Some new weighing matrices using sequences with zero autocorrelation function

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Dedicated to the memory of Alan Rahilly, 1947 – 1992

Abstract

We verify the skew weighing matrix conjecture for orders $2^t.13$, $t \geq 5$, and give new results for $2^t.15$ proving the conjecture for $t \geq 3$.

1 Introduction

An *orthogonal design* A, of order n, and type $(s_1, s_2, \ldots, s_u)$, denoted $OD(n; s_1, s_2, \ldots, s_u)$ on the commuting variables $(\pm x_1, \pm x_2, \ldots, \pm x_u, 0)$ is a square matrix of order n with entries $\pm x_k$ where each x_k occurs s_k times in each row and column such that the distinct rows are pairwise orthogonal.

In other words

$$AA^T = (s_1 x_1^2 + \ldots + s_u x_u^2) I_n$$

where I_n is the identity matrix. It is known that the maximum number of variables in an orthogonal design is $\rho(n)$, the Radon number, where for $n = 2^a b$, b odd, set $a = 4c + d, 0 \leq d < 4$, then $\rho(n) = 8c + 2^d$.

A weighing matrix $W = W(n, k)$ is a square matrix with entries 0, ± 1 having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an othogonal design $OD(n; k)$. The number k is called the *weight* of W.

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Weighing matrices have long been studied because of their use in weighing experiments as first studied by Hotelling [8] and later by Raghavarao [9] and others.

There are a number of conjectures concerning weighing matrices:

Conjecture 1 (Wallis [13]) *There exists a weighing matrix* $W(4t, k)$ *for* $k \in \{1, \dots, 4t\}$.

This conjecture was proved true for orders $n = 2^t$, t a positive integer by Geramita, Pullman and (Seberry) Wallis [3]. Later the conjecture was made stronger by Seberry until it appeared in the following forms.

Conjecture 2 (Seberry) *When* $n \equiv 4 (mod\ 8)$, *there exist a skew-weighing matrix (also written as an* $OD(n; 1, k)$*) when* $k \leq n - 1$, $k = a^2 + b^2 + c^2$, a, b, c *integers except that* $n - 2$ *must be the sum of two squares.*

Conjecture 3 (Seberry) *When* $n \equiv 0 (mod\ 8)$, *there exist a skew-weighing matrix (also written as an* $OD(n; 1, k)$*) for all* $k \leq n - 1$.

This conjecture was established for $n = 2^t.3, 2^t.5, 2^t.9$ by Geramita and (Seberry) Wallis [4, 5], by Eades and (Seberry) Wallis [1] for $t \geq 3$ and for $n = 2^t.15$ and $2^t.21$, $t \geq 4$ by Seberry [10, 11]. The result for $2^t.15$ is improved to $t \geq 3$ in this paper and the results are given for $2^t.13$, for $t \geq 5$.

Given the sequence $A = \{a_1, a_2, ..., a_n\}$ of length n the *non-periodic autocorrelation function* $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, ..., n-1. \tag{1}$$

If $A(z) = a_1 + a_2 z + \cdots + a_n z^{n-1}$ is the associated polynomial of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}), \ z \neq 0. \tag{2}$$

Given A as above of length n the *periodic autocorrelation function* $P_A(s)$ is defined, reducing $i + s$ modulo n, as

$$P_A(s) = \sum_{i=1}^{n} a_i a_{i+s}, \quad s = 0, 1, ..., n-1. \tag{3}$$

2 Preliminary Results

We make extensive use of the book of Geramita and Seberry [6]. We quote the following theorems, giving their reference from the aforementioned book, that we use:

Lemma 1 [6, Lemma 4.11] *If there exists an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$ *then there exists an orthogonal design* $OD(2n; s_1, s_1, es_2, \ldots, es_u)$ *where* $e = 1$ *or* 2.

Lemma 2 [6, Lemma 4.4] *If* A *is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$ *on the commuting variables* $(\pm x_1, \pm x_2, \ldots, \pm x_u, 0)$ *then there is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_i + s_j, \ldots, s_u)$ *and* $OD(n; s_1, s_2, \ldots, s_{j-1}, s_{j+1}, \ldots, s_u)$ *on the* $u-1$ *commuting variables* $(\pm x_1, \pm x_2, \ldots, \pm x_{j-1}, \pm x_{j+1}, \ldots, \pm x_u, 0)$.

