On weighing matrices

Christos Koukouvinos
Department of Statistics
Athens University of Economics
76 Patission St
104 34 Athens
Greece

and

Jennifer Seberry
Department of Computer Science
University of Wollongong
Wollongong
NSW 2522
Australia

Abstract

We give new sets of $\{0, 1, -1\}$ sequences with zero autocorrelation function, new constructions for weighing matrices and review the weighing matrix conjecture for orders $4t$, $t \in \{1, \ldots, 25\}$ establishing its veracity for orders 52, 68 and 76. We give the smallest known lengths for sequences with zero autocorrelation function and weights ≤ 100.

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1 Introduction

Definition 1 Let W be a (1, -1, 0)-matrix of order n satisfying $WW^T = kI_n$. We call W a *weighing matrix* of order n with weight k, denoted by $W(n,k)$.

There are a number of conjectures concerning weighing matrices:

Conjecture 1 *There exists a weighing matrix $W(4t,k)$ for $k \in \{1, \ldots, 4t\}$.*

Conjecture 2 *When $n \equiv 4(mod\ 8)$, there exist a skew-weighing matrix (also written as an $OD(n;1,k)$) when $k \leq n-1$, $k = a^2 + b^2 + c^2$, a, b, c integers except that $n-2$ must be the sum of two squares.*

Conjecture 3 *When $n \equiv 0(mod\ 8)$, there exist a skew-weighing matrix (also written as an $OD(n;1,k)$) for all $k \leq n-1$.*

The reader in referred to Geramita and Seberry [17] for all other undefined terms.

In Geramita and Seberry [17] the status of the weighing matrix conjecture is given for $W(4t,k), k \in \{1, \ldots, 4t-1\}$ and $t \in \{1, \ldots, 21\}$. We give new results including resolving the conjecture in the affirmative for 52, 68 and 76. Further we give the length of the smallest n for which $4-CS(n,w)$ are known for all $w \leq 100$.

Given the sequence $A = \{a_1, a_2, ..., a_n\}$ of length n the non-periodic autocorrelation function $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, ..., n-1. \tag{1}$$

If $A(z) = a_1 + a_2 z + \cdots + a_n z^{n-1}$ is the associated polynomial of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}),\ z \neq 0. \quad (2)$$

Given A as above of length n the periodic autocorrelation function $P_A(s)$ is defined, reducing $i+s$ modulo n, as

$$P_A(s) = \sum_{i=1}^{n} a_i a_{i+s}, \quad s = 0, 1, ..., n-1. \quad (3)$$

Notation 1 We use 0_t to represent the sequence of t zeros, $\bar{1}$ for -1, and J for the matrix with every element 1. We call $g = 2^a 10^b 26^c$, a, b, c non-negative integers, *Golay numbers*. If $A = \{a_1, a_2, \cdots, a_{n-1}, a_n\}$ is a sequence of n elements we will use $A^* = \{a_n, a_{n-1}, \cdots, a_2, a_1\}$ to be the reverse sequence and $\bar{A} = \{-a_1, -a_2, \cdots, -a_{n-1}, -a_n\}$ to be the sequence with all the elements negated. We use the notation (A/B) for the sequence $\{a_1, b_1, ..., a_n, b_n\}$ and (A, B) for the sequence $\{a_1, ..., a_n, b_1, ..., b_n\}$.

If $A^* = \{a_n, ..., a_1\}$ is the reversed sequence, then

$$A^*(z) = z^{n-1} A(z^{-1}). \quad (4)$$

Base, Turyn, Golay and normal sequences are finite sequences, with zero autocorrelation function, useful in constructing orthogonal designs and Hadamard matrices [17], in communications engineering [39], in optics and signal transmission problems [19, 21], etc.

If $A = \{a_1, ..., a_n\}$, $B = \{b_1, ..., b_n\}$ are two binary $(1, -1)$ sequences of length n and

$$N_A(s) + N_B(s) = 0 \quad \text{for } s = 1, ..., n-1 \quad (5)$$

then A, B are called *Golay sequences* of length n (abbreviated GS(n)). See [12, 17, 19].

