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NEW WEIGHING MATRICES*

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SUMMARY. New weighing matrices and skew weighing matrices are given for many orders $4t \leq 100$. These are constructed by finding new sequences with zero autocorrelation. These results enable us to determine for the first time that for $4t \leq 84$ a $W(4t,k)$ exists for all $k = 1, \ldots, 4t-1$ and also that there exists a skew-weighing matrix (also written as an $OD(4t;1,k)$) for $4t \leq 80$, t odd, $k = a^2+b^2+c^2, a, b, c$ integers except $k = 4t-2$ must be the sum of two squares.

1. Introduction

An *orthogonal design* of order n and type $(s_1, s_2, \ldots, s_u)$ $(s_i > 0)$, denoted $OD(n; s_1, s_2, \ldots, s_u)$, on the commuting variables $x_1, x_2, \ldots, x_u$ is an $n \times n$ matrix A with entries from $\{0, \pm x_1, \pm x_2, \ldots, \pm x_u\}$ such that

$$AA^T = (\sum_{i=1}^{u} s_i x_i^2) I_n$$

Alternatively, the rows of A are formally orthogonal and each row has precisely s_i entries of the type $\pm x_i$. In [3], where this was first defined, it was mentioned that

$$A^T A = (\sum_{i=1}^{u} s_i x_i^2) I_n$$

and so our alternative description of A applies equally well to the columns of A. It was also shown in [3] that $u \leq \rho(n)$, where $\rho(n)$ (Radon's function) is defined by $\rho(n) = 8c + 2^d$, when $n = 2^a b$, b odd, $a = 4c + d$, $0 \leq d < 4$.

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A *weighing matrix* $W = W(n,k)$ is a square matrix with entries $0, \pm 1$ having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an orthogonal design $OD(n;k)$. The number k is called the *weight* of W.

Given the sequence $A = \{a_1, a_2, \ldots, a_n\}$ of length n the *non-periodic autocorrelation function* $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, \ldots, n-1, \qquad \ldots (1)$$

If $A(z) = a_1 + a_2 z + \ldots + a_n z^{n-1}$ is the associated polynomial of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}), z \neq 0. \quad \ldots (2)$$

Given A as above of length n the *periodic autocorrelation function* $P_A(s)$ is defined, reducing $i+s$ modulo n, as

$$P_A(s) = \sum_{i=1}^{n} a_i a_{i+s}, \quad s = 0, 1, \ldots, n-1. \qquad \ldots (3)$$

Sequences with zero periodic autocorrelation function or zero non-periodic autocorrelation function, of length t, are used to form the first rows of four circulant matrices which are then used in the Goethals-Seidel array to form matrices of order $4t$ of the required type (if orthogonal designs) or weight (if weighing matrices). In the case of sequences with zero non-periodic autocorrelation function, the sequences are first padded with sufficient zeros added to the end to make their length t.

The results used are:

Theorem 1 [5, Theorem 4.49]. *If there exist four circulant matrices A_1, A_2, A_3, A_4 of order n satisfying $\sum_{i=1}^{4} A_i A_i^T = fI$ where f is the quadratic form $\sum_{j=1}^{u} s_j x_j^2$, then there is an orthogonal design $OD(4n; s_1, s_2, \ldots, s_u)$.*

Corollary 1. *If there are four $\{0, \pm 1\}$-sequences of length n and weight w with zero periodic or non-periodic autocorrelation function then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form $OD(4n; w)$ or a $W(4n, w)$. If one of the sequences is skew-type then they can be used similarly to make an $OD(4n; 1, w)$. We note that if there are sequences of length n with zero non-periodic autocorrelation function then there are sequences of length $n + m$ for all $m \geq 0$.*

Weighing matrices have long been studied because of their use in weighing experiments as first studied by Hotelling [7] and later by Raghavarao [12] and others. For more applications of weighing matrices see Banerjee [1] and Harwit and Sloane [6]. There are a number of conjectures concerning weighing matrices:

Conjecture 1. *There exists a weighing matrix* $W(4t,k)$ *for* $k \in \{1,\ldots,4t\}$.

