ELSEVIER

Journal of Statistical Planning and Inference 62 (1997) 91–101

journal of statistical planning and inference

Weighing matrices and their applications

Christos Koukouvinos[a], Jennifer Seberry[b,*,1]

[a] Department of Mathematics, National Technical University of Athens, Zografou 157 73, Athens, Greece

[b] Department of Computer Science, University of Wollongong, Wollongong, NSW 2522, Australia

Received 6 July 1994; revised 9 October 1995

Abstract

Three major applications of weighing matrices are discussed. New weighing matrices and skew weighing matrices are given for many orders $4t \leqslant 100$. We resolve the skew-weighing matrix conjecture in the affirmative for $4t \leqslant 88$.

AMS classification: primary 62K05; 62K10; secondary 05B20

Keywords: Weighing matrix; Weighing experiment; Skew weighing matrix

1. Introduction

Definition 1. Let W be a $(1,-1,0)$-matrix of order n satisfying $WW^{\mathrm{T}} = kI_n$. Then, we call W a *weighing matrix* of order n with weight k, denoted by $W(n,k)$.

There are a number of conjectures concerning weighing matrices.

Conjecture 1. *There exists a weighing matrix* $W(4t,k)$ *for* $k \in \{1,\ldots,4t\}$.

Conjecture 2. *When* $n \equiv 4 \pmod 8$, *there exists a skew-weighing matrix* (*also written as an* $\mathrm{OD}(n;1,k)$) *when* $k \leqslant n-1$, $k = a^2+b^2+c^2, a, b, c$ *integers except that* $n-2$ *must be the sum of two squares.*

Conjecture 3. *When* $n \equiv 0 \pmod 8$, *there exists a skew-weighing matrix* (*also written as an* $\mathrm{OD}(n;1,k)$) *for all* $k \leqslant n-1$.

The reader in referred to Geramita and Seberry (1979) for all other undefined terms.

* Corresponding author.

1 Research funded by ARC grant A48830241.

PII S0378-3758(96)00172-3

In Geramita and Seberry (1979) the status of the weighing matrix conjecture is given for $W(4t,k)$, $k \in \{1,\dots,4t\}$ and $t \in \{1,\dots,21\}$. We give new results including resolving the skew-conjecture in the affirmative for all $4t \leqslant 88$.

2. Hadamard matrices and balanced incomplete block designs

The best-known examples of weighing matrices are *Hadamard matrices* of order $4t$ which are $W(4t,4t)$. These matrices are intimately related to SBIBD as shown by the next theorem. However weighing matrices are also related to designs, especially PBIBD and GDD.

Theorem 4. *The existence of an Hadamard matrix of order* $4t$ *is equivalent to the existence of* SBIB *designs with the parameters,*

(i) $v = b = 4t - 1,\ r = k = 2t - 1,\ \lambda = t - 1,$

(ii) $v = b = 4t - 1,\ r = k = 2t,\ \lambda = t.$

Proof. Let H be a normalized Hadamard matrix of order $4t$ and let A be the core of H. Then $B = \frac{1}{2}(J + A)$, $C = \frac{1}{2}(J - A)$ are incidence matrices of SBIB designs with parameters (i) and (ii), respectively. It is clear that the above processes are reversible. □

Corollary 5. *The existence of an Hadamard matrix of order* $4t$ *implies the existence of* BIB *designs with the parameters,*

(i) $v = 2t - 1,\ b = 4t - 2,\ r = 2t - 2,\ k = t - 1,\ \lambda = t - 2,$

(ii) $v = 2t,\ b = 4t - 2,\ r = 2t - 1,\ k = t,\ \lambda = t - 1,$

(iii) $v = 2t - 1,\ b = 4t - 2,\ r = 2t,\ k = t,\ \lambda = t.$

Proof. Let B be as in the construction of 4. Rearrange (if necessary) rows of B and bring it to the form

$$\begin{bmatrix} \mathbf{1} & D \\ \mathbf{0} & E \end{bmatrix}.$$

Then D and E are the incidence matrices for designs with the parameters in (i) and (ii) above. It can be easily shown that $F = \frac{1}{2}(J - D)$ is the incidence matrix for a design with the parameters in (iii). □

3. Weighing matrices and optimal weighing designs

Suppose we are given p objects to be weighed in n weighings with a chemical balance (two-pan balance) having no bias. Let

$x_{ij} = 1$ if the jth object is placed in the left pan in the ith weighing,

$x_{ij} = -1$ if the jth object is placed in the right pan in the ith weighing,

$x_{ij} = 0$ if the jth object is not weighed in the ith weighing.

