On Amicable Orthogonal Designs of Order 8

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Abstract

Some new amicable orthogonal designs of order 8 are found as part of a complete search of the equivalence classes for orthogonal designs OD$(8;1,1,1,1)$, OD$(8;1,1,1,4)$, OD$(8;1,1,2,2)$, OD$(8;1,1,1,2)$, OD $(8;1,1,2,4)$, OD$(8;1,1,1,3)$, OD$(8;1,1,2,3)$, OD$(8;1,1,1,5)$ and OD$(8;1,1,3,3)$.
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1 Motivation

Two Hadamard matrices H_1 and H_2 are called *equivalent*(or H-*equivalent*) if one can be obtained from the other by a sequence of row negations, row permutations, column negations and column permutations. Their usefulness has been shown in many areas. For example, H-equivalent Hadamard matrices can be applied to improve correlation characteristics of bipolar of quadri-phase sequences for channel separation in direct sequence code division multiple access(DS CDMA) ststems [?][?][?]. The application of H-equivalent Hadamard matrices is also helpful in statistical analysis such as screening designs [?][?][?][?] which are used for situations where a large number of factors (n) is examined but only a few (k) of these are expected to be important. This leads us to believe that the study of equivalent and especially inequivalent orthogonal designs will give us very different properties for communication systems and other applications.

2 Introduction

Definition 2.1 Let $x_1, x_2, \cdots, \ x_t$ be commuting variables. An orthogonal design X of order n and type $(s_1, s_2, \cdots, \ s_t)$ denoted $OD(n; s_1, \cdots, \ s_t)$, where s_i are positive integers, is a matrix of order n with entries from $\{0, \pm x_1, \cdots, \ \pm x_t\}$, such that

$$XX^T = (\sum_{i=1}^{t} s_i x_i^2) I_n,$$

where X^T denotes the transpose of X and I_n is the identity matrix of order n.

Alternatively, each row of X has s_i entries of the type $\pm x_i$ and the rows are pairwise-orthogonal under the Euclidean inner product [**?**]. The above description of X applies to the columns of X as well.

Definition 2.2 Let X be an $OD(n; u_1, \cdots, \ u_s)$ on the variables $\{x_1, \cdots, \ x_s\}$ and Y an $OD(n; v_1, \cdots, v_t)$ on the variables $\{y_1, \cdots, y_t\}$. It is said that X and Y are *amicable orthogonal designs* $AODs(n; u_1, \cdots, u_s; v_1, \cdots, v_t)$ if $XY^T = YX^T$.

Two orthogonal designs are called *equivalent*(or A-*equivalent*) if one can be obtained from the other by a sequence of the following operations

1. multiply one row (one column) by -1.
2. swap two rows (columns).
3. rename or negate a variable throughout the design

We note that the operation 3 means that our definition of equivalence is different from the usual Hadamard equivalence(H-*equivalence*) which uses only operations 1 and 2. The discussion of orthogonal design equivalence is very difficult because of the lack of a nice canonical form. It also means that it is quite difficult to decide whether or not two given orthogonal designs of same order are equivalent. The next section discusses the classification of orthogonal designs introduced to construct amicable orthogonal designs.

Amicable orthogonal designs are a useful tool in constructing orthogonal designs, and applicable in space-time block codes [**?**]. The most recent existence or non-existence results about amicable orthogonal designs of order 8 are summarized by Street[?]. The tables of [?] leave a number of undecided cases which can not be excluded by general theorems of non-existence [?].

In [?], Street gave results about the existence of amicable orthogonal designs of order 8. The cases for $AOD(8;1,1,1,2)$ left undecided by Street are given in Table 2.1.