From this definition and relation (2) we conclude that two $(1, -1)$ sequences of length n are GS(n) if and only if

$$A(z)A(z^{-1}) + B(z)B(z^{-1}) = 2n, \quad z \neq 0. \quad (6)$$

Golay sequences GS(n) exist for $n = 2^a 10^b 26^c$ (Golay numbers) where a, b, c are non-negative integers and do not exist when n has any factor $\equiv 3 (\text{mod } 4)$ [12, 19, 20].

Definition 2 m sequences $A_1, A_2, \ldots, A_m$ of length n with entries $0, +1, -1$ are called *m-complementary sequences* of *weight* w, written $m - CS(n, w)$, if:

$$N_{A_1}(s) + \ldots + N_{A_m}(s) = \begin{cases} 0, & s = 1, ..., n-1 \\ w, & s = 0 \end{cases} \quad (7)$$

which can be replaced by

$$A_1(z)A_1(z^{-1}) + \ldots + A_m(z)A_m(z^{-1}) = w, \quad z \neq 0 \quad (8)$$

Setting $z = 1$ in (8) we obtain

$$a_1^2 + \ldots + a_m^2 = w \tag{9}$$

where $a_1, \ldots, a_m$ are the sums of the elements of $A_1, \ldots, A_m$ respectively.

If there exist 4-$CS(m,w)$ $\{A\}$, $\{B\}$, $\{C\}$ and $\{D\}$ which actually have lengths m_1, m_2, m_1 and m_2 and weight w then $\{A,B\}$, $\{A,-B\}$, $\{C,D\}$ and $\{C,-D\}$ are 4-$CS(m_1+m_2, 2w)$.

Example 1 $\{1\ 0\ 1\}$ and $\{1\ 1\ {-1}\}$ are 2-CS(3,5).
$\{1\ 1\ 1\ 0\ \bar{1}\ 1\ 1\ 0\ \bar{1}\ 1\ \bar{1}\}$ and $\{1\ 0\ 1\ 0\ 0\ 0\ \bar{1}\ 0\ 0\ 0\ 1\}$ are 2-CS(11,13).
$\{1\ 0\ \bar{1}\}$, $\{1\ 1\ \bar{1}\}$, $\{1\ 1\ 1\}$, $\{1\ \bar{1}\ 1\}$ are 4-CS(3,11).
$\{1\ 1\ 0\ \bar{1}\ 1\}$, $\{1\ 1\ 1\ 1\ \bar{1}\}$, $\{1\ \bar{1}\ 1\}$, $\{\bar{1}\ 1\ 1\}$ are 4-CS(5,15).

We note, in the language of this paper, one very important result:

Theorem 1 *Suppose there exist 4-CS(n,w). Then there exists a W(4n,w). If there exist 2-CS(n,w) there exists a W(2n,w).*

Proof. The sequences are used to make circulant matrices which are then used in the Goethals-Seidel or other appropriate array to obtain the result. □

Definition 3 $2k$ sequences $A_1, A_2, \ldots, A_{2k}$ of length n with entries $0, +1, -1$ (where A_{2k} may be the sequence of n zeros to ensure an even number of sequences) are called ***2k-disjoint complementary sequences*** of ***weight*** w, written $2k - DCS(n,w)$, if they have zero nonperiodic autocorrelation function, total weight w, $A_i \pm A_{i+1}$, are also $0, +1, -1$ sequences with maximum length n and zero nonperiodic autocorrelation function for $i = 1, 3, \ldots, 2k-1$. These also satisfy equations (7), (8), and (9).

We note that 2-DCS(m,w) always give 2-CS$(m,2w)$.

4-DCS(m,w), A, B, C, D of lengths m, m, n and n, where $m \geq n$ will be called *suitable sequences* and denoted SS$(m,n;w)$.

4-CS$(m, 2m+2n)$ with lengths m, m, n, n and entries $+1, -1$, are called *base sequences.* Base sequences of lengths $n+1, n+1, n, n$ are denoted by BS$(2n+1)$ and we get

$$a^2 + b^2 + c^2 + d^2 = 4n + 2 \tag{10}$$

where a, b, c, d are the sum of the elements of A, B, C, D respectively.