Then the $n \times p$ matrix $X = (x_{ij})$ completely characterizes the weighing experiment.

Let us write $w_1, w_2, \ldots, w_p$ for the true weights of the p objects, and $y_1, y_2, \ldots, y_n$ for the results of n weighings (so that the readings indicate that the weight of the left pan exceeds that of the right pan by y_i in the weighing of i), denote the column vectors of w's and y's by W and Y, respectively.

Then the readings can be represented by the linear model

$$Y = XW + e,$$

where e is the column vector of $e_1, e_2, \ldots, e_n$ and e_i is the error between observed and expected readings. We assume that e is a random vector distributed with mean zero and covariance matrix $\sigma^2 I$. This is a reasonable assumption in the case where the objects to be weighed have small mass compared to the mass of the balance.

We assume X to be a non-singular matrix. Then the best-linear unbiased estimator of W is

$$\hat{W} = (X'X)^{-1}X'Y$$

with covariance of $\hat{W}$

$$\mathrm{Cov}(\hat{W}) = \sigma^2(X'X)^{-1}.$$

It has been shown by Hotelling (1944) that for any weighing design the variance of $\hat{w}_i$ cannot be less than σ^2/n. Therefore, we shall call a weighing design X optimal if it estimates each of the weights with this minimum variance, σ^2/n. Kiefer (1975) proved that an optimal weighing design in our sense is actually optimal with respect to a very general class of criteria. It can be shown that X is optimal if and only if $X'X = nI$. This means that a chemical balance weighing design X is optimal if it is an $n \times p$ matrix of ± 1 whose columns are orthogonal. Thus we have:

Theorem 6. *Any p ($\leqslant n$) columns of an Hadamard matrix of order n constitute an $n \times p$ optimal chemical balance weighing design.*

An optimal weighing design is clearly A-, D-, and E-optimal in the following sense. A weighing design X is said to be A-optimal if the trace of $(X'X)^{-1}$ is minimum, D-optimal if the determinant of $(X'X)^{-1}$ is minimum, and E-optimal if the maximum eigenvalue of $(X'X)^{-1}$ is minimum among the class of all $n \times p$ weighing designs and for the model of response specified above. If the balance has not been corrected for the bias we may assume the bias to be one of the p objects, say the first one. Hadamard matrices can also be used for weighing objects using a spring balance (a one-pan balance). The spring balance problem is different from the chemical balance problem in that the elements x_{ij} of the matrix X are restricted to the values of 0 and 1. In this case no design exists with $\mathrm{Var}(\hat{w}_i) = \sigma^2/n$ because $X'X$ never becomes nI. However, a D-optimal spring balance design can be constructed for $p = n - 1$ objects using $n - 1$ measurements if an Hadamard matrix of order $4n$ exists.

The procedure is easy. Let $X = C$, the incidence matrix of the BIB design in (ii) of 4. In this case the variance of each estimated weight is $4(n-1)\sigma^2/n$. Thus we have:

Theorem 7. *The existence of an Hadamard matrix of order n implies the existence of a saturated D-optimal spring balance design for $n-1$ objects using $n-1$ weighings.*

The weighing design

$$\begin{bmatrix} 1 & \mathbf{1}' \\ \mathbf{1} & J-C \end{bmatrix}$$

can be used as a biased spring balance design, where the bias corresponds to the first object. With such a design the variance of estimated weights of objects $2,3,\ldots,n$ will be $4\sigma^2/n$. This is the minimum possible variance as has been shown by Moriguti (1954). Therefore we have:

Theorem 8. *The existence of an Hadamard matrix of order n implies the existence of a saturated A-optimal biased spring balance design for $n-1$ objects using n weighings.*

In Theorems 7 and 8 the term saturated implies that in the usual analysis of variance no degrees of freedom will be left for the errors. For a more detailed study of optimal weighing designs, the reader should consult Mood (1946), Raghavarao (1959, 1960, 1971) and Banerjee (1975).