Lemma 3 [6, Corollary 5.2] *If all orthogonal designs* $OD(n; 1, k)$, $k = 1, 2, \cdots, n-1$, *exist then all orthogonal design* $OD(2n; 1, j)$, $j = 1, 2, \cdots, 2n-1$, *exist.*

Theorem 1 [6, Theorems 2.19 and 2.20] *Suppose* $n \equiv 0 (mod\ 4)$. *Then the existence of a* $W(n, n-1)$ *implies the existence of a skew-symmetric* $W(n, n-1)$. *The existence of a skew-symmetric* $W(n, k)$ *is equivalent to the existence of an* $OD(n; 1, k)$.

Theorem 2 [6, Proposition 3.54 and Theorem 2.20] *An orthogonal design* $OD(n; 1, k)$ *can only exist in order* $n \equiv 4 (mod\ 8)$ *if* k *is the sum of three squares. An orthogonal design* $OD(n; 1, n-2)$ *can only exist in order* $n \equiv 4 (mod\ 8)$ *if* $n-2$ *is the sum of two squares.*

Theorem 3 *Orthogonal designs* $OD(n; 1, k)$ *exist for* $k = 1, 2, \cdots, n-1$ *in orders* $n = 2^t, 2^{t+3}.3, 2^{t+3}.5, 2^{t+3}.7, 2^{t+3}.9, 2^{t+4}.15$ *and* $2^{t+4}.21$, $t \geq 0$ *an integer.*

Theorem 4 [6, Theorem 4.49] *If there exist four circulant matrices* A_1, A_2, A_3, A_4 *of order* n *satisfying*

$$\sum_{i=1}^{4} A_i A_i^T = fI$$

where f *is the quadratic form* $\sum_{j=1}^{u} s_j x_j^2$, *then there is an orthogonal design* $OD(n; s_1, s_2, \ldots, s_u)$.

Corollary 1 *If there are four* $\{0, \pm 1\}$*-sequences of length* n *and weight* w *with zero periodic or non-periodic autocorrolation function then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form* $OD(4n; w)$ *or a* $W(4n, w)$. *If one of the sequences is skew-type then they can be used similarly to make an* $OD(4n; 1, w)$. *We note that if there are sequences of length* n *with zero non-periodic autocorrelation function then there are sequences of length* $n + m$ *for all* $m \geq 0$.

Theorem 5 [6, Theorems 4.124 and 4.41] *Let* q *be a prime power then there is a circulant* $W = W(q^2 + q + 1, q^2)$. *Let* $p \equiv 1 (mod\ 4)$ *then there are two circulant symmetric matrices* R, S *of order* $(p+1)/2$ *satisfying*

$$RR^T + SS^T = pI.$$

Lemma 4 [6, Proof of Lemma 4.34] *Let* q *be a prime. Then there is a circulant matrix* Q *which satisfies* $QQ^T = qI - J$, $QJ = JQ = 0$, $Q^T = (-1)^{(q-1)/2} Q$.

Corollary 2 *There exists a circulant* $W = W(13,9)$. *There exist two circulant symmetric matrices* R *and* S *or order 13 satisfying* $RR^T + SS^T = 25I$. *There exists a circulant symmetric matrix* Q *of order 13 satisfying* $QQ^T = 13I - J$.

Lemma 5 [6, **Lemmas 4.21 and 4.22**] *Let* A *and* B *be circulant matrices of order* n *and* $R = (r_{ij})$ *where* $r_{ij} = 1$ *if* $i + j - 1 = n$ *and 0 otherwise, then* $A(BR)^T = (BR)A^T$.

