SOME GOOD ORTHOGONAL BIPOLAR SPREADING SEQUENCES OF LENGTHS 12 AND 20

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Abstract: *We present an evaluation, from the viewpoint of DS CDMA applications, of the sets of 12-chip spreading sequences based on Paley's matrix, and sequences of length 20 based on Golay complementary sequences of length 10.*

1. Introduction

Orthogonal spreading sequences are used in direct sequence code division multiple access (DS CDMA) systems for channel separation and to provide a spreading gain, e.g. [1]. The most popular class of such spreading sequences are the sets of Walsh-Hadamard sequences [2], which are easy to generate. However, the cross-correlation between two Walsh-Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them. Moreover, due to their very regular structure, Walsh-Hadamard sequences are characterized with very poor auto-correlation properties. Another important drawback of using Walsh-Hadamard sequences or modified Walsh-Hadamard sequences [3] is the fact that sequence length must be equal to the integer power of 2. This is not the case if orthogonal sequences based on Paley's matrices [4], or on complementary Golay sequences [5] are used instead of Walsh-Hadamard sequences.

In the paper, we present construction and the important correlation characteristics of the orthogonal bipolar spreading sequences of lengths 12 and 20. The orthogonal bipolar spreading sequences of the lengths not being powers of 2 can be very useful when different spreading ratios are required but the orthogonality must be preserved.

The paper is organised as follows. Section 2 describes construction techniques used to design Hadamard matrices based on complementary Golay sequences and also present a Hadamard matrix of order 12 obtained by the use of Paley's construction. In Section 3, we describe the sequence modification method, and show the correlation parameters for some sequence sets of lengths 12 and 20. Section 5 concludes the paper.

2. Construction of Hadamard Matrices of orders 20 and 12

In [4], Lipski and Marek indicated that there is a common expectation that bipolar orthogonal (Hadamard) matrices exist for any $n \equiv 0 \pmod 4$, with the first doubtful case being $n = 428$. In this Section, we introduce two different constructions allowing us to design Hadamard matrices for some values n not being power of 2 but $n \equiv 0 \pmod 4$.

The first such a construction is based on the use of Golay complementary sequences. For a pair of Golay complementary sequences S_1, and S_2, the sum of their aperiodic autocorrelation functions equals to zero, except for the zero shift [6]:

$$c_{S_1}(\tau) + c_{S_2}(\tau) = 0 \qquad \tau \neq 0 \tag{1}$$

where $c_{S_1}(\tau)$ and $c_{S_2}(\tau)$ denote the aperiodic autocorrelation functions [6].

It can be proven [5] that if matrices **A** and **B** are the circulant matrices created from a pair of Golay complementary sequences S_1, and S_2 then the matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathbf{T}} & -\mathbf{A}^{\mathbf{T}} \end{bmatrix} \tag{2}$$

is a Hadamard matrix. The matrix **G** can be used to generate spreading sequences for DS CDMA applications.

In [7] there are two pairs of Golay complementary sequences of length 10 that can be used to generate orthogonal spreading sequences of length 20. These are:

$S_{\text{10-1-A}}$= [+++++−+−−+] $S_{\text{10-1-B}}$= [++−−+++−+−]

$S_{\text{10-2-A}}$= [++−+−+−−++] $S_{\text{10-2-B}}$= [++−+++++−−]

Both pairs can be used to generate sets of orthogonal spreading sequences using formula (2), and the matrix $\mathbf{G}_{20\text{-}1}$ obtained using the first pair is given by:

$$\mathbf{G}_{20-1} = \left[\begin{array}{c}
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\end{array}\right]$$

Another technique to create Hadamard matrices for values n not being power of 2, but $n \equiv 0 \pmod 4$, is based on second Paley's construction technique described in details in [4]. Here, we present a matrix of order 12 obtained using this method.

$$\mathbf{P}_{12} = \begin{bmatrix} + & + & + & + & + & + & - & + & + & + & + & + \\ + & + & + & - & - & + & + & - & + & - & - & + \\ + & + & + & + & - & - & + & + & - & + & - & - \\ + & - & + & + & + & - & + & - & + & - & + & - \\ + & - & - & + & + & + & + & - & - & + & - & + \\ + & + & - & - & + & + & + & + & - & - & + & - \\ - & + & + & + & + & + & - & - & - & - & - & - \\ + & - & + & - & - & + & - & - & - & + & + & - \\ + & + & - & + & - & - & - & - & - & - & + & + \\ + & - & + & - & + & - & - & + & - & - & - & + \\ + & - & - & + & - & + & - & + & + & - & - & - \\ + & + & - & - & + & - & - & - & + & + & - & - \end{bmatrix}$$

3. Sequence Modification Method

The correlation characteristics of the spreading sequences defined by the matrices $\mathbf{G}_{20\text{-}1}$ and $\mathbf{P}_{12}$ are reasonable for such short sequences. Further improvement to the values of correlation parameters of the sequence sets can be obtained using the method introduced in [3].

