illustrated in Figure 1.b. In this model, the received signal r is given by

$$r = sq \qquad (9)$$

and estimation of the channel gain for performing ML decision involves estimation of just 4 real values that is half of what is required in the classical approach. From formulae (8) and (9), the relationship between the classical and the proposed models can be easily found, i.e.

$$\mathrm{Re}\{h_{hh}\} = \mathrm{Re}\{h_{vv}\} = q_1$$
$$\mathrm{Im}\{h_{hh}\} = -\mathrm{Im}\{h_{vv}\} = q_2$$
$$-\mathrm{Re}\{h_{vh}\} = \mathrm{Re}\{h_{hv}\} = q_3$$
$$\mathrm{Im}\{h_{vh}\} = \mathrm{Im}\{h_{hv}\} = q_4 \qquad (10)$$

where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ mean real and imaginary values, respectively.

The ML decision, when the proposed model is used, means finding a signal $\tilde{s}$ in a constellation S that minimizes, over all possible values of $s \in S$, the following decision metric

$$|r - sq|^2 = (r - sq)(r - sq)^Q \qquad (11)$$

where $(\cdot)^Q$ denotes the quaternion transform. For a given quaternion number a, its quaternion transform a^Q is

$$a^Q = a_1 - a_2 i - a_3 j - a_4 k \qquad (12)$$

which, based on (3), (4), and (5), yields

$$a a^Q = a^Q a = |a|^2 = a_1^2 + a_2^2 + a_3^2 + a_4^2 \qquad (13)$$

The proposed model also simplifies simulation of transmission systems when dual-polarized antennas are employed. In the most general case of independent flat Rayleigh fading [9] for both polarizations and random cross-polar scattering, simulation involves just generation of four independent, zero-mean, Gaussian random variables, or in other words a random quaternion Gaussian variable. It is easy to notice, from equations (9) and (5) that at the receiver, each of the 4 signal amplitudes is then a zero-mean Gaussian random variable, which means that the envelopes of the received signal follow Rayleigh distribution for both polarizations. Some examples of channels, simulated using the proposed model, are presented in the next section.

4. Simulation Results

We used the developed channel model to simulate a Rayleigh fading environment for three different scenarios:

   i) System utilizing a vertically polarized transmit antenna and a dual-polarized receive antenna with transmission involving both Rayleigh fading and random cross-polar scattering.

   ii) System utilizing dual-polarized transmit and receive antennas but transmission involving just Rayleigh fading without any cross-polar scattering.

iii) System utilizing dual-polarized transmit and receive antennas and transmission involving both random Rayleigh fading and random cross-polar scattering.

In all cases, we assumed a signal energy normalized to 1 and consisting of both real and imaginary components. The channel coefficient was also normalized to have no effect on the received channel energy, i.e. the sum of variances of q_1, q_2, q_3, and q_4 being equal to 1. For cases (i) and (iii) that meant all q_1, q_2, q_3, and q_4 being i.i.d. Gaussian random variables with zero mean and variances equal to 0.25, while for case (i) $q_3 = q_4 = 0$, and q_1, and q_2 being i.i.d. Gaussian random variables with zero mean and variances equal to 0.5. The simulation was repeated 1000 times, and the resulting variations in the received signal envelope magnitudes for both polarizations are plotted in Figures 2.a., 3.a., and 4.a. for a horizontal polarization and Figure 2.b., 3.b.,
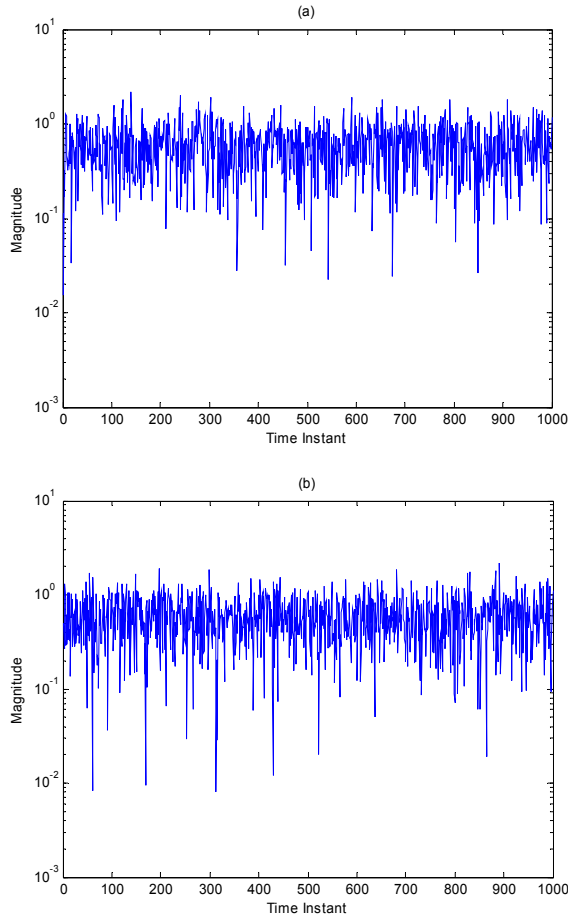