3 Notation

I is the identity matrix with the order taken from the context;

J is the matrix of ones with the order taken from the context;

X is the backcirculant matrix with first row {a b 0_{10} $\bar{b}$ } where 0_{10} is a sequence of 10 zeros and a and b are commuting variables;

Y is the circulant matrix with first row {0 b 0_{10} b} where 0_{10} is a sequence of 10 zeros and b is a commuting variable;

W is the backcirculant matrix with first row {0 1 0 1 1 0 0 − − 1 1 − 1} where − is used for -1, and W is a $W(13,9)$;

R and S are circulant symmetric matrices satisfying $RR^T + SS^T = 25I$;

Q is the circulant symmetric matrix of order 13 satisfying $QQ^T = 13I - J$;

A, B, C, D are circulant symmetric matrices satisfying $AA^T + BB^T + CC^T + DD^T = 52I$ (these are Williamson matrices see [12, pp511, 541].

$I + K$, L, M, N are circulant matrices where K is skew-symmetric, $(cI + dK)'$ is the backcirculant matrix with the same first row as $cI + dK$, and L, M and N are symmetric satisfying $KK^T + LL^T + MM^T + NN^T = 51I$ (these are good matrices see [12, pp492].

4 Sequences with Zero Autocorrelation

Tables 1 to 4 give sequences of lengths 13 and 15 with zero non-periodic and periodic auto-correlation function.

Length=13	Sequences with zero non-periodic autocorrelation function
1,34	{+ 0 0 + − − *a* + + − 0 0−}, {0 0 0 + + + + 0 + − + −+},
	{0 − 0 + 0 + + 0 + 0 0 − +}, {− 0 0 − + + − 0 + + + −−}
1,37	{0 + − + + 0 *a* 0 − − + −0}, {+ + − + − + 0 + + + + + −},
	{− + 0 − + + 0 − − + + 0 0}, {− − − 0 + 0 + 0 0 0 + +−}
1,1,40	{+ 0 + + + − *a* + − − − 0−}, {+ 0 + − − + *b* − + + − 0−},
	{+ 0 + + + − 0 − + + + 0+}, {+ 0 + − − + 0 + − − + 0+}
1,45	{+ + + − − + *a* − + + − − −}, {+ + − + − + 0 + − + + + +},
	{− + − + + − + + + + − 0 0}, {+ + − + + 0 − − − − + 0 0}
48	{+ + + − + + + + − + − + 0}, {+ + + − + + − − + − + − 0}
	{+ + + − − − + + − + + − 0}, {+ + + − − − − − + − − + 0}

Table 1: Sequences of length 13 with zero non-periodic autocorrelation function

Length=13	Sequences with zero periodic autocorrelation function
1,42	{− + + − − 0 *a* 0 + + − −+}, {− + − 0 − + 0 + + + + + +},
	{+ + + + − + − 0 + 0 0 + −}, {+ + − − − + 0 0 + + + −−}
1,46	{− − − + + − *a* + − − + + +}, {− + + + − + 0 + + + − + +},
	{+ + + + + + 0 − −0 − +−}, {+ + − − 0 − + 0 + + − +−}
1,48	{+ + + − + + *a* − − + − − −}, {− + + + + + 0 − + + − + −},
	{− + 0 + − + − − + + + + +}, {+ + + + − + − − + + 0 + −}
1,49	{+ + − − + − *a* + − + + − −}, {+ + + + + + 0 + − + − + −}
	{+ + − − + + + + + + − − −}, {+ + + + − − 0 − + + − + −}

Table 2: Sequences of length 13 with zero periodic autocorrelation function

Length=15	Sequences with zero non-periodic autocorrelation function
49	{− + 0 + + 0 0 + + + 0 + − 0+}, {+ − − − + − + + + + − + + + −}
	{− 0 + + 0 + + − 0 0 − + 0 + −}, {+ − − − − + + 0 + − + + − −+}
1,53	{0 + + + − + + *a* − − + − − − 0}, {+ + + + 0 − + + + − + − + + −},
	{+ − + + − + + − + − 0 − + + +}, {+ − − − + + + + + − − + −0 0}
1,56	{+ − + − − − − *a* + + + + − +−}, {− + − + + + + 0 + + + + − +−},
	{+ + − − + + + + + − − + + − 0}, {+ − − + + − + − + + − − + + 0}