BS$(2n+1)$ for all decompositions of $4n+2$ into four squares for $n = 1, 2, \ldots, 24$ are given in [2, 23, 26], BS$(2n+1)$ for $n = 25, 26, 29$ and $n = 2^a 10^b 26^c$ (Golay numbers) are given in Yang [46] and for 27, 28, 30 in [37]. SS$(47,24;71)$ are given in [25, 28], which are equivalent to base sequences 47, 47, 24, 24 with total weight 142.

Remark 1 Since base sequences BS$(2n+1)$ exist for all $n \leq 30$ [37], they give us 4-CS$(n+1, 4n+2)$ and 4-DCS$(n+1, 2n+1)$ for each of these n. Also using them as {A,C}, $\{A,\bar{C}\}$, {B,D}, $\{B,\bar{D}\}$ gives 4-CS$(2n+1, 8n+4)$.

The base sequences m, m, n, n may be given in the form {Z,W}, $\{Z,\bar{W}\}$, {X}, {Y}, with weight $2w$, in this case {Z}, {W}, {(X+Y)/2}, {(X−Y)/2} are 4-CS(M,w) where $M = \max\{|Z|, |W|, n\}$. In the case of 47, 47, 24, 24 noted above the shorter sequences have lengths 24, 23, 24, 18 and give 4-CS(24,71). The paper [25] gives 4-CS(20,59) by the same observation.

Example 2 $\{1\ 0\ 1\ 1\ 1\ \bar{1}\}$ and $\{1\ 0\ 1\ \bar{1}\ \bar{1}\ 1\}$ are 2-CS(6,10) and they give $\{1\ 0\ 1\ 0\ 0\ 0\}$ and $\{0\ 0\ 0\ 1\ 1\ \bar{1}\}$ which are 2-DCS(6,5).
$\{1\ 0\ 1\ \bar{1}\ 1\ \bar{1}\ \bar{1}\ 1\ 1\ \bar{1}\ 1\ 1\ 1\ 1\}$ and $\{\bar{1}\ 0\ \bar{1}\ \bar{1}\ 1\ \bar{1}\ 1\ 1\ 1\ \bar{1}\ \bar{1}\ 1\ 1\ 1\}$ are 2-CS(14,26) and they give $\{1\ 0\ 1\ 0\ 0\ 0\ \bar{1}\ 0\ 0\ 0\ 1\ 0\ 0\ 0\}$ and $\{0\ 0\ 0\ \bar{1}\ 1\ \bar{1}\ 0\ 1\ 1\ \bar{1}\ 0\ 1\ 1\ 1\}$ which are 2-DCS(14,13).
$(G+H)/2$ and $(G-H)/2$ where G and H are Golay sequences of length g are 2-DCS(g,g).
$\{1\ 1\ \bar{1}\ 1\ 0\ 1\}$, $\{\bar{1}\ \bar{1}\ 1\ 1\ 0\ 1\}$, $\{1\ 1\}$, $\{1\ \bar{1}\}$ are SS(6,2;14). □

In this paper we give some special examples of $m-CS(n,w)$. Also we give constructions, based on T-sequences and TW-sequences, which form weighing matrices.

Definition 4 The four sequences X, Y, Z, W of length n with entries $0, 1, -1$ are called *T-sequences* if

(i) $|x_i| + |y_i| + |z_i| + |w_i| = 1, \quad i = 1, ..., n$

(ii) $N_X(s) + N_Y(s) + N_Z(s) + N_W(s) = \begin{cases} 0, & s = 1, ..., n-1 \\ n, & s = 0 \end{cases}$

Yang [44] gives another name for T-sequences and calls them four-symbol δ-codes. He also calls the quadruple $Q = X + Y$, $R = X - Y$, $S = Z + W$, $T = Z - W$ regular δ-code of length n, where X, Y, Z, W are T-sequences of length n.