AOD(8;1,1,1,2;1,1,4)	undecided
AOD(8;1,1,1,2;1,2,4)	undecided
AOD(8;1,1,1,2;2,2,2)	undecided
AOD(8;1,1,1,2;2,4)	undecided
AOD(8;1,1,1,2;3,4)	undecided
AOD(8;1,1,1,2;1,5)	undecided
AOD(8;1,1,1,2;1,6)	undecided
AOD(8;1,1,1,2;2,5)	undecided
AOD(8;1,1,1,2;6)	undecided
AOD(8;1,1,1,2;7)	undecided

Table 2.1 unsolved amicable pairs of AOD(8;1,1,1,2)

In section 4, we show some of these undecided amicable pairs do in fact exist. We need to point out that in Table 1 [?], $AOD(8;1,1,1,2;1,2,2,2)$ was indicated to be non-existent but we find this pair does exist after an exhaustive search.

3 Preliminaries

It is easy to check that there are only one equivalence class for $OD(4;1,1,1)$ and

some orthogonal designs $OD(8;1,1,1,2)$ can be normalized as

$$C1:\left(\begin{array}{c|c} OD(4;1,1,1) & OD(4;2) \\ \hline -OD(4;2)^T & OD(4;1,1,1) \end{array}\right).$$

Due to the symmetric structure of C1, $OD(4;1,1,1)$ can be fixed as

$$\begin{pmatrix} a & b & 0 & c \\ -b & a & c & 0 \\ 0 & -c & a & b \\ -c & 0 & -b & a \end{pmatrix},$$

and only the $OD(4;2)$ can be changed without loss of orthogonality. The following two $OD(8;1,1,1,2)$s are all of the form C1 but they are inequivalent.

$$\begin{bmatrix} a & b & 0 & c & d & 0 & d & 0 \\ -b & a & c & 0 & 0 & -d & 0 & -d \\ 0 & -c & a & b & -d & 0 & d & 0 \\ -c & 0 & -b & a & 0 & d & 0 & -d \\ -d & 0 & d & 0 & a & b & 0 & c \\ 0 & d & 0 & -d & -b & a & c & 0 \\ -d & 0 & -d & 0 & 0 & -c & a & b \\ 0 & d & 0 & d & -c & 0 & -b & a \end{bmatrix}; \begin{bmatrix} a & b & 0 & c & 0 & d & 0 & d \\ -b & a & c & 0 & d & 0 & d & 0 \\ 0 & -c & a & b & 0 & d & 0 & -d \\ -c & 0 & -b & a & d & 0 & -d & 0 \\ 0 & -d & 0 & -d & a & b & 0 & c \\ -d & 0 & -d & 0 & -b & a & c & 0 \\ 0 & -d & 0 & d & 0 & -c & a & b \\ -d & 0 & d & 0 & -c & 0 & -b & a \end{bmatrix}$$

Similarly, some orthogonal designs $OD(8;1,1,1,2)$ can be normalized as

$$C2: \left(\begin{array}{c|c} OD(4;1,1,1,1) & OD(4;1) \\ \hline -OD(4;1)^T & OD(4;1,1,1,1)^T \end{array}\right),$$

where $OD(4;1,1,1,1)$ has only one equivalence class and it is easy to check out that all $OD(8;1,1,1,2)$ of form C2 can be normalized to one equivalence class.

We claim that every $OD(8;1,1,1,2)$ is of form C1 or C2 after a sequence of equivalence operations.

4 Amicability of OD(8;1,1,1,2)s

Given an orthogonal design over s variables, we can get new designs of the same order but different types by setting variables equal to one another or zero. The concept of "Equating and Killing variables" was first stated as Lemma 4.4 in [**?**] and we rewrite it as follows.

Lemma 4.1 *Let X be an orthogonal design of type $(u_1, \cdots, u_s)$ on the variables $x_1, \cdots, x_s$ and has a form of*

$$X = A_1 x_1 + \cdots + A_s x_s$$

where $A_i s$ are coefficient matrices for each variable, then
(i) $X' = A_1x_1 + \cdots + A_{i-1}x_{i-1} + (A_i + A_j)x_i + A_{i+1}x_{i+1} + \cdots + A_{j-1}x_{j-1} + A_{j+1}x_{j+1} + \cdots + A_s x_s$ *is an orthogonal design of type* $(u_1, \cdots, u_i + u_j, \cdots, u_s)$ *on variables* $x_1, \cdots, \hat{x_j}, \cdots, x_s$*;*
(ii) $X'' = A_1x_1 + \cdots + A_{j-1}x_{j-1} + A_{j+1}x_{j+1} + \cdots + A_s x_s$ *is an orthogonal design of type* $(u_1, \cdots, u_{j-1}, u_{j+1}, \cdots, u_s)$ *on variables* $x_1, \cdots, \hat{x_j}, \cdots, x_s$*;*

The description of Equating and Killing variables applies as well to amicable orthogonal designs.