Table 3: Sequences of length 15 with zero non-periodic autocorrelation function

Length=15	Sequences with zero periodic autocorrelation function
1,42	{0 + 0 − + − − *a* + + − + 0 − 0}, {0 + − + − + − − 0 + + + + + +},
	{+ + − 0 0 + + 0 − + + 0 0 + −}, {+ + 0 0 0 − 0 − − + + + 0 0 −}
1,54	{+ + + − − + − *a* + − + + − −−}, {+ + − 0 + − − + + + − + + + 0},
	{0 + + + − + − + + + − 0 + −+}, {− + + + + + + 0 − + − + − −−}
1,57	{− − + − + − − *a* + + − + − ++}, {+ + − + − + − − + + − + + + +},
	{− + + + + + − − + + + 0 − +−}, {+ + + 0 − + − − − + + + + +−}

Table 4: Sequences of length 15 with zero periodic autocorrelation function

Length=17	Sequences with zero non-periodic autocorrelation function
63	$\{+-+-+-++\ 0+++++++--\}$,
	$\{++-+-++-++-++--++\}$,
	$\{0-++-0----+++-+++\}$,
	$\{+-+++---+++0-+--0\}$,

Table 5: Sequences of length 17 with zero non-periodic autocorrelation function

Length=17	Sequences with zero periodic autocorrelation function
1,61	$\{---0++-+a-+--0+++\}$,
	$\{-+-+++++++++-+--0\}$,
	$\{++-+-+++-++-++--0\}$,
	$\{++--0+-+++-0-++-+\}$,
1,65	$\{a+--+----++++-++-\}$,
	$\{++--+++-+++++-+-++\}$,
	$\{0+++-+-+-+++-++--\}$,
	$\{0-++------+-++++-+\}$

Table 6: Sequences of length 17 with zero periodic autocorrelation function

Length=18	Sequences with zero non-periodic autocorrelation function
1,66	$\{0+++-+-++a--+-+---\}$,
	$\{0+++-+-++0++-+-+++\}$,
	$\{+++-0+---+++--+--+\}$,
	$\{+++-0+-------++-++-\}$

Table 7: Sequences of length 18 with zero non-periodic autocorrelation function

5 Results in Orders Divisible by 13

We recall that orthogonal designs $OD(52;1,k)$ can only exist if k is the sum of three squares. We see $52-2 = 5^2 + 5^2 = 7^2 + 1^2$ so the other condition is satisfied. Hence we have that $OD(52;1,k)$ cannot exist for $k = 4^a(8b+7)$, ie $k \in \{7, 15, 23, 28, 31, 39, 47\}$.

Theorem 6 *Orthogonal designs* $OD(52;1,k)$ *exist for* $k \in \{x : x = a^2 + b^2 + c^2\}$. *In other words the necessary conditions are sufficient for the existence of an* $OD(52;1,k)$. *All are constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. From [6, Theorem 4.149] we get the result for $k \neq 34, 37, 42, 45, 46, 48$ or 49. Tables 1 and 2 give 4 sequences which can be used in Corollary 1 to give all these values.

Corollary 3 $W(52,k)$ *exist for all* $k = 1, 2, \ldots, 52$.

Proof. From the theorem we only have to consider $k \in \{7, 15, 23, 28, 31, 39, 47\}$ as all other values of k have an $OD(52; 1, k)$: setting the first variable zero gives the required weighing matrix. For these other values we consider $OD(52; 1, k-1)$ and equate the variables to give the result. □

Corollary 4 *Orthogonal designs* $OD(104; 1, k)$ *exist for* $k = 1, 2, \ldots, 103$ *with the possible exception of* 94 *and* 95 *which are undecided.*

Proof. We use Lemma 1 to construct $OD(104; 1, 1, k, k)$ for k given in the previous Theorem. This assures us of the existence of all $OD(1, j)$ with the possible exception of $j = 56, 57, 62, 63, 78, 79, 94$ and 95. We replace the variables of the $OD(8; 1, 1, 1, 1, 1, 1, 1, 1)$ as given in Table 8 to get the orthogonal designs indicated there:

Variables Replaced By								Design Constructed
cI	dI	X	Y	eA	eB	eC	eD	OD(104;1,1,1,4,52)
aI	bI	cI	dW	eA	eB	eC	eD	OD(104;1,1,1,9,52)
aI	bI	cS	cR	dA	dB	dC	dD	OD(104;1,1,25,52)
X	Y	cS	cR	dS	dR	eS	eR	OD(104;1,4,25,25,25)

Table 8: Construction of Orthogonal Designs in Order 104.