It is sometimes not possible to weigh all the objects simultaneously in one weighing. Accordingly, one asks about weighing designs in which exactly k objects are weighed each time. In the case of a design with as many weighings as objects, so that the matrix is square, one asks for a matrix X of order p with exactly k entries ± 1 per row and the rest zero, such that $X'X$ is diagonal. In the case of an unbiased balance it is best to have exactly k entries ± 1 per column, so one seeks a matrix X with entries $0, -1$ and 1, which satisfies

$$X'X = kI.$$

It has been conjectured, see J. (Seberry) Wallis (1972), that such a matrix exists whenever $0 \leqslant k \leqslant p \equiv 0 \bmod 4$. Considerable work has been done on this conjecture recently. For a survey of the results see (Eades and Seberry Wallis (1976), Geramita et al. (1974), Geramita and Seberry (1979), Koukouvinos (1993), Koukouvinos and Seberry (1993a, b) and Seberry (1980, 1982).

4. Weighing designs and optical multiplexing

The connection between weighing designs and multiplexing optics is now straightforward. In the optical case the unknowns w_i represent intensities of individual spatial and/or spectral elements in a beam of radiation. In contrast to scanning instruments which measure the intensities one at a time, the multiplexing optical system measures (i.e. weighs) several intensities (or w_i's) simultaneously. The y_i's now represent the readings of the detector (instead of the reading of the balance). Finally, the weighing design itself, X, is represented by a mask. More precisely, one row of X, which specifies which objects are present in a single weighing, corresponds to the row of transmitting, absorbing or reflecting elements. We usually refer to such a row as a mask configuration.

The two types of weighing designs – chemical and spring balance designs – are realized by masks which contain either transmitting, absorbing and reflecting elements (for the chemical balance design) or simply open and close slots (for the spring balance design). Note that the former case requires two detectors, whereas in the latter case the reference detector can be omitted. In Hadamard transform spectrometry the separated light is sent to a mask. Various parts of the mask will be clear, allowing the light to pass through, reflective (sending light to a secondary detector), or opaque. Let us represent clear, reflective and opaque by $1, -1$ and 0 respectively. Then the configuration of the mask is represented by a sequence of elements $1, -1$ and 0.

Suppose k measurements are to be made, and suppose it is convenient to measure the intensity of light at n points of the spectrum. Then the experiment will involve k masks, which can be thought of as $n \times k$ matrix of entries $1, 0$, and -1. The efficiency of the experiment is the same as the efficiency of the matrix as a weighing design. The best systems of mask are thus derived from weighing matrices.

In closing this section it should be mentioned that the designs used in the weighing problems are applicable to any problem of measurements, in which the measure of a combination is a known linear function of the separate measures with numerically equal coefficient and a homoscedastic error term; see Sloane and Harwit (1975). We refer the reader to the book of Harwit and Sloane (1979) for more details and other applications of weighing matrices.

5. Constructions for some new sequences with zero autocorrelation

The constructions of this paper are based on formulations of a theorem of Goethals and Seidel which may be used in the following form.

Theorem 9 (Geramita and Seberry, 1975, Theorem 4.49). *If there exist four circulant matrices A_1, A_2, A_3, A_4 of order n satisfying*

$$\sum_{i=1}^{4} A_i A_i^{\mathrm{T}} = fI,$$

where f is the quadratic form $\sum_{j=1}^{u} s_j x_j^2$, then there is an orthogonal design $\mathrm{OD}(4n; s_1, s_2, \ldots, s_u)$.

Corollary 10. *If there are four* $\{0, \pm 1\}$*-sequences of length n and weight w with zero periodic or non-periodic autocorrelation function then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals–Seidel array to form* $\mathrm{OD}(4n; w)$ *or a* $W(4n, w)$. *If one of the sequences is skew-type then they can be used similarly to make an* $\mathrm{OD}(4n; 1, w)$. *We note that if there are sequences of length n with zero non-periodic autocorrelation function then there are sequences of length* $n + m$ *for all* $m \geqslant 0$.

