Necessary and sufficient conditions for two variable orthogonal designs in order 44: Addendum

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Abstract

In our recent paper *Necessary and sufficient conditions for some two variable orthogonal designs in order* 44, Koukouvinos, Mitrouli and Seberry leave 7 cases unresolved. Using a new algorithm given in our paper *A new algorithm for computer searches for orthogonal designs* by the present four authors we are able to finally resolve all these cases.

This note records that the necessary conditions for the existence of two variable designs constructed using four circulant matrices are sufficient. In particular of 484 potential cases 404 cases have been found, 68 cases do not exist and 12 cases cannot be constructed using four circulant matrices.

Key words and phrases: Autocorrelation, construction, sequence, orthogonal design.

AMS Subject Classification: Primary 05B15, 05B20, Secondary 62K05.

1 Introduction

Throughout this paper we will use the definition and notation of Koukouvinos, Mitrouli, Seberry and Karabelas [4].

We note from [5] that we have to test $\frac{1}{4}n^2 = 484$ cases. This note records that 404 cases have been found, 68 cases do not exist and establish 12 cases cannot be constructed using four circulant matrices.

Theorem 1 **[3]** *An* $OD(44; s_1, s_2)$ *cannot exist for the following* $2-$*tuples* (s_1, s_2)*:*

$(1,7)$	$(3,5)$	$(4,23)$	$(6,10)$	$(8,14)$	$(10,24)$	$(12,20)$	$(15,20)$
$(1,15)$	$(3,13)$	$(4,28)$	$(6,26)$	$(8,30)$	$(11,13)$	$(12,21)$	$(15,25)$
$(1,23)$	$(3,20)$	$(4,31)$	$(7,9)$	$(9,15)$	$(11,16)$	$(12,29)$	$(16,19)$
$(1,28)$	$(3,21)$	$(4,39)$	$(7,16)$	$(9,23)$	$(11,20)$	$(13,19)$	$(16,23)$
$(1,31)$	$(3,29)$	$(5,11)$	$(7,17)$	$(9,28)$	$(11,21)$	$(13,27)$	$(16,28)$
$(1,39)$	$(3,37)$	$(5,12)$	$(7,25)$	$(9,31)$	$(11,29)$	$(14,18)$	$(17,23)$
$(1,42)$	$(3,40)$	$(5,19)$	$(7,28)$	$(10,17)$	$(12,13)$	$(15,16)$	$(19,20)$
$(2,14)$	$(4,7)$	$(5,27)$	$(7,33)$	$(10,22)$	$(12,15)$	$(15,17)$	$(19,21)$
$(2,30)$	$(4,15)$	$(5,35)$	$(7,36)$				

Remark. A computer search, which we believe was exhaustive, was carried out which leads us to believe that

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1. there are no 4-$NPAF(2,41)$ sequences of length 11. This means that there are also no 4-$NPAF(1,2,41)$ sequences of length 11.

2. there are no 4-$NPAF(6,37)$ sequences of length 11.

Lemma 1 **[3]** *The following $OD(44;1,a,43-a)$ and $OD(44;a,43-a)$ cannot be constructed using four circulant matrices in the Goethals-Seidel array:*

$(5,38)$	$(8,35)$	$(12,31)$	$(14,29)$	$(16,27)$	$(20,23)$
$(1,5,38)$	$(1,8,35)$	$(1,12,31)$	$(1,14,29)$	$(1,16,27)$	$(1,20,23)$
$(6,37)$	$(10,33)$	$(13,30)$	$(15,28)$	$(19,24)$	$(21,22)$
$(1,6,37)$	$(1,10,33)$	$(1,13,30)$	$(1,15,28)$	$(1,19,24)$	$(1,21,22)$

2 New Results

Theorem 2 **Main Theorem** *The sequences given herein together with those in [3] can be used to construct the appropriate designs to establish that the necessary conditions for the existence of an $OD(44;s_1,s_2)$ are sufficient, except possibly for the following 12 cases which the Geramita-Verner Theorem and the Sum-Fill Theorem show cannot be constructed from four circulant matrices;*

$(5,38)$	$(6,37)$	$(8,35)$	$(10,33)$	$(12,31)$	$(13,30)$
$(14,29)$	$(15,28)$	$(16,27)$	$(19,24)$	$(20,23)$	$(21,22)$.