Theorem 4.1 *Let* X, Y *be orthogonal designs with same order ,* X'*,* X'' *be orthogonal designs obtained from* X *in Lemma 4.1. If* X *and* Y *are amicable, then so are* X' *and* Y*, and* X'' *and* Y*.*

Proof: X and Y are amicable gives

$$A_i B_\ell^T = B_\ell A_i^T$$

so we have

$$(A_i + A_j)B_\ell^T = A_i B_\ell^T + A_j B_\ell^T = B_\ell A_i^T + B_\ell A_j^T = B_\ell (A_i + A_j)^T$$

So X' and Y are amicable and obviously X'' and Y are amicable. □

Suppose X and Y are amicable orthogonal designs in order n, where X is of type $(u_1, ..., u_s)$ on the variables $x_1, ..., x_s$ and Y is of type $(v_1, ..., v_t)$ on the variables $y_1, ..., y_t$ and the $x_i' s$ and $y_j' s$ are distinct. Let $\{A_i\}_{i=1}^s$ and $\{B_j\}_{j=1}^t$ be the coefficient matrices for X and Y, respectively. The real matrices $\{A_i\}$ and $\{B_j\}$ of order n satisfy the following conditions:

$$\begin{aligned} &A_i * A_j = 0, \ for\ i \neq j, \ B_k * B_\ell = 0, \ for\ k \neq \ell \\ &A_i A_i^T = u_i I_n \ \forall i, \ B_j B_j^T = v_j I_n \ \forall j \\ &A_i A_j^T = -A_j A_i^T \ for\ i \neq j, \ B_k B_\ell^T = -B_\ell B_k^T \ for\ k \neq \ell \\ &A_i B_j^T = B_j A_i^T, \ \forall i, j \end{aligned} \tag{1}$$

where $*$ denotes the Hadamard product. The conditions in (1) are necessary and sufficient for the existence of amicable pairs of order n [**?**]. For each $OD(8; 1, 1, 1, 2)$ from different inequivalence classes, we use those conditions to quicken our search for its amicable pair.

Example 4.1 The $OD(8;1,1,1,2)$ of the form C2 is given below.

$$X = \begin{bmatrix} a & b & c & d & d & 0 & 0 & 0 \\ -b & a & d & -c & 0 & d & 0 & 0 \\ -c & -d & a & b & 0 & 0 & d & 0 \\ -d & c & -b & a & 0 & 0 & 0 & d \\ -d & 0 & 0 & 0 & a & -b & -c & -d \\ 0 & -d & 0 & 0 & b & a & -d & c \\ 0 & 0 & -d & 0 & c & d & a & -b \\ 0 & 0 & 0 & -d & d & -c & b & a \end{bmatrix}$$

Write $X = aA_1 + bA_2 + cA_3 + dA_4$, it is easy to observe that A_1 is an identity matrix and A_2, A_3, A_4 are all skew-symmetric. For (X, Y) to be amicable orthogonal designs, Y must be symmetric and A_iB_js are anti-commuting except for A_1B_js. The patient reader will then discover that position (1,4) in Y is zero. Other relations of positions in Y such as $y_{1,1} = y_{1,8} = -y_{2,2} = -y_{3,3} = y_{4,4} = y_{8,1} = -y_{7,2} = y_{6,3} = y_{3,6} = -y_{2,7} = -y_{5,4} = -y_{4,5} = -y_{5,5} = y_{6,6} = y_{7,7} = -y_{8,8}$ can be easily concluded as well.