So by equating variables and setting variables to zero we have constructed $OD(104; 1, i)$, for $i = 56, 57, 62, 63, 78$ and 79 giving the result. □

Corollary 5 *Orthogonal designs* $OD(208; 1, k)$ *exist for* $k = 1, 2, \ldots, 207$ *with the possible exception of* 189 *and* 191 *which are undecided. All* $W(208, k)$ *exist,* $k = 1, 2, \ldots, 208$.

Proof. We use Lemma 1 to construct $OD(208; 1, 1, k, k)$ for k given in the previous Corollary. This assures us of the existence of all $OD(1, j)$ with the possible exception of $j = 188, 189, 190$ and 191. We replace the variables of the $OD(16; 1, 1, 1, 1, 1, 1, 5, 5)$ by aI, bW, cQ, $dI + cQ$, $dI - cQ$, cJ, $eI + cQ$, $eI - cQ$ to obtain an $OD(208; 1, 2, 9, 10, 169)$ and hence equating and killing variables the $OD(208; 1, i)$, $i = 188$ and 190 giving the result. □

Corollary 6 *Orthogonal designs* $OD(416; 1, k)$ *exist for* $k = 1, 2, \ldots, 415$. *All* $W(416, k)$ *exist,* $k = 1, 2, \ldots, 416$.

Proof. We use Lemma 1 to construct $OD(416; 1, 1, k, k)$ for k given in the previous Corollary. This assures us of the existence of all $OD(1, j)$ with the possible exception of 378, 379, 382 and 383. We replace the variables of the following designs in order 32 (i) OD(32;1,1,3,3,3,3,9,9) by aI, bI, $(cI + dK)'$, dL, dM, dN, eR and eS to obtain the $OD(416; 1, 1, 3, 153, 225)$ giving the result for 378, 379 and 382, and (ii) OD(32;1,1,1,1,2,2,3,3,9,9) by aI, bI, $dI + cQ$, $dI - cQ$, $c(J - I)$, cQ, eR, eS, $fI + cQ$ and $fI - cQ$ to obtain the $OD(416; 1, 1, 2, 18, 75, 288)$ design which gives by equating variables the $OD(416; 1, 1, 2, 381)$ giving the result for 383. □

Hence using Lemma 3 we have

Theorem 7 *Orthogonal designs* $OD(2^t.13; 1, k)$ *exist for* $k = 1, 2, \ldots, 2^t.13 - 1$ *for all* $t \geq 5$. *All* $W(2^t.13, k)$ *exist,* $k = 1, 2, \ldots, 2^t.13$ *for all* $t \geq 5$.

6 Results in Orders Divisible by 15

We recall that orthogonal designs $OD(60; 1, k)$ can only exist if k is the sum of three squares. We see $60-2 = 7^2 + 3^2$ so the other condition is satisfied. Hence we have that $OD(60; 1, k)$ cannot exist for $k = 4^a(8b+7)$, ie $k \in \{7, 15, 23, 28, 31, 39, 47, 55\}$.

Theorem 8 *Orthogonal designs* $OD(60; 1, k)$ *exist for* $k \in \{x : x = a^2 + b^2 + c^2\}$ *except possibly for* $k = 48$ *or* 49 *which are undecided. In other words the necessary conditions are sufficient for the existence of an* $OD(60; 1, k)$ *except possibly for* $k = 48$ *or* 49 *which are undecided. All, except the* $OD(60; 1, 46)$*, are constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. From [6, Theorem 4.149] we have the result for $k \neq 34, 37, 42, 45, 46, 48, 49, 53, 54, 56$ or 57. Tables 1,3 and 4 give 4 sequences which can be used in Corollary 1 to give all these values except 46, 48 and 49. We replace the variables of the $OD(12; 1, 1, 5, 5)$ by aI, bI, $c(J-2I)$, dQ to give the $OD(60; 1, 1, 45)$ and hence the $OD(60; 1, 46)$.

Corollary 7 $W(60, k)$ *exist for all* $k = 1, 2, \ldots, 60$.

