Minimal Critical Set of a Room Square of order 7

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A *Room square* R of order r is an $r \times r$ array each of whose cells may either be empty or contain an unordered pair of objects $0, 1, 2, ..., r$, subject to the following conditions :

(i) each of the objects $0, 1, 2, ..., r$ occurs precisely once in each row of R and precisely once in each column of R, and

(ii) every possible unordered pair of objects occurs precisely once in the whole array.

A *critical set* $Q = [Q_1, Q_2, Q_3, ..., Q_c]$, $|Q| = c$, in a Room square R of order r, is a set of quadruples $Q_a = [i, j; k, l]$ such that if any Q_a is removed from the set, it can no longer be uniquely completed. In Q_a, (i, j) shows the position of the pair (k, l) in the square. A *minimal critical set (min. cs)* of a Room square R of order r is a critical set of minimum cardinality.

Through computer search, we have found a critical set of size 10 for a Room square of order 7. We believe that critical set of size 10 given below is the minimal because critical set of size 9 could not be found. We also note that its size is less than $r^2/4$.

Example A minimal critical set in a Room square of order 7 and its completion.

0 7	* *	3 4	5 6	* *	* *	* *
* *	1 7	* *	* *	* *	* *	3 5
* *	* *	2 7	* *	* *	* *	* *
* *	* *	* *	* *	1 5	* *	* *
* *	0 3	* *	* *	* *	* *	* *
* *	* *	* *	2 4	0 6	* *	* *
* *	* *	* *	* *	* *	* *	* *

0 7	- -	3 4	5 6	- -	- -	1 2
4 6	1 7	- -	- -	- -	0 2	3 5
- -	4 5	2 7	0 1	- -	3 6	- -
- -	2 6	- -	3 7	1 5	- -	0 4
2 5	0 3	1 6	- -	4 7	- -	- -
1 3	- -	- -	2 4	0 6	5 7	- -
- -	- -	0 5	- -	2 3	1 4	6 7

where "*" shows the unknown pair positions and "–" shows empty positions in the Room square.

References

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