By using the above search method, it can be shown that $AOD(8;1,1,1,2;1,2,2,2)$ exists for $OD(8;1,1,1,2)$ of the form C2 (we use $\bar{a}$ for $-a$, $\bar{b}$ for $-b$ and so on).

$$X : \begin{bmatrix} a & b & c & d & d & 0 & 0 & 0 \\ \bar{b} & a & d & \bar{c} & 0 & d & 0 & 0 \\ \bar{c} & \bar{d} & a & b & 0 & 0 & d & 0 \\ \bar{d} & c & \bar{b} & a & 0 & 0 & 0 & d \\ \bar{d} & 0 & 0 & 0 & a & \bar{b} & \bar{c} & \bar{d} \\ 0 & \bar{d} & 0 & 0 & b & a & \bar{d} & c \\ 0 & 0 & \bar{d} & 0 & c & d & a & \bar{b} \\ 0 & 0 & 0 & \bar{d} & d & \bar{c} & b & a \end{bmatrix}; \quad Y : \begin{bmatrix} x & y & w & 0 & z & \bar{w} & y & x \\ y & \bar{x} & 0 & \bar{w} & w & z & \bar{x} & y \\ w & 0 & \bar{x} & y & \bar{y} & x & z & w \\ 0 & \bar{w} & y & x & \bar{x} & \bar{y} & \bar{w} & z \\ z & w & \bar{y} & \bar{x} & \bar{x} & \bar{y} & \bar{w} & 0 \\ \bar{w} & z & x & \bar{y} & \bar{y} & x & 0 & w \\ y & \bar{x} & z & \bar{w} & \bar{w} & 0 & x & \bar{y} \\ x & y & w & z & 0 & w & \bar{y} & \bar{x} \end{bmatrix}$$

Using Theorem 4.1 and knowing existence of $AOD(8;1,1,1,2;1,2,2,2)$, We claim more new amicable pairs listed in Table 4.1. Finally we summarize the existence and non-existence results after a systematic search of all equivalence classes of $OD(8;1,1,1,2)$s by hand.

AOD(8;1,1,1,2;1,2,2,2)	Exist
AOD(8;1,1,1,2;1,1,4)	Not-exist
AOD(8;1,1,1,2;1,2,4)	Exist
AOD(8;1,1,1,2;2,2,2)	Exist
AOD(8;1,1,1,2;2,4)	Exist
AOD(8;1,1,1,2;3,4)	Exist
AOD(8;1,1,1,2;1,5)	Not-exist
AOD(8;1,1,1,2;1,6)	Exist
AOD(8;1,1,1,2;2,5)	Exist
AOD(8;1,1,1,2;6)	Exist
AOD(8;1,1,1,2;7)	Exist

Table 4.1 search results with respect to table 2.1

5 All new amicable pairs

There are many amicable orthogonal designs which can not be constructed by using general theorems. A complete search of exactly which amicable orthogonal design may exist in order 8 has not yet been made. We list all new amicable pairs as a result of our study on equivalent orthogonal designs.

$\underline{AOD(8;1,1,2,2;4,4)}$:

$$X: \begin{bmatrix} a & b & 0 & 0 & c & c & d & d \\ b & \bar{a} & c & d & 0 & d & 0 & \bar{c} \\ 0 & \bar{c} & \bar{a} & c & b & 0 & \bar{d} & d \\ 0 & \bar{d} & \bar{c} & \bar{a} & d & 0 & b & \bar{c} \\ c & 0 & \bar{b} & \bar{d} & \bar{a} & d & \bar{c} & 0 \\ c & \bar{d} & 0 & 0 & \bar{d} & \bar{a} & c & b \\ d & 0 & d & \bar{b} & c & \bar{c} & \bar{a} & 0 \\ d & c & \bar{d} & c & 0 & \bar{b} & 0 & \bar{a} \end{bmatrix}; \quad Y: \begin{bmatrix} x & \bar{y} & x & y & \bar{y} & \bar{y} & x & x \\ y & x & \bar{y} & x & \bar{x} & x & \bar{y} & y \\ \bar{x} & \bar{y} & x & y & y & y & \bar{x} & x \\ \bar{y} & x & y & \bar{x} & \bar{x} & \bar{x} & \bar{y} & y \\ y & \bar{x} & y & \bar{x} & \bar{x} & x & \bar{y} & \bar{y} \\ y & x & y & \bar{x} & x & x & y & y \\ \bar{x} & \bar{y} & \bar{x} & \bar{y} & \bar{y} & y & x & x \\ \bar{x} & y & x & y & \bar{y} & y & x & \bar{x} \end{bmatrix}$$