Theorem 11. *Let X and Y be two sequences of length g with zero non-periodic autocorrelation function which can be used to form an* $\mathrm{OD}(2m; a, b)$, $m \geqslant g$. *Then the following four sequences of length* $n \geqslant 4g + 3$ *and zero non-periodic autocorrelation function can be used to form an* $\mathrm{OD}(4n; 1, 4, 8u, 8v)$:

$$X\ 0\ \bar{Y}^*\ a\ Y\ 0\ \bar{X}^*,\quad X\ 0\ \bar{Y}^*\ 0\ \bar{Y}\ 0\ X^*,\quad Y\ b\ X^*\ 0\ X\ \bar{b}\ Y^*,$$
$$Y\ b\ X^*\ 0\ \bar{X}\ b\ \bar{Y}^*.$$

Corollary 12. *There exist*
(i) $\mathrm{OD}(4n; 1, 4, 8, 32)$, $n \geqslant 15$,
(ii) $\mathrm{OD}(4n; 1, 4, 32, 32)$, $n \geqslant 19$,
(iii) $\mathrm{OD}(4n; 1, 4, 16, 64)$, $n \geqslant 27$,
(iv) $\mathrm{OD}(4n; 1, 4, 80, 80)$, $n \geqslant 43$.

Theorem 13. *Let X and Y be two sequences of length g with zero non-periodic autocorrelation function zero which can be used to form an* $\mathrm{OD}(2m; u, v)$, $m \geqslant g$. *Let A and B be two sequences of length h with zero non-periodic autocorrelation function zero which can be used to form an* $\mathrm{OD}(2n; a, b)$, $n \geqslant h$. *Then the following four sequences of length* $\ell \geqslant g + h$ *and zero non-periodic autocorrelation function can be used to form an* $\mathrm{OD}(4n; 2a, 2b, 2u, 2v)$:

$$X\ A,\quad X\ \bar{A},\quad Y\ B,\quad Y\ \bar{B}.$$

Corollary 14. *There exist*
(i) $\mathrm{OD}(4n; 40, 40, 10, 10)$, $n \geqslant 26$,
(ii) $\mathrm{OD}(4n; 26, 26, 20, 20)$, $n \geqslant 24$,
(iii) $\mathrm{OD}(4n; 32, 32, 10, 10)$, $n \geqslant 22$,
(iv) $\mathrm{OD}(4n; 26, 26, 10, 10)$, $n \geqslant 20$.

6. Some new sequences with zero autocorrelation

In this section we give new sequences which can be used in Corollary 10 to form the designs indicated in the first column (see Tables 1–3).

Table 1
Sequences of length 21 with zero periodic autocorrelation function

Length = 21	Sequences with zero periodic autocorrelation function
1,70	{− + 0 − − + − − − − *a* + + + + − + + 0 − +},
	{− + 0 + + − + + + + 0 − − + + − + + 0 − +},
	{− − 0 + + 0 + − + + 0 + − − − 0 + − + + +},
	{+ + 0 + + 0 + − + + 0 + − + + 0 − + − − −}
1,78	{− + + + + + − + − − *a* + + − + − − − − − +},
	{+ − − + − + − + − − 0 + + − + + − − + + +},
	{+ 0 + + − + + + + + 0 + − − + − + + + − −},
	{− 0 − + + + − + + + 0 + + − + + − − − + +}

Table 2
Sequences of length 23 with zero periodic autocorrelation function

Length = 23	Sequences with zero periodic autocorrelation function
1,91	{+ + − + − + + − − − − *a* + + + + − − + − + − −},
	{+ + − − − + − + + + − + + + + + + + − + + + +},
	{+ + − + + + − + + + − − − − + − − − + + + + −},
	{− + − + − − + − − + + + + − − − + + − + + − +}
1,1,90	{+ − + + + − − − + − − *a* + + − + + + − − − + −},
	{+ − − − − − + + − − − *b* + + + − − + + + + + −},
	{+ − − + + − + + − + − + + − + − + + − + + − −},
	{+ + − + − − − − + − − − − − − + − − − − + − +}