Conjecture 2. *When* $n \equiv 4 (mod\ 8)$, *there exists a skew-weighing matrix (also written as an* $OD(n;1,k)$) *when* $k \leq n-1$, $k = a^2+b^2+c^2, a, b, c$ *integers except that* $n-2$ *must be the sum of two squares.*

Conjecture 3. *When* $n \equiv 0 (mod\ 8)$, *there exists a skew-weighing matrix (also written as an* $OD(n;1,k)$) *for all* $k \leq n-1$.

The reader is referred to Geramita and Seberry [5] for all other undefined terms.

In Geramita and Seberry [5] the status of the weighing matrix conjecture is given for $W(4t,k), k \in \{1,\ldots,4t\}$ and $t \in \{1,\ldots,21\}$. We give new results including resolving the conjecture in the affirmative for all $4t \leq 84$.

2. Some new sequences with zero autocorrelation

Tables 1 to 6 give new results which allow us to settle in the affirmative Conjecture 2 for orders 68 and 76. They also eliminate most unsolved cases for Conjecture 1 in order 92 and Conjecture 2 in order 84.

TABLE 1. SEQUENCES OF LENGTH 19 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=19	Sequences with zero non-periodic autocorrelation function
1,62	$\{0+++-+-0+a-0+-+---0\}$,
	$\{+-+++--++0--+-0++++\}$,
	$\{+-+++--++0++-+0----\}$,
	$\{0+++-+-0+0+0-+-+++0\}$
1,66	$\{0+++-+-++a--+-+---0\}$,
	$\{0+++-+-++0++-+-+++0\}$,
	$\{+++-0+------++-++-0\}$,
	$\{+++-0+---+++--+--+0\}$
1,68	$\{++-+0+---a+++-0-+--\}$,
	$\{+++-++-++0++-+--+++\}$,
	$\{+++-++-++0--+-++---\}$,
	$\{++-+0+---0---+0+-++\}$
1,72	$\{--+++++-+a-+-----++\}$,
	$\{+-+-++-++0+-+++--++\}$,
	$\{+-+-++-++0-+---++--\}$,
	$\{--+++++-+0+-+++++--\}$

3. Numerical consequences

We use the tables of Appendix H and extend Theorem 4.149 of Geramita and Seberry [5].

We note that sequences with periodic autocorrelation function zero exist for $W(4t, 4t-1)$, $W(4t, 4t)$ for all $t \in \{1, \ldots, 31\}$ [15]. We use the $(OD; 1, 1, 64)$ for $n \geq 17$ and the $(OD; 1, 1, 80)$ for $n \geq 21$ from [11]. Hence using the results of [8], [9], [10], [11] and those given in Tables 1, 2 and 3, we have:

TABLE 2. SEQUENCES OF LENGTH 19 WITH ZERO PERIODIC AUTOCORRELATION FUNCTION

Length=19	Sequences with zero periodic autocorrelation function
1,61	{0 0 − + + − + − − *a* + + − + − − + 0 0},
	{+ + + + + + − − − + − + − + + + 0 0 0},
	{+ + − + + + − − + + − + − + 0 0 + 0 0},
	{− + 0 − − − + + − + + + + + − − − 0 0}
1,67	{+ − − − 0 + + − + *a* − + − − 0 + + + −},
	{0 + + + + + − + + + 0 + − + + − − + −},
	{− + + + + − + − + + − − − + − + 0 + 0},
	{− 0 + + − + + + − + + + + − − − − + 0}
1,69	{0 + − − + + + − + *a* − + − − − + + − 0},
	{− + + + + + − − − + − + − − + 0 0 + −},
	{+ + − 0 + − + − − + − − − + + + + +−},
	{− 0 + + + + + + + + − + − + − + + +−},
1,70	{+ + + + + − + − + *a* − + − + − − − −−},
	{+ 0 − + − + + − − + − + + + − + + ++},
	{+ + − + + + − − 0 + + + − − + + − + 0},
	{− + + + 0 − + + − + + + − 0 − − − ++}
1,1,74	{+ + − + − + + + − *a* + − − − + − + −−},
	{− + − − + − − − − *b* + + + + − + + −+},
	{− + + − − − − − + + + − − − − − + + −},
	{− + + − + − + + + + + + + − + − + + −},