$\underline{AOD(8;1,1,2,2;1,1,4)}$:

$$X:\begin{bmatrix} a & b & 0 & 0 & \bar{c} & c & d & d \\ b & \bar{a} & 0 & 0 & \bar{c} & \bar{c} & d & \bar{d} \\ 0 & 0 & \bar{a} & b & \bar{d} & d & \bar{c} & \bar{c} \\ 0 & 0 & \bar{b} & \bar{a} & \bar{d} & \bar{d} & \bar{c} & c \\ c & \bar{c} & \bar{d} & \bar{d} & a & b & 0 & 0 \\ c & c & \bar{d} & d & b & \bar{a} & 0 & 0 \\ d & \bar{d} & c & c & 0 & 0 & \bar{a} & b \\ d & d & c & \bar{c} & 0 & 0 & \bar{b} & \bar{a} \end{bmatrix};\ Y:\begin{bmatrix} x & y & 0 & 0 & w & \bar{w} & w & w \\ \bar{y} & x & 0 & 0 & w & w & w & \bar{w} \\ 0 & 0 & \bar{x} & y & w & \bar{w} & \bar{w} & \bar{w} \\ 0 & 0 & y & x & w & w & \bar{w} & w \\ w & \bar{w} & \bar{w} & \bar{w} & y & x & 0 & 0 \\ w & w & \bar{w} & w & \bar{x} & y & 0 & 0 \\ \bar{w} & w & \bar{w} & \bar{w} & 0 & 0 & \bar{y} & x \\ \bar{w} & \bar{w} & \bar{w} & w & 0 & 0 & x & y \end{bmatrix}$$

$\underline{AOD(8;2,2,2,2;2,2,2,2)}$:

$$X:\begin{bmatrix} a & a & b & b & c & d & c & d \\ a & \bar{a} & b & \bar{b} & d & \bar{c} & d & \bar{c} \\ b & b & \bar{a} & \bar{a} & c & d & \bar{c} & \bar{d} \\ b & \bar{b} & \bar{a} & a & d & \bar{c} & \bar{d} & c \\ \bar{d} & \bar{c} & \bar{d} & \bar{c} & b & b & a & a \\ \bar{c} & d & \bar{c} & d & b & \bar{b} & a & \bar{a} \\ \bar{d} & \bar{c} & d & c & a & a & \bar{b} & \bar{b} \\ \bar{c} & d & c & \bar{d} & a & \bar{a} & \bar{b} & b \end{bmatrix};\ Y:\begin{bmatrix} x & x & y & y & w & z & w & z \\ x & \bar{x} & y & \bar{y} & \bar{z} & w & \bar{z} & w \\ \bar{y} & \bar{y} & x & x & w & z & \bar{w} & \bar{z} \\ \bar{y} & y & x & \bar{x} & \bar{z} & w & z & \bar{w} \\ w & z & w & z & \bar{x} & \bar{x} & \bar{y} & \bar{y} \\ \bar{z} & w & \bar{z} & w & \bar{x} & x & \bar{y} & y \\ w & z & \bar{w} & \bar{z} & y & y & \bar{x} & \bar{x} \\ \bar{z} & w & z & \bar{w} & y & \bar{y} & \bar{x} & x \end{bmatrix}$$

