Critical Sets in Orthogonal Arrays

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Abstract

A critical set consists of the minimum amount of information needed to recreate combinatorial structures uniquely. A *minimal* critical set is a critical set of minimum cardinality. In this paper we consider orthogonal arrays constructed from Mutually Orthogonal Latin Squares (MOLS) and obtain bounds on the possible sizes of the *minimal* critical sets. If there exists a set of k MOLS of order n and each Latin Square has a *minimal* critical set of size atmost c_l, then we show that there is a minimal critical set of size c_{OA}, of the corresponding $OA(n^2, k+2, n, 2)$ satisfying $c_{OA} \leq kc_l$. This bound is shown to be exact for $n = 3$ by actual construction of a *minimal* critical set. However, for $n \geq 4$ this bound can be improved upon. Using the fact that for $n = 4$ the Latin Square representing the elementary abelian 2-group has a *minimal* critical set of size 5, and only this form or its isotopic has orthogonal mates, we exhibit a *minimal* critical set of size 11 for an $OA(16, 5, 4, 2)$. For $n > 4, n$ odd, we consider $OA(n^2, k+2, n, 2)$ made from a back circulant Latin Square which is a particular Latin Square having initial row in the standard form and subsequent rows formed by translating the previous row one element to the left. We prove by actual construction, that if the minimal critical set for a back circulant latin square is of size c_l, then any of its orthogonal mate can be uniquely reconstructed given information on $c_l - 1$ cells only. Interestingly enough, for $n = 5$, starting with a minimal critical set of size 6, we construct a minimal critical set of size 18 for a complete set of MOLS and the corresponding orthogonal array. Identification of a critical set for $n = 7$ is also discussed.

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