Figure 2: Plots of the magnitudes of signal envelopes for scenario (i); (a) horizontal polarization, (b) vertical polarization.
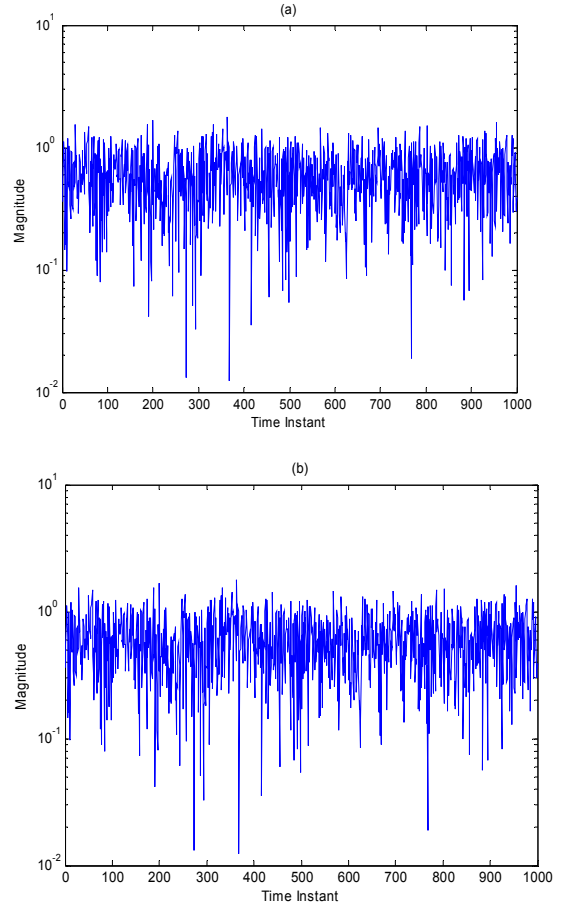

Figure 3: Plots of the magnitudes of signal envelopes for scenario (ii); (a) horizontal polarization, (b) vertical polarization.

and 4.b., for the vertical polarization for scenarios (i), (ii), and (iii), respectively. It is interesting to notice that the correlation coefficients between the values of envelope magnitudes for the horizontal and vertical polarizations for scenarios (i), (ii), and (iii) are 0.0144, 1.0, and 0.0095, respectively.

5. Conclusions

In the paper we proposed a simple mathematical model for wireless fading channels where dual-polarized antennas are used. The model is based on describing the dual-polarized signal by means of quaternion representation, which leads to a channel described by its gain being a single quaternion variable. The model is very useful as it allows for a simple accommodation of any polarization twist between the transmit and receive antennas, as well as, modeling random fading in both polarizations with or without cross-polar scattering.
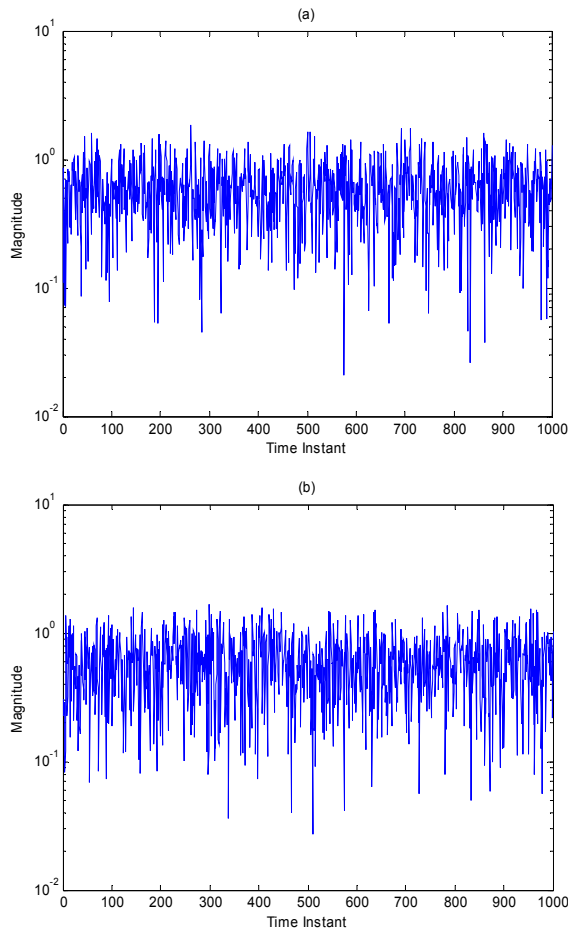
Figure 4: Plots of the magnitudes of signal envelopes for scenario (iii); (a) horizontal polarization, (b) vertical polarization.

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