$\underline{AOD(8;1,1,1,3;2,3)}$:

$$X:\begin{bmatrix} a & b & c & 0 & d & d & d & 0 \\ \bar{b} & a & 0 & c & d & \bar{d} & 0 & d \\ \bar{c} & 0 & a & \bar{b} & d & 0 & \bar{d} & \bar{d} \\ 0 & \bar{c} & b & a & 0 & d & \bar{d} & d \\ \bar{d} & \bar{d} & \bar{d} & 0 & a & b & c & 0 \\ \bar{d} & d & 0 & \bar{d} & \bar{b} & a & 0 & c \\ \bar{d} & 0 & d & d & \bar{c} & 0 & a & \bar{b} \\ 0 & \bar{d} & d & \bar{d} & 0 & \bar{c} & b & a \end{bmatrix};\ Y:\begin{bmatrix} x & \bar{x} & 0 & 0 & y & y & y & 0 \\ \bar{x} & \bar{x} & 0 & 0 & y & \bar{y} & 0 & y \\ 0 & 0 & \bar{x} & x & y & 0 & \bar{y} & \bar{y} \\ 0 & 0 & x & x & 0 & y & \bar{y} & y \\ y & y & y & 0 & x & \bar{x} & 0 & 0 \\ y & \bar{y} & 0 & y & \bar{x} & \bar{x} & 0 & 0 \\ y & 0 & \bar{y} & \bar{y} & 0 & 0 & \bar{x} & x \\ 0 & y & \bar{y} & y & 0 & 0 & x & x \end{bmatrix}$$

$\underline{AOD(8;1,1,1,5;4)}$ is given below:

$$X:\begin{bmatrix} a & b & c & d & d & d & d & d \\ b & \bar{a} & d & \bar{c} & d & \bar{d} & \bar{d} & d \\ c & \bar{d} & \bar{a} & b & d & d & \bar{d} & \bar{d} \\ d & c & \bar{b} & \bar{a} & d & \bar{d} & d & \bar{d} \\ d & \bar{d} & \bar{d} & \bar{d} & \bar{a} & b & c & d \\ d & d & \bar{d} & d & \bar{b} & \bar{a} & \bar{d} & c \\ d & d & d & \bar{d} & \bar{c} & d & \bar{a} & \bar{b} \\ d & \bar{d} & d & d & \bar{d} & \bar{c} & b & \bar{a} \end{bmatrix};\ Y:\begin{bmatrix} x & \bar{x} & x & 0 & \bar{x} & 0 & 0 & 0 \\ x & x & 0 & \bar{x} & 0 & x & 0 & 0 \\ \bar{x} & 0 & x & \bar{x} & 0 & 0 & x & 0 \\ 0 & \bar{x} & \bar{x} & \bar{x} & 0 & 0 & 0 & x \\ x & 0 & 0 & 0 & x & \bar{x} & x & 0 \\ 0 & x & 0 & 0 & \bar{x} & \bar{x} & 0 & x \\ 0 & 0 & x & 0 & x & 0 & \bar{x} & x \\ 0 & 0 & 0 & x & 0 & x & x & x \end{bmatrix}$$

$\underline{AOD(8;1,1,1,5;6)}$:

$$X:\begin{bmatrix} a & b & c & d & d & d & d & d \\ b & \bar{a} & d & \bar{c} & d & \bar{d} & \bar{d} & d \\ c & \bar{d} & \bar{a} & b & d & d & \bar{d} & \bar{d} \\ d & c & \bar{b} & \bar{a} & d & \bar{d} & d & \bar{d} \\ d & \bar{d} & \bar{d} & \bar{d} & \bar{a} & b & c & d \\ d & d & \bar{d} & d & \bar{b} & \bar{a} & \bar{d} & c \\ d & d & d & \bar{d} & \bar{c} & d & \bar{a} & \bar{b} \\ d & \bar{d} & d & d & \bar{d} & \bar{c} & b & \bar{a} \end{bmatrix};\ Y:\begin{bmatrix} x & \bar{x} & x & 0 & 0 & x & x & x \\ x & x & 0 & \bar{x} & x & 0 & \bar{x} & x \\ \bar{x} & 0 & x & \bar{x} & x & x & 0 & \bar{x} \\ 0 & \bar{x} & \bar{x} & \bar{x} & x & \bar{x} & x & 0 \\ 0 & x & x & x & x & \bar{x} & x & 0 \\ \bar{x} & 0 & x & \bar{x} & \bar{x} & \bar{x} & 0 & x \\ \bar{x} & \bar{x} & 0 & x & x & 0 & \bar{x} & x \\ \bar{x} & x & \bar{x} & 0 & 0 & x & x & x \end{bmatrix}$$