The modification is achieved by multiplying the original Hadamard matrix ($\mathbf{G}_{20\text{-}1}$, or $\mathbf{P}_{12}$) by a diagonal matrix $\mathbf{D}$ of the respective order and the elements $d_{i,j}$ fulfilling the condition:

$$\left| d_{m,n} \right| = \begin{cases} 0 & \text{for} \quad m \neq n \\ k & \text{for} \quad m = n \end{cases}; \qquad m, n = 1, \ldots, N \tag{3}$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for} \quad m \neq n \\ \exp(j\phi_m) & \text{for} \quad m = n \end{cases}; \qquad m, n = 1, \ldots, N \tag{4}$$

where the phase coefficients ϕ_m; $m = 1, 2, \ldots, N$, are real numbers taking their values from the interval $[0, 2\pi)$. The values of ϕ_m; $m = 1, 2, \ldots, N$, can be optimised to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic cross-correlation functions.

We applied this technique to both of the sequence sets defined by $\mathbf{G}_{20\text{-}1}$ and $\mathbf{P}_{12}$ trying to minimize peaks in the aperiodic cross-correlation functions between any pairs of the sequences. The results are shown in Fig. 1 and Fig. 2. It is clearly visible that after applying the modification method, we achieved significant reduction (about 40%) in the values of peaks in the aperiodic cross correlation functions. Such a reduction translates directly on the reduction in multi-access interference level and reduction in the BER of the DS CDMA system utilising such sequences.

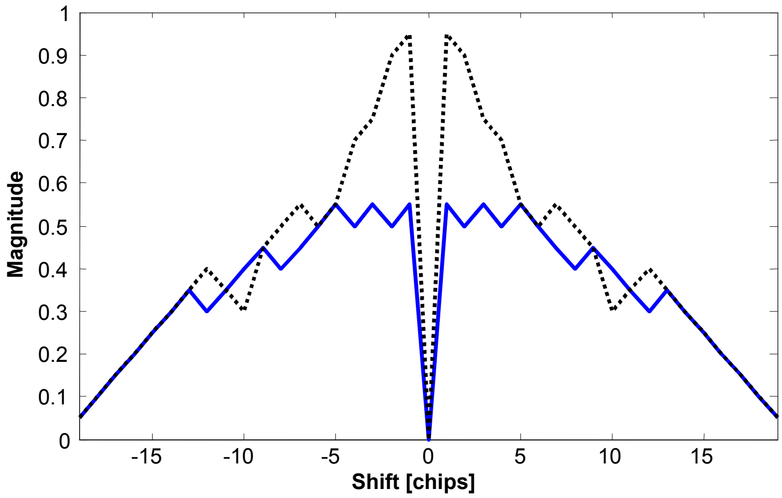

Figure 1: Peak magnitude of aperiodic cross-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix $\mathbf{G}_{20\text{-}1}$, solid line - modified sequences

4. Conclusions

In the paper, we presented two constructions allowing for design of orthogonal spreading sequences of lengths not being necessary the power of 2. We expect that such spreading sequences might be useful for some special applications where different spreading ratios are required. The correlation characteristics of the presented sequences are good compared to the Walsh-Hadamard sequences of length 16.

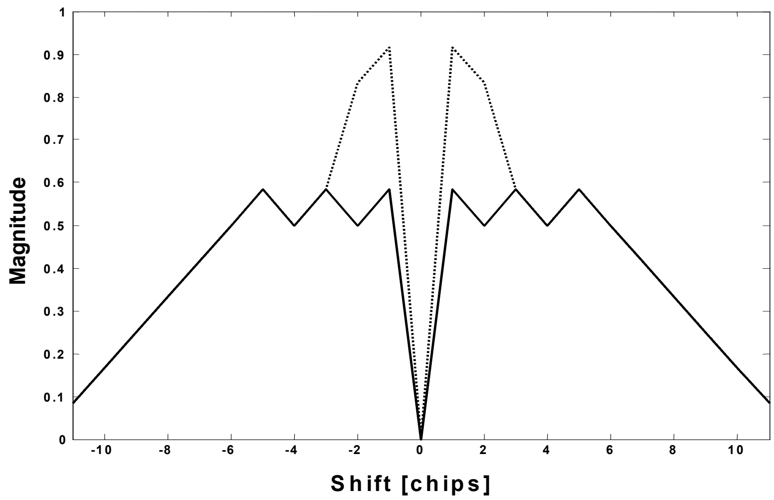

Figure 2: Peak magnitude of aperiodic cross-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Paley matrix $\mathbf{P}_{12}$, solid line - modified sequences

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