Corollary 1 *The necessary conditions for the existence of $OD(44;s_1,s_2)$ constructed from four circulant matrices in the Goethals-Seidel Theorm are sufficient.*

Tuple	Type	Sequences										
(5,37)	NPAF	a	-a	a	-b	a	0	-b	-a	-a	-a	a
		a	a	a	-a	-a	b	a	-a	-a	a	-a
		a	-a	a	a	-a	a	a	a	-a	-a	-a
		a	a	a	b	a	0	-b	a	-a	a	a
(7,32)	NPAF	a	-a	-a	b	-a	0	-a	b	a	a	a
		a	-a	a	-a	-a	a	a	b	0	-a	0
		a	-a	a	a	a	-a	a	-b	0	a	0
		a	a	a	b	a	b	-a	-b	a	a	-a
(8,31)	PAF	b	-a	-a	a	a	-b	a	0	a	a	0
		b	a	-a	-a	-a	b	-a	0	-a	a	0
		a	-a	a	a	-a	-a	a	a	b	a	b
		a	-a	a	-a	-a	a	a	a	b	0	-b
(9,30)	PAF	b	0	-b	-b	-c	-b	b	c	b	b	0
		b	-b	-b	b	b	-b	-b	-b	c	0	-c
		b	-b	-b	-c	c	-b	-b	-b	0	-b	0
		b	-c	b	-b	-b	b	-b	b	-b	-c	-c
(9,33)	PAF	a	-a	-c	a	a	0	a	a	-c	-a	a
		a	a	c	a	-a	0	-a	a	c	a	a
		a	-a	c	a	a	-a	-a	-a	-c	a	-a
		a	a	-c	a	-a	-c	a	-a	c	-a	-a
(1,11,30)	PAF	a	a	-a	-a	-a	a	c	-a	c	c	0
		a	a	-a	-a	-a	c	-a	-c	-a	a	c
		a	a	a	a	-c	c	-c	a	c	c	0
		a	-a	-a	a	-a	a	-a	a	a	-a	-b
(13,29)	PAF	a	a	a	-b	-a	-a	a	-a	-b	-a	0
		a	a	-a	-a	-a	a	-a	-a	a	-a	-b
		a	a	b	b	a	-b	-b	b	-b	b	b
		a	-a	a	-a	-a	-a	b	-a	b	-a	0
(15,26)	PAF	b	b	b	b	-b	-b	-b	b	-b	-b	c
		b	-b	-b	b	b	-b	b	-b	b	-c	0
		b	0	b	-c	c	b	c	b	-c	-c	-c
		b	b	-c	c	b	0	c	c	-c	c	c

Remark. There are 484 possible 2−tuples. Table 1 lists the 404 which correspond to designs which exist in order 44: 68 2-tuples correspond to designs eliminated by number theory (NE). For 12 cases, if the designs exist, they cannot be constructed using circulant matrices (Y).

P indicates that 4-PAF sequences with length 11 exist; n indicates 4-$NPAF$ sequences with length n exist.

s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n
1	1	1	2	10	3	3	21	NE	4	34	10	6	15	7	7	34	P
1	2	1	2	11	5	3	22	7	4	35	11	6	16	7	7	35	P
1	3	1	2	12	5	3	23	7	4	36	10, 11	6	17	7	7	36	NE
1	4	2	2	13	5	3	24	7	4	37	P	6	18	7	7	37	11
1	5	2	2	14	NE	3	25	7	4	38	11	6	19	7	8	8	5
1	6	3	2	15	5	3	26	9	4	39	NE	6	20	7	8	9	5
1	7	NE	2	16	5	3	27	9	4	40	11	6	21	7	8	10	5
1	8	3	2	17	5	3	28	9	5	5	3	6	22	7	8	11	5
1	9	3	2	18	5	3	29	NE	5	6	3	6	23	9	8	12	5
1	10	3	2	19	7	3	30	9	5	7	3	6	24	8	8	13	7
1	11	3	2	20	6	3	31	10	5	8	5	6	25	9	8	14	NE
1	12	4	2	21	7	3	32	9	5	9	5	6	26	NE	8	15	7
1	13	5	2	22	7	3	33	9	5	10	5	6	27	9	8	16	7
1	14	5	2	23	7	3	34	10	5	11	NE	6	28	9	8	17	7
1	15	NE	2	24	7	3	35	11	5	12	NE	6	29	P	8	18	7
1	16	5	2	25	9	3	36	11	5	13	5	6	30	9	8	19	9
1	17	5	2	26	7	3	37	NE	5	14	5	6	31	10	8	20	7
1	18	5	2	27	9	3	38	11	5	15	5	6	32	10	8	21	9
1	19	5	2	28	8	3	39	11	5	16	7	6	33	P, 20	8	22	8
1	20	7	2	29	9	3	40	NE	5	17	7	6	34	10	8	23	9
1	21	7	2	30	NE	3	41	11	5	18	7	6	35	P	8	24	9
1	22	7	2	31	9	4	4	2	5	19	NE	6	36	11	8	25	9
1	23	NE	2	32	9	4	5	3	5	20	7	6	37	Y	8	26	9
1	24	7	2	33	9	4	6	3	5	21	7	6	38	11	8	27	P
1	25	7	2	34	9	4	7	NE	5	22	9	7	7	4	8	28	9
1	26	9	2	35	10	4	8	3	5	23	7	7	8	6	8	29	P
1	27	7	2	36	10, 11	4	9	5	5	24	9	7	9	NE	8	30	NE
1	28	NE	2	37	11	4	10	5	5	25	9	7	10	5	8	31	P
1	29	9	2	38	10, 11	4	11	5	5	26	9	7	11	7	8	32	10
1	30	11	2	39	11	4	12	5	5	27	NE	7	12	7	8	33	P
1	31	NE	2	40	11	4	13	5	5	28	9	7	13	5	8	34	11
1	32	9	2	41	P	4	14	5	5	29	9	7	14	7	8	35	Y
1	33	9	2	42	11	4	15	NE	5	30	10	7	15	7	8	36	11
1	34	11	3	3	2	4	16	5	5	31	9	7	16	NE	9	9	5
1	35	11	3	4	3	4	17	7	5	32	10	7	17	NE	9	10	5
1	36	11	3	5	NE	4	18	7	5	33	10	7	18	7	9	11	5
1	37	11	3	6	3	4	19	7	5	34	P	7	19	8	9	12	7
1	38	11	3	7	3	4	20	7	5	35	NE	7	20	9	9	13	6
1	39	NE	3	8	3	4	21	7	5	36	11	7	21	7	9	14	7
1	40	11	3	9	3	4	22	7	5	37	11	7	22	9	9	15	NE
1	41	11	3	10	5	4	23	NE	5	38	Y	7	23	9	9	16	7
1	42	NE	3	11	5	4	24	7	5	39	11	7	24	9	9	17	7
1	43	11	3	12	5	4	25	9	6	6	3	7	25	NE	9	18	7
2	2	1	3	13	NE	4	26	8	6	7	5	7	26	9	9	19	7
2	3	2	3	14	5	4	27	9	6	8	5	7	27	9	9	20	9
2	4	2	3	15	5	4	28	NE	6	9	5	7	28	NE	9	21	9
2	5	3	3	16	7	4	29	9	6	10	NE	7	29	9	9	22	9
2	6	2	3	17	5	4	30	9	6	11	5	7	30	P	9	23	NE
2	7	3	3	18	7	4	31	NE	6	12	5	7	31	10	9	24	9
2	8	3	3	19	7	4	32	9	6	13	7	7	32	11	9	25	9
2	9	5	3	20	NE	4	33	10	6	14	5	7	33	NE	9	26	9