$\underline{AOD(8;1,1,3,3;6)}$:

$$X:\begin{bmatrix} a & a & a & b & b & b & c & d \\ a & \bar{a} & b & \bar{a} & b & \bar{b} & d & \bar{c} \\ a & \bar{b} & \bar{a} & a & d & c & \bar{b} & \bar{b} \\ b & a & \bar{a} & \bar{a} & c & \bar{d} & \bar{b} & b \\ b & \bar{b} & \bar{d} & \bar{c} & \bar{a} & a & b & a \\ b & b & \bar{c} & d & \bar{a} & \bar{a} & a & \bar{b} \\ c & \bar{d} & b & b & \bar{b} & \bar{a} & \bar{a} & a \\ d & c & b & \bar{b} & \bar{a} & b & \bar{a} & \bar{a} \end{bmatrix};\ Y:\begin{bmatrix} x & 0 & 0 & x & x & x & \bar{x} & x \\ 0 & \bar{x} & x & 0 & x & \bar{x} & \bar{x} & \bar{x} \\ 0 & x & x & 0 & x & \bar{x} & x & x \\ \bar{x} & 0 & 0 & x & x & x & x & \bar{x} \\ \bar{x} & x & x & x & \bar{x} & 0 & \bar{x} & 0 \\ \bar{x} & \bar{x} & \bar{x} & x & 0 & \bar{x} & 0 & x \\ x & \bar{x} & x & x & \bar{x} & 0 & x & 0 \\ \bar{x} & \bar{x} & x & \bar{x} & 0 & x & 0 & x \end{bmatrix}$$

$\underline{AOD(8;1,1,2,3;4)}$:

$$X:\begin{bmatrix} a & b & c & c & 0 & d & d & d \\ b & \bar{a} & \bar{c} & c & \bar{d} & 0 & \bar{d} & d \\ c & c & \bar{a} & \bar{b} & \bar{d} & \bar{d} & d & 0 \\ c & \bar{c} & b & \bar{a} & \bar{d} & d & 0 & \bar{d} \\ 0 & \bar{d} & \bar{d} & \bar{d} & a & b & c & c \\ d & 0 & d & \bar{d} & b & \bar{a} & \bar{c} & c \\ d & d & \bar{d} & 0 & c & c & \bar{a} & \bar{b} \\ d & \bar{d} & 0 & d & c & \bar{c} & b & \bar{a} \end{bmatrix};\ Y:\begin{bmatrix} x & 0 & 0 & 0 & 0 & x & x & x \\ 0 & x & 0 & 0 & \bar{x} & 0 & \bar{x} & x \\ 0 & 0 & 0 & x & \bar{x} & \bar{x} & x & 0 \\ 0 & 0 & x & 0 & \bar{x} & x & 0 & \bar{x} \\ 0 & x & x & x & x & 0 & 0 & 0 \\ \bar{x} & 0 & \bar{x} & x & 0 & x & 0 & 0 \\ \bar{x} & \bar{x} & x & 0 & 0 & 0 & 0 & x \\ \bar{x} & x & 0 & \bar{x} & 0 & 0 & x & 0 \end{bmatrix}$$

6 Conclusion

It has been believed that equivalence of orthogonal design is an area worthy of study. In this work, we find some new amicable orthogonal designs of order 8 by searching all equivalence classes of an orthogonal design with certain type.

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