Table 3
Sequences of length 27 with zero periodic autocorrelation function

Length = 27	Sequences with zero periodic autocorrelation function
1,90	{+ + + + − + + + − + − 0 0 *a* 0 0 + − + − − − + − − − −},
	{− + − − − − + − − + + 0 0 0 0 0 − − + + + + + + − − +},
	{+ + − − + + − − − − + 0 0 0 0 − − − − + + + − − + + −},
	{+ + + − + − + + − + + 0 0 0 0 + + − + + − + − + − + +},
1,1,104	{ *a* + + + + − + + − − + − + − + − + − + + − − + − − − −},
	{ 0 + + + + − + + − − + − + + + + − + − − + + − + + + +},
	{ *b* + + + − − + + + − + − − + − + + − + − − − + + − − −},
	{ 0 + + + − − + + + − + − − − − − − + − + + + − − + + +},

7. Numerical consequences

We use the tables of Appendix H and extend Theorem 4.149 of Geramita and Seberry (1979).

We note that sequences with periodic autocorrelation function zero exist for $W(4t, 4t-1)$, $W(4t, 4t)$ for all $t \in \{1, \ldots, 31\}$ Seberry and Yamada (1992). We use the (OD; 1, 1, 64) for $n \geqslant 17$ and the (OD; 1, 1, 80) for $n \geqslant 21$ from Koukouvinos and Seberry (1993b). Hence, using the results of Koukouvinos (1993, 1994) and Koukouvinos and

Seberry (1993a, b, to appear) and those given in Tables 1–3, we have the following two theorems.

Theorem 15. *There exists an orthogonal design* OD($4n; 1, k$) *when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9$ *with* $k \in \{x\colon x \leq 4t - 1,\ x = a^2 + b^2 + c^2\}$,

(ii) *for* $n \geq 11$, *with* $k \in \{x\colon x \leq 43,\ x = a^2 + b^2 + c^2,\ x \neq 42\}$,

(iii) *for* $n \geq t$, $t = 13, 15$ *with* $k \in \{x\colon x \leq 4t - 1,\ x = a^2 + b^2 + c^2\}$,

(iv) *for* $n \geq 17$, *with* $k \in \{x\colon x \leq 67,\ x = a^2 + b^2 + c^2,\ x \neq 61, 66\}$,

(v) *for* $n \geq 19$, *with* $k \in \{x\colon x \leq 75,\ x = a^2 + b^2 + c^2,\ x \neq 61\}$,

(vi) *for* $n \geq 21$, *with* $k \in \{x\colon x \leq 83,\ x = a^2 + b^2 + c^2,\ x \neq 61, 77, 78, 82\}$,

(vii) *for* $n \geq 23$, *with* $k \in \{x\colon x \leq 91,\ x = a^2+b^2+c^2,\ x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91\}$,

(viii) *for* $n \geq 25$, *with* $k \in \{x\colon x \leq 99,\ x = a^2+b^2+c^2,\ x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91, 93, 94, 97, 98, 99\}$.

All are constructed by using four circulant matrices in the Goethals–Seidel array.

Remark 1. We note that 79, 87, 92, 95, 103 are of the form $4^a(8b + 7)$ and so none of these numbers is the sum of three squares and hence an OD($4n; 1, k$), n odd, cannot exist for these values of k.

Theorem 16. *There exists a* $W(4n; k)$ *when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9, 11, 13, 15, 17, 19$, *with* $k \in \{x\colon x \leq 4t\}$,

(ii) *for* $n \geq 21$, *with* $k = 1, \ldots, 78, 80, 81, 82$,

(iii) *for* $n \geq 23$, *with* $k = 1, \ldots, 78, 80, \ldots, 86, 88, 89, 90, 92$.

All are constructed by using four circulant matrices in the Goethals–Seidel array.