Theorem 2. *There exists an orthogonal design $OD(4n; 1, k)$ when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9$ *with* $k \in \{x : x \leq 4t - 1,\ x = a^2 + b^2 + c^2\}$;

(ii) *for* $n \geq 11$, *with* $k \in \{x : x \leq 43,\ x = a^2 + b^2 + c^2,\ x \neq 42\}$;

(iii) *for* $n \geq 13$, *with* $k \in \{x : x \leq 51,\ x = a^2 + b^2 + c^2\}$;

(iv) *for* $n \geq 15$, *with* $k \in \{x : x \leq 59,\ x = a^2 + b^2 + c^2\}$;

(v) *for* $n \geq 17$, *with* $k \in \{x : x \leq 67,\ x = a^2 + b^2 + c^2,\ x \neq 61, 66\}$;

(vi) *for* $n \geq 19$, *with* $k \in \{x : x \leq 75,\ x = a^2 + b^2 + c^2,\ x \neq 61\}$.

(vii) *for* $n \geq 21$, *with* $k \in \{x : x \leq 83,\ x = a^2 + b^2 + c^2,\ x \neq 61, 77, 78, 82\}$.

TABLE 3. SEQUENCES OF LENGTH 21 WITH ZERO PERIODIC AUTOCORRELATION FUNCTION

Length=21	Sequences with zero periodic autocorrelation function
1,57	{− − − − + − 0 0 0 − a + 0 0 0 + − + + + +},
	{+ + 0 0 0 + + + − + + + − + − 0 0 0 − + +},
	{+ 0 0 0 − + + + 0 + 0 − + + − − − + 0 − 0},
	{+ + − 0 − + 0 + + 0 + − − 0 0 − + 0 0 + −}
1,61	{+ + + − + 0 − + 0 + a − 0 − + 0 − + − − −},
	{+ − + − + + − − + 0 + + + − + + 0 + 0 0 0},
	{+ + + − + 0 + + + + − − + 0 − − + 0 0 0 0},
	{+ + 0 0 − 0 0 0 − + + 0 − − − − + − 0 + +}
1,62	{− − − − + 0 − 0 − 0 a 0 + 0 + 0 − + + + +},
	{+ 0 + + − + − − 0 + 0 + 0 + + + − + − + +},
	{+ + + + 0 − − + − 0 0 + − 0 + − 0 + − 0 +},
	{0 + + − − + + 0 − 0 0 − + − + − 0 + + + −}
1,67	{0 − − + + + − 0 + − a + − 0 + − − − + + 0},
	{+ 0 − + − + − + 0 + 0 + 0 + + − − + + + +},
	{+ + 0 + − + − − 0 0 − − + + + + − + + − 0},
	{0 0 − − − + + + + + + − + + − + − + 0 − 0}
1,73	{+ + + + + + − + − 0 a 0 + − + − − − − − −},
	{− + + − 0 + + + − + 0 + + + − 0 − + + + −},
	{+ + − 0 + + − + + 0 − + − + 0 − + + + + −},
	{− + − + + + − − − − + + − − + − 0 + 0 + +}
1,74	{+ + − − − − + − + − a + − + − + + + + − −},
	{− + + + + + − − + 0 0 + − + + + + + − + −},
	{+ 0 0 0 − + + + + + + + − + − − − + − − +},
	{+ + − + − + 0 0 + + − − − + + − 0 + 0 − +}
1,75	{+ + − + − + + + + 0 a 0 − − − − + − + − −},
	{− 0 + 0 + + − − − + + + − + + + − + + + −},
	{− + + + + + + − + + − − − + − + + 0 0 + −},
	{+ + − + − + + − + + + + + − − + − + 0 0 −}
1,76	{+ + − − − + − + + − a + − − + − + + + − −},
	{− 0 − 0 + + + + + + + + − + − − + + 0 + −},
	{− + + + + − + − + + − + − + − + + + 0 + −},
	{+ 0 − 0 + − − − + + − − − + − + + + 0 + +}
1,77	{− + + + + + − + 0 − a + 0 − + − − − − − +},
	{− 0 + + + + − + − + + + − + + − − + + + −},
	{+ + 0 + + + 0 + − + − + + − + − + + − − −},
	{− + − + + 0 − − + + − − + − + + + + + + −}
79	{+ + − + 0 − − − + + − + + + + + 0 + − + +},
	{0 + + + − − − + + + − + 0 + + − + + − − +},
	{− + − + + + − + + + + − − − + − − + 0 + −},
	{+ − + + − − + − − + − + − + − − − + + + +}
1,1,82	{− − + − − + + + − − a + + − − − + + − + +},
	{+ + + + − − − + − − b + + − + + + − − − −},
	{− + − + + − + − + − + − + − + − + + − + −},
	{− + − − + + + + + + + + + + + + + − − + −}