We now slightly extend the definition of T-sequences

Definition 5 Four sequences A, B, C, D of length t, with elements 0, $+1$, -1, total number of non-zero elements (the weight) w, with zero non-periodic autocorrelation function and which further satisfy $A \pm B \pm C \pm D$ (where addition or subtraction of sequences means adding or subtracting them element by element) are also sequences of 0, $+1$, or -1 with zero non-periodic autocorrelation function will be called *TW-sequences* and denoted $TW(t;w)$.

Definition 6 A triple $(F;G,H)$ of sequences is said to be a set of *normal sequences* for length n (abbreviated as NS(n)) if the following conditions are satisfied.

(i) $F = (f_k)$ is a $(1,-1)$ sequence of length n.

(ii) $G = (g_k)$ and $H = (h_k)$ are sequences of length n with entries $0, 1, -1$, such that $G + H = (g_k + h_k)$ is a $(1,-1)$ sequence of length n.

(iii) $N_F(s) + N_G(s) + N_H(s) = 0$, $s = 1, ..., n-1$.

We note they are also 3-CS$(n, 2n)$, the quadruple $(F, 0_n; G, H)$ gives 4-DCS$(n, 2n)$, while the quadruple $\{F, G+H\}$, $\{F, -G\}$, $\{F, -H\}$, $\{0, G-H\}$, are 4-CS$(2n; 6n)$. □

In [25, Theorem 1] it was shown that the sequences G and H of Definition 6 are *quasi-symmetric*, i.e. if $g_k = 0$, then $g_{n+1-k} = 0$ and also if $h_k = 0$, then $h_{n+1-k} = 0$. This means that $G \pm H^*$ and $G^* \pm H$ are also $1,-1$ sequences.

Definition 7 The sequences E, F, G, H of lengths $2m$, $2m-1$, $2m$, $2m$ respectively, with entries 0, ± 1 are called *near normal sequences* for length $n = 4m+1$ (abbreviated as NN(n)) if the following conditions are satisfied

(i) $E = (X/0, 1)$, $F = (Y/0)$, where X and Y are (± 1)-sequences of length m, $0 = 0_{m-1}$ the sequences of zeros of length $m-1$, and $X/0 = \{x_1, 0, x_2, 0, \cdots, x_{m-1}, 0, x_m\}$.

(ii) G and H are quasi-symmetric supplementary (0, ± 1)-sequences of length $2m$, ie $G + H$ is a (± 1)-sequence of length $2m$ and zeros appear symmetrically in G and H ie $g_i = 0$ iff $g_{2m+1-i} = 0$, $h_i = 0$ iff $h_{2m+1-i} = 0$, $g_i = 0$ iff $h_i \neq 0$, $g_i \neq 0$ iff $h_i = 0$ (this condition is necessarily implied by (i) and (iii)).

(iii) The sequences E, F, G, H have zero nonperiodic autocorrelation sum. The sequences X, Y, G, H are called *Yang quasi-symmetric sequences.*

All Yang's theorems [46, 43, 44, 45, 47, 37, 27] use *Yang quasi-symmetric sequences.*

Also in [25] it is shown that from Golay sequences of length n we can always obtain at least two sets of normal sequences NS(n) and that normal sequences of length n always give base sequences BS($2n+1$). In fact BS($2n+1$) give NS(n) and vice versa.

Normal sequences do not exist for $n = 2^{2a-1}(8b+7)$, a, b non-negative integers as the numbers $4^a(8b+7)$ can only be written as the sum of four squares. In particular $n \neq 14, 30, 46, 56, 62, 78, 94, \ldots$ It is known that NS(6) does not exist. Yang [46] has given these sequences for $n = 3, 5, 7, 9, 11, 12, 13, 15, 25, 29$ and he notes they exist for $n = 2^a 10^b 26^c$ (Golay numbers).

NS(n) have not been found for $n \geq 20$ (except $n = 25, 29$ and g, g a Golay number). We can construct T-sequences of length $(2n+1)t$, if a new set of NS(n) can be found, where $t = 2s+1$ is the length of base sequences (BS($2s+1$)).