Proof. From the theorem we only have to consider $k \in \{7, 15, 23, 28, 31, 39, 47, 48, 49, 55\}$ as all other values of k have an $OD(60; 1, k)$: setting the first variable zero gives the required weighing matrix. The sequences that can be used to give weights 48 and 49 are given in Tables 1 and 3 (note that for sequences with zero non-periodic autocorrelation function the appropriate number of zeros can be added to the end of each sequence to give the required length). For the other values we consider $OD(60; 1, k-1)$ and equate the variables to give the result. □

Corollary 8 *Orthogonal designs* $OD(120; 1, k)$ *exist for* $k = 1, 2, \ldots, 119$. *All* $W(120, k)$*,* $k = 1, 2, \ldots, 120$ *exist.*

Proof. We use Lemma 1 to construct $OD(120; 1, 1, k, k)$ for k given in the previous Theorem. This assures us of the existence of all $OD(1, j)$ with the possible exception of $j = 47, 62, 63, 78, 79, 94, 95, 96, 97, 98, 99, 110$, and 111.

I_n is the identity matrix with the order n taken from the context;

J_n is the matrix of ones with the order n taken from the context;

Write $K = J - 2I$ and $L = J - I$;

X is the backcirculant matrix with first row $\{a\ b\ 0\ 0\ \bar{b}\}$ where a and b are commuting variables;

Y is the circulant matrix with first row {0 b 0 0 b} where b is a commuting variable;

A is the backcirculant matrix with first row {a b $\bar{b}$ } where a and b are commuting variables;

B is the backcirculant matrix with first row {a b b $\bar{b}$ $\bar{b}$ } where a and b are commuting variables;

Q is the circulant symmetric matrix of order 5 with first row {0 1 - - 1} satisfying $QQ^T = 5I - J$;

$I+E$, F, G, H are circulant matrices where E is skew-symmetric, $(cI+dE)'$ is the backcirculant matrix with the same first row as $cI+dE$, and F, G and H are symmetric satisfying $EE^T + FF^T + GG^T + HH^T = 19I$ (these are good matrices see [12, pp492].

We replace the variables of the indicated OD in orders 24 and 40 as given in Table 9 to get the orthogonal designs indicated there:

Variables In	Variables Replaced By						Design Constructed
OD(24;4,4,1,1,5,5)	eJ-2eI	eQ	aI	bI	cI	dI	OD(120;1,1,5,5,36)
OD(24;4,4,1,5,5,1)	eJ-2eI	eQ	cI	X	Y	dI	OD(120;1,1,5,20,36)
OD(24;6,1,1,1,9,6)	eJ-2eI	aI	bI	cI	dI	eQ	OD(120;1,1,1,9,54)
OD(24;4,4,4,4,1,2)	$(fI+eE)'$	eF	eG	eH	aI	bI	OD(120;1,2,4,76)
OD(24;4,4,4,4,1,3)	$(fI+eE)'$	eF	eG	eH	aI	bI	OD(120;1,3,4,76)
OD(24;9,1,12,1,1)	cJ-2cI	cJ-cI	cQ	aI	bI		OD(120;1,1,97)
OD(24;5,1,1,2,15)	cJ-2cI	X	Y	cJ	cQ		OD(120;1,4,95)
OD(24;9,9,1,1,1,3)	dI+cQ	dI-cQ	cL	aI	bI	cJ	OD(120;1,1,18,91)
OD(24;1,2,12,8,1)	bI+bQ	bI-bQ	cK	bQ	B		OD(120;1,111)
OD(40;18,19,1)	bJ-2bI	bJ-bI	A				OD(120;1,94)
OD(40;19,19,1,1)	bJ-2bI	bJ-bI	aI	cI			OD(120;1,1,95)

Table 9: Construction of Orthogonal Designs in Order 120.

Setting variables equal to each other or to zero gives all the remaining cases. □

Hence using Lemma 3 we have

Theorem 9 *Orthogonal designs* $OD(2^t.15; 1, k)$ *exist for* $k = 1, 2, \ldots, 2^t.15 - 1$ *for all* $t \geq 3$. *All* $W(2^t.15, k)$ *exist,* $k = 1, 2, \ldots, 2^t.15$, $t \geq 3$.

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