(viii) *for* $n \geq 23$, *with* $k \in \{x : x \leq 91,\ x = a^2 + b^2 + c^2,\ x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91\}$.

(ix) *for* $n \geq 25$, *with* $k \in \{x : x \leq 99,\ x = a^2 + b^2 + c^2,\ x \neq 61, 77, 78, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 98, 99\}$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. We have the results for $n \geq 13$ for all k except 46 and 49 from [11]. The $OD(52; 1, 46)$ is given in [10, Table 2]. An $OD(52; 1, 49)$ is given in [5, Theorem 8.38]. The $OD(4n; 1, 46)$ and $OD(4n; 1, 49)$ for $n \geq 15$ are given in Table 1 of [8].

TABLE 4. SEQUENCES OF LENGTH 21 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=21	Sequences with zero non-periodic autocorrelation function
1,68	{+ + 0 + + − + 0 + − a + − 0 − + − − 0 − −},
	{− + + − + + + 0 − − 0 + + 0 − − − + + − +},
	{− + + − + + + 0 − − 0 − − 0 + + + − − + −},
	{+ + 0 + + − + 0 + − 0 − + 0 + − + + 0 + +}
1,72	{0 + − + + − − + + + a − − − + + − − + − 0},
	{+ 0 + − + − + + − + 0 − − − + + + + + 0 +},
	{+ 0 + − + − + + − + 0 + + + − − − − − 0 −},
	{0 + − + + − − + + + 0 + + + − − + + − + 0}
74	{− + 0 + + + + 0 + + 0 − + − + 0 − − + + +},
	{− + 0 + + + + 0 + + 0 + − + − 0 + + − − −},
	{− + − + + − − + + − 0 + + − − − − + − + +},
	{− + − + + − − + + − 0 − − + + + + − + − −}
1,76	{+ + + 0 − − + − − + a − + + − + + 0 − − −},
	{− + + + + − + + + + 0 + + − − + − + − + −},
	{− + + + + − + + + + 0 − − + + − + − + − +},
	{+ + + 0 − − + − − + 0 + − − + − − 0 + + +}

The existence of $OD(4n; 1, 57)$ $n \geq 15$ and $OD(4n; 1, 67)$ for $n \geq 17$ is established in [9].

For $n \geq 17$ [11] gives the result except for k = 46, 49, 61, 62. The result for 62 is given in Table 1. An $OD(68; 1, 66)$ does not exist as its existence would imply the existence of an $OD(68; 1, 1, 66)$ by the Geramita-Verner Theorem [5, Theorem 2.20] and the existence of that design requires that 66 should be the sum of two squares which it is not. Hence the $OD(68; 1, 66)$ does not exist. The value 60 was erroneously included in [11] Theorem 8: it should not have been included as 60 is a number of the form $4^a(8b + 7)$ and so it cannot be written as three squares as required by the conditions of Conjecture 2.

$OD(4n; 1, k)$ for $k = 70$ exist for $n = 19$ from Table 2, $k = 21$ from [9], and for $n \geq 23$ from Table 5.