Therefore we know these sequences

(i) exist for $n \in \{1, 2, \ldots, 5, 7, 8, \ldots, 13, 15, 16, 18, 19, 20, 25, 26, 29, 32, \ldots\}$;

(ii) do not exist for $n \in \{6, 14, 17, 21, 22, 23, 30, 46, 56, 62, 78, 94, \ldots\}$.

(iii) $n \in \{24, 27, 28, 31, \ldots,\}$ are undecided.

From corollary 5.24 of [37] we have T-sequences or T-matrices for all $t \leq 100$ except possibly 73, 79, 83, 89, and 97. Hence we have using the construction of Cooper and (Seberry) Wallis [3] that $OD(4t; t, t, t, t)$ exist for these t. Setting the first three variables equal to 1 and the fourth equal to zero we have

Lemma 1 *There exists a $W(4t, 3t)$ for all $t \leq 100$ except possibly for* 73, 79, 83, 89, *and* 97.

Lemma 2 *If there exist T-sequences of length t, T_1, T_2, T_3, T_4 then*

$\{T_1 + T_2 + T_3\}$, $\{-T_1 + T_2 + T_4\}$, $\{-T_1 + T_3 - T_4\}$, $\{T_2 - T_3 - T_4\}$

are $4 - CS(t, 3t)$. (We note that, similarly, there will be sequences with periodic autocorrelation function zero if T-matrices were used.) In particular $4 - CS(t, 3t)$ exist for all $t \leq 71$ [37]. Also if $TW(t; w)$ exist the same construction gives $4 - CS(t, 3w)$.

Example 3 $\{11-0000\}, \{0001010\}, \{0000100\}, \{0000001\}$ are T-sequences of length 7 and so

$$\{11-1110\}, \{--11011\}, \{--1010-\}, \{0001-1-\}$$

are the required $4-CS(7,21)$.

This means there are $4-CS(19,57)$, $4-CS(21,63)$, $4-CS(23,69)$, $4-CS(25,75)$, $4-CS(29,87)$, $4-CS(31,93)$, $4-CS(33,99)$. □

Remark 2 Turyn gave a construction for T-sequences using six sequences X, Y, Z, Z, W, W of lengths n, n, n, n, $n-1$, $n-1$. The known results are summarized in Table 1. Hence we have

Lemma 3 *If there exist six sequences X, Y, Z, Z, W, W, of lengths n, n, n, n, $n-1$ and $n-1$ with non-periodic auto-correlation function zero then writing $U = (X + Y)/2$ and $V = (X - Y)/2$ of length n*

$$\{\ Z,\ W,\ U\}, \{\ Z,\ -W,\ -V\}, \{\ Z,\ 0_{n-1},\ -U+V\}, \{\ 0_n,\ W,\ -U-V\}$$

are $4-CS(3n-1; 9n-3)$. □

Corollary 1 $4-CS(3n-1;9n-3)$ *exist for even $n = 2, 4, \cdots 24$. In particular* $4-CS(11;33)$, $4-CS(17;51)$, $4-CS(23;69)$, *and* $4-CS(29;87)$ *exist.*

Lengths	Sums of Squares	X, Y, Z, W of lengths n, n, n, $n-1$
3,2	$0^2+0^2+3^2+1^2$	$X = Y = (+-), Z = (++), W = (+)$
7,4	$2^2+0^2+3^2+3^2$	$X = (-+++), Y = (-+-+)$,
		$Z = (-++-), W = (+++)$
11,6	$4^2+4^2+1^2+1^2$	$X = (++-+++), Y = (+-++++)$,
		$Z = (--+-++), W = (-+++-)$
	$2^2+2^2+5^2+1^2$	$X = (+--+++), Y = (+-++-+)$,
		$Z = (-+++-+), W = (++++-)$
15,8	$6^2+0^2+3^2+1^2$	$X = (-+++++++), Y = (+---+++-)$,
		$Z = (-+-++++-), W = (+-++--+)$
19,10	$2^2+2^2+7^2+1^2$	$X = (----++++++)$,
		$Y = (-+-++-+-++)$,
		$Z = (+++--++-++)$,
		$W = (-+++-+-++)$
	$0^2+0^2+7^2+3^2$	$X = (---+-++++-)$,
		$Y = (+---+++--+)$,
		$Z = (-+--++-+++)$,
		$W = (++-+-++++)$
23,12	$4^2+2^2+7^2+1^2$	$X = (--+++-+-++++)$,
		$Y = (-++--++-+-++)$,
		$Z = (++++--++-+-+)$,
		$W = (-++++++--+-)$