Lemma 17. *There exists an* OD($84; 1, k$) *for* $k \in \{x\colon x \leq 83,\ x = a^2 + b^2 + c^2\}$. *All may be constructed using four circulant matrices in the Goethals–Seidel array.*

Proof. Use the results of Koukouvinos and Seberry (to appear) and Table 1. □

Lemma 18. *The necessary conditions are sufficient for the existence of* OD($4n; 1, k$) *for* $n = 3, 5, \ldots, 21$ *and* $k \leq 4n - 1$. *All are constructed from four circulant matrices in the Goethals–Seidel array.*

Proof. Use the results of Koukouvinos and Seberry (to appear) and the previous lemma. □

Lemma 19. *There exists a* $W(4n, k)$ *for* $k \in \{x\colon 0 \leq x \leq 4n\}$ *with* $n = 1, 3, \ldots, 21$. *All are constructed from four circulant matrices in the Goethals–Seidel array.*

Lemma 20. *There exists an* OD($92; 1, k$) *for all* k *where* $k \in \{x\colon x \leq 91,\ x = a^2 + b^2 + c^2\}$, *except possibly for* $k \in \{61, 77, 78, 82, 85, 86, 89\}$ *which are undecided.*

Table 4
Summary of the conjectures

Order	Applicable conjecture	Unresolved cases	Applicable conjecture	Unresolved cases
4	1	True	2	True
8	1	True	3	True
12	1	True	2	True
16	1	True	3	True
20	1	True	2	True
24	1	True	3	True
28	1	True	2	True
32	1	True	3	True
36	1	True	2	True
40	1	True	3	True
44	1	True	2	True[a]
48	1	True	3	True
52	1	True	2	True
56	1	True	3	True
60	1	True	2	True
64	1	True	3	True
68	1	True	2	True[a]
72	1	True	3	True
76	1	True	2	True
80	1	True	3	True
84	1	True	2	True
88	1	True	3	True
92	1	79, 87	2	61, 77, 78, 82, 85, 86, 89
96	1	True	3	True
100	1	79, 87, 91, 93, 95	2	61, 77, 78, 82, 85, 86, 89,90,91,93,94,97,98
104	1	95	3	94, 95
108	1	79, 87, 93, 95, 99 101, 103 3	2	61, 77, 78, 82, 85, 86, 89, 91, 93, 94, 97, 98, 99, 100, 101, 102, 106
112	1	True	3	True
120	1	True	3	True

Note: True signifies the conjecture is verified.
[a] $OD(n; 1, n-2)$ is not possible as $n-2$ is not the sum of two squares.

Proof. The results for $k = 90$ and 91 are given in Table 2. The remaining results are given in Koukouvinos (1994). □

Lemma 21. *There exists a* $W(92, k)$ *for all* k *except possibly* 79, 87, *which are undecided. All are constructed by using four circulant matrices in the Goethals–Seidel array.*

Lemma 22. *There exists an* $OD(100; 1, k)$ *for all* k *where* $k \in \{x\colon x \leqslant 99,\ x = a^2 + b^2 + c^2\}$, *except possibly for* $k \in \{61, 77, 78, 82, 85, 86, 89, 90, 91, 93, 94, 97, 98\}$ *which are undecided.*

Lemma 23. *There exists a* $W(100,k)$ *for all* k *except possibly* 79, 87, 91, 93, 95 *which are undecided. All are constructed by using four circulant matrices in the Goethals–Seidel array.*

Lemma 24. *There exists an* $\mathrm{OD}(108;1,k)$ *for all* k *where* $k \in \{x\colon x \leqslant 107,\ x = a^2 + b^2 + c^2\}$, *except possibly for* $k \in \{61, 77, 78, 82, 85, 86, 89, 91, 93, 94, 97, 98, 99, 100, 101, 102, 106\}$ *which are undecided.*

Proof. The results for $k = 90, 104$ and 105 are given in Table 3. The remaining results are given in Koukouvinos (1994) or Koukouvinos and Seberry (to appear). □

Lemma 25. *There exists a* $W(108,k)$ *for all* k *except possibly* 79, 87, 93, 95, 99, 101, 103 *which are undecided. All are constructed by using four circulant matrices in the Goethals–Seidel array.*

Proof. The result for $k = 102$ follows because there are four sequences of $0, \pm 1$ with length 13 and weight 51 from Koukouvinos and Seberry (1993b). The remaining results follow from Koukouvinos and Seberry (1993b) and the previous lemma. □

8. Summary

See Table 4 for the summary of conjectures.