TABLE 5. SEQUENCES OF LENGTH 23 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=23	Sequences with zero non-periodic autocorrelation function
1,70	{0 − + 0 + + + + − − + a − + + − − − − 0 − + 0},
	{− + + 0 + 0 + 0 + − + 0 − + + 0 − + + 0 + − −},
	{− + + 0 + 0 + 0 + − + 0 + − − 0 + − − 0 − + +},
	{0 − + 0 + + + + − − + 0 + − − + + + + 0 + − 0}
1,74	{− 0 − 0 0 + + − + + − a + − − + − − 0 0 + 0 +},
	{− + + + + 0 + + + − + 0 − + − + + + − − − + +},
	{− + + + + 0 + + + − + 0 + − + − − − + + + − −},
	{− 0 − 0 0 + + − + + − 0 − + + − + + 0 0 − 0 −}
1,84	{+ − + + − + + + − − − a + + + − − − + − − + −},
	{+ + − + + 0 − + + + + 0 − + − + − 0 + + + + −},
	{+ + − + + 0 − + + + + 0 + − + − + 0 − − − − +},
	{+ − + + − + + + − − − 0 − − − + + + − + + − +}
1,88	{− + + − − + − + + + + a − − − − + − + + − − +},
	{+ − − + + + − + + + + 0 − + − + − + + + + − −},
	{+ − − + + + − + + + + 0 + − + − + − − − − + +},
	{− + + − − + − + + + + 0 + + + + − + − − + + −}

$OD(4n;1,k)$ for $k = 68$, 72 and $n \geq 19$ are given in Table 1, the result for $k = 66$ and 68 with $n \geq 19$ is given in [5, Table H.2]. $OD(4n;1,76)$ for $n \geq 21$ is given in Table 4 and $OD(4n;1,k)$ for $k = 68$, 72 and 83 are given in [9]. The $OD(4n;1,k)$ for $k = 84, 88$ and $n \geq 23$ are given in Table 4. The $OD(4n;1,75)$ for $n = 21$ is from Table 3 and for $n \geq 23$ from Table 5.

The $OD(4n;1,k)$ for $k = 69$, 74 and 75 exist for n =19 from Table 2 and for $n \geq 21$ from [9]. The $OD(76;1,73)$ is given in [8] and the $OD(4n;1,73)$, $n \geq 21$, in [9].

The $OD(4n;1,96)$, $n \geq 25$ is given in Table 6.

TABLE 6. SEQUENCES OF LENGTH 25 WITH ZERO NON-PERIODIC AUTOCORRELATION FUNCTION

Length=25	Sequences with zero non-periodic autocorrelation function
1,96	{+ − + + − + + + + − − − a + + + − − − − + − − + −},
	{+ + + − + − + + + − + + 0 + − − − + + − + + + − +},
	{+ + + − + − + + + − + + 0 − + + + − − + − − − + −},
	{+ − + + − + + + + − − − 0 − − − + + + + − + + − +},

Theorem 3. *There exists a* $W(4n;k)$ *when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9, 11, 13, 15, 17, 19$ *with* $k \in \{x : x \leq 4t\}$;

(ii) *for* $n \geq 21$, *with* $k = 1, \ldots, 78, 80, 81, 82$;

(iii) *for* $n \geq 23$, *with* $k = 1, \ldots, 78, 80, 81, 82, 85, 86, 88, 89, 90, 92$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. We use Theorem 2, the $W(4t, 4t)$ and $W(4t, 4t-1)$ from [15] and Theorem 3 from [11].

The result for 75 in (i) comes from Theorem 1, Tables 3 and 5 and for 77, in (ii) from Theorem 1 and from Table 4. The result for 78, 80, 81, 82 and 84 in (ii) comes from [11, Table 8] as does the result for 86, 88, 90 and 92 in (iii). Table 5 gives 71, 85 and 89 of (iii). ▱

Lemma 1. *The necessary conditions are sufficient for the existence of* $OD(4n; 1, k)$ *for* $n = 3, 5, \dots, 19$ *and* $k \le 4n - 1$. *All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Use Theorem 2 for all the results for $n \le 11$.