Table 1: The existence of $OD(44; s_1, s_2)$.

s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n
9	27	9	10	31	P	12	15	NE	13	25	P	15	21	9	17	25	P
9	28	NE	10	32	11	12	16	7	13	26	P	15	22	P	17	26	P
9	29	P	10	33	Y	12	17	9	13	27	NE	15	23	P	17	27	P
9	30	$P, 20$	10	34	11	12	18	8	13	28	P	15	24	P	18	18	9
9	31	NE	11	11	6	12	19	9	13	29	P	15	25	NE	18	19	P
9	32	$P, 15$	11	12	7	12	20	NE	13	30	Y	15	26	P	18	20	10
9	33	$P, 20$	11	13	NE	12	21	NE	13	31	P	15	27	$P, 20$	18	21	P
9	34	P	11	14	7	12	22	9	14	14	7	15	28	Y	18	22	10
9	35	P	11	15	7	12	23	NE	14	15	P	15	29	P	18	23	P
10	10	5	11	16	NE	12	24	9	14	16	8	16	16	8	18	24	11
10	11	7	11	17	7	12	25	P	14	17	P	16	17	9	18	25	P
10	12	7	11	18	9	12	26	P	14	18	NE	16	18	9	18	26	P
10	13	7	11	19	9	12	27	20	14	19	9	16	19	NE	19	19	P
10	14	7	11	20	NE	12	28	10	14	20	9	16	20	9	19	20	NE
10	15	7	11	21	NE	12	29	NE	14	21	9	16	21	11	19	21	NE
10	16	7	11	22	9	12	30	$P, 13$	14	22	9	16	22	10	19	22	P
10	17	NE	11	23	9	12	31	Y	14	23	P	16	23	NE	19	23	P
10	18	7	11	24	9	12	32	11	14	24	P	16	24	10	19	24	Y
10	19	P	11	25	9	13	13	7	14	25	P	16	25	P	19	25	P
10	20	8	11	26	P	13	14	9	14	26	10	16	26	11	20	20	10
10	21	9	11	27	P	13	15	7	14	27	P	16	27	Y	20	21	P
10	22	NE	11	28	P	13	16	10	14	28	$P, 12$	16	28	NE	20	22	11
10	23	9	11	29	NE	13	17	9	14	29	Y	17	17	9	20	23	Y
10	24	NE	11	30	P	13	18	9	14	30	P	17	18	9	20	24	11
10	25	9	11	31	P	13	19	NE	15	15	9	17	19	9	21	21	11
10	26	9	11	32	P	13	20	9	15	16	NE	17	20	11	21	22	Y
10	27	P	11	33	P	13	21	9	15	17	NE	17	21	P	21	23	P
10	28	10	12	12	7	13	22	P	15	18	9	17	22	P	22	22	11
10	29	P	12	13	NE	13	23	9	15	19	9	17	23	NE			
10	30	10	12	14	7	13	24	P	15	20	NE	17	24	P			

Table 1(Cont): The existence of $OD(44; s_1, s_2)$.

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