References

Banerjee, K.S. (1975). *Weighing designs for Chemistry, Medicine, Economics, Operations Research and Statistics.* Marcel Dekker, New York.

Cooper, J. and J. Wallis (1972). A construction for Hadamard arrays. *Bull. Austr. Math. Soc.* **7**, 269–277.

Eades, P. and J. Seberry Wallis (1976). An infinite family of skew-weighing matrices. In: *Combinatorial Mathematics IV*, Lecture Notes in Math., Vol. 560 Springer, Berlin, 27–60.

Geramita, A.V., N.J. Pullman and J. Seberry Wallis (1974). Family of weighing matrices. *Bull. Austral. Math. Soc.* **10**, 119–122.

Geramita, A.V. and J. Seberry (1979). *Orthogonal Designs: Quadratic Forms and Hadamard Matrices.* Marcel Dekker, New York.

Harwit, M. and N.J.A. Sloane (1979). *Hadamard Transform Optics.* Academic Press, New York.

Hotelling, H. (1994). Some improvements in weighing and other experimental techniques. *Ann. Math. Statist.* **15**, 297–306.

Kiefer, J. (1975). Construction and optimality of generalized Youden designs. In: J.N. Srivastara, Ed., *Statistical Design and Linear Models.* North-Holland, Amsterdam, 333–353.

Kishen, K. (1944). On the design of experiments for weighing and making other types of measurements. *Ann. Math. Statist.* **16**, 294–300.

Koukouvinos, C. (1993). Construction of some new weighing matrices. *Utilitas Math.* **44**, 51–55.

Koukouvinos, C. (1994). Some new weighing matrices. *Utilitas Math.* **46**, 81–89.

Koukouvinos, C. and J. Seberry (1993a). Some new weighing matrices using sequences with zero autocorrelation function. *Austral. J. Combin.* **8**, 143–152.

Koukouvinos, C. and J. Seberry (1993b). On weighing matrices. *Utilitas Math.* **43**, 101–127.

Koukouvinos, C. and J. Seberry, New weighing matrices. *Sankhya Ser. A* (to appear).

Mood, A. (1946). On Hotelling's weighing problem. *Ann. Math. Statist.* **17**, 432–446.

Moriguti, S. (1954). Optimality of orthogonal designs. *Rep. Statist. Appl. Res. Un. Japan Sci. Eng.* **3**, 75–98.

Raghavarao, D. (1959). Some optimum weighing designs. *Ann. Math. Statist.* **30**, 295–303.

Raghavarao, D. (1960). Some aspects of weighing designs. *Ann. Math. Statist.* **31**, 878–884.

Raghavarao, D. (1971). *Constructions and Combinatorial Problems in Design of Experiments.* Wiley, New York.

Seberry, J. (1980). An infinite family of skew-weighing matrices. *Ars Combin.* **10**, 323–329.

Seberry, J. (1982). The skew-weighing matrix conjecture. *Univ. Indore Res. J. Sci.* **7**, 1–7.

Seberry, J. and M. Yamada (1992). Hadamard matrices, sequences and block designs. In: J. Dinitz and D.R. Stinson, Eds., *Contemporary Design Theory – a Collection of Surveys.* Wiley, New York, 431–560.

Seberry Wallis, J. (1972). Hadamard matrices, Part IV. In: W.D. Wallis, A.P. Street and J. Seberry Wallis, Eds., *Combinatorics: Room Squares, Sum Free Sets and Hadamard Matrices*, Lecture Notes in Math., Vol. 292. Springer, Berlin.

(Seberry) Wallis, J. (1972). Orthogonal (0, 1, −1)-matrices. In: J. Wallis and W.D. Wallis Eds., *Proc. 1st Australian Conf. on Combinatorial Mathematics.* TUNRA Ltd, Newcastle, Australia, 61–84.

Sloane, N.J.A. and M. Harwit (1975). Masks for Hadamard transform optics and weighing designs. *Appl. Opt.* **15**, 107–114.