The $OD(52; 1, 46)$ and $OD(60; 1, 57)$ required are given in Tables 2 and 4 of [10].

The $OD(68; 1, k)$ for k = 57, 61, and 67 are given in [11, Table 6] and [10, Table 6]. An $OD(68; 1, 66)$ does not exist as was shown in the proof of Theorem 2 above. The value 60 was erroneously included in [11] : an $OD(68; 1, 60)$ does not exist.

The $OD(76; 1, k)$ for k = 57, 61, 66, 67, 69, 70, 73, 74 and 75 are given in Tables 2 and 3 of [8], and Table 2 of this paper. This gives us the result of the enunciation. ▱

Lemma 2. *There exists an* $OD(84; 1, k)$ *for* $k \in \{x : x \le 83,\ x = a^2 + b^2 + c^2,\ x \ne 70, 78\}$ *which are undecided. All may be constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. The results for k = 57, 61, 67, 73, 75, 77, 82 and 83 which may be constructed using four circulants in the Goethals-Seidel array are given in Table 3 of this paper. ▱

Lemma 3. *There exists an* $OD(92; 1, k)$ *for* $n \ge 23$, *with* $k \in \{x : x \le 91,\ x = a^2 + b^2 + c^2,\ x \ne 61, 77, 78, 82, 85, 86, 89, 90, 91\}$ *which are undecided.*

Proof. Use Theorem 2. ▱

Lemma 4. *There exists a* $W(4n, k)$ *for* $k \in \{x : 0 \le x \le 4n\}$ *with* $n = 1, 3, \dots, 21$. *All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Follows from [11], Lemma 1 and Tables 1 and 2. $k = 71$ is given in [9]. The sequences for $W(84, 79)$ and $W(84, 77)$ are given in Tables 3 and 4. ▱

Lemma 5. *There exists a* $W(92, k)$ *for all* k *except possibly* 79, 87, *which are undecided. All are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof. The paper [11] gives all k except possibly 71, 73, 75, 77, 79, 83, 85, 87 and 89. Koukouvinos [9] gives $k = 83$. The sequences for 71, 73, 75, 77, 85, and 89 are given in Tables 2, 4 and 5. ▱

Lemma 6. *There exists a* $W(100, k)$ *for all* k *except possibly* 87, 91, 93, 95 *which are undecided. All are constructed by using four circulant matrices* *in the Goethals-Seidel array.*

Proof. The paper [11] gives all k except possibly 71, 77, 83, 87, 89, 91, 93, 95, 97. Koukouvinos [9] gives $k = 83$. The sequences for 71, 77 and 89 are given in Tables 4 and 5. The $OD(100; 1, 96)$ in Table 6 gives $k = 97$.

TABLE 7. SUMMARY OF THE CONJECTURES
TRUE SIGNIFIES THE CONJECTURE IS VERIFIED

Order	Applicable Conjecture	Unresolved Cases	Applicable Conjecture	Unresolved Cases
4	1	true	2	true
8	1	true	3	true
12	1	true	2	true
16	1	true	3	true
20	1	true	2	true
24	1	true	3	true
28	1	true	2	true
32	1	true	3	true
36	1	true	2	true
40	1	true	3	true
44	1	true	2	true*
48	1	true	3	true
52	1	true	2	true
56	1	true	3	true
60	1	true	2	true
64	1	true	3	true
68	1	true	2	true*
72	1	true	3	true
76	1	true	2	true
80	1	true	3	true
84	1	true	2	70,78
88	1	true	3	true
92	1	79,87	2	61,77,78,82 85,86,89,90,91
96	1	true	3	true
100	1	87,91,93,95	2	61,77,78,82,85, 86,89,90,91,92, 93,94,97,98*
104	1	95	3	94,95
112	1	true	3	true
120	1	true	3	true

$*OD(n; 1, n-2)$ is not possible as $n-2$ is not the sum of two squares

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