Table 1: $\{X\}$, $\{Y\}$, $\{Z,W\}$ and $\{Z,-W\}$ of lengths n, n, $2n-1$, $2n-1$ have zero non-periodic autocorrelation function, ie $CS(n, 2n-1; 6n-2)$.

Lengths	Sums of Squares	X, Y, Z, W of lengths n, n, n, $n-1$
23,12	$4^2+2^2+5^2+5^2$	$X = (- - + + + + + - + - + +)$,
		$Y = (- + + + - - + - - + + +)$,
		$Z = (- - + - + + - + + + - -)$,
		$W = (+ + + + - + + + - + -)$
27,14	$4^2+4^2+7^2+1^2$	$X = (- - - + + + + + - + + + - +)$,
		$Y = (- + + - - + - + + + + + - +)$,
		$Z = (+ + + + - - - + + - + + - +)$,
		$W = (+ + + - + + + - - + - + -)$
	$4^2+4^2+5^2+5^2$	$X = (- + + + - + - + + + - + + -)$,
		$Y = (+ + - - + + + + - + - + - +)$,
		$Z = (- - - - + + + + - + + - - +)$,
		$W = (+ + + - + + + + - + - - +)$
	$6^2+6^2+3^2+1^2$	$X = (- - + - - - - - + - - - + +)$,
		$Y = (+ - - + + - + - - - - - - -)$,
		$Z = (- + - + + - + - - - + + - -)$,
		$W = (+ - + + + - + - - - - + +)$
		can be obtained by changing the signs of
		odd elements of each sequence in
		$2t = 82 = 1^2+4^2+4^2+7^2$
31,16	$4^2+2^2+7^2+5^2$	$X = (+ + - + + + + + - + + - - - + -)$,
		$Y = (- + - + - - - + + + + - + + - +)$,
		$Z = (+ - + + + + - - - + + + + + - +)$,
		$W = (- + + + - + + + - - + - - + -)$
	$0^2+6^2+7^2+3^2$	$X = (+ - - - + - + - - - + + - + + +)$,
		$Y = (- - - - - + - - + - + + + - - -)$,
		$Z = (+ + + - + - - + - - + - + - - -)$,
		$W = (- - + - - - + - - + + + - - -)$
		can be obtained by changing the signs of
		odd elements of each sequence in
		$2t = 94 = 2^2+4^2+5^2+7^2$
35,18	$10^2+2^2+1^2+1^2$	$X = (+ + + + - - + + + + + + + - + + + -)$,
		$Y = (+ - + - + + - - - + + + + - + - + -)$,
		$Z = (- - - + - + + - + + - + - - + + + -)$,
		$W = (+ - - - - - + + - + + + - + + + -)$
	$4^2+8^2+5^2+1^2$	$X = (+ - + - - + + - + - + - + + + - + +)$,
		$Y = (+ + + + + - - + - - + - + + + + + +)$,
		$Z = (- + - - - - + + + - - - - + + - + +)$,
		$W = (+ + - + - + + - - - + - - - + - -)$
		can be obtained by changing the signs of
		odd elements of each sequence in
		$2t = 106 = 1^2+1^2+2^2+10^2$
	$2^2+2^2+7^2+7^2$	$X = (+ + + + + + - - - + - - + + + - - -)$,
		$Y = (- - + - + + - - + + + + - - + + - +)$,
		$Z = (- + - - + + + - + + - + + + - - - -)$,
		$W = (+ + + + - + + - + - + + + - + - +)$

Table 1 (continued): $\{X\}$, $\{Y\}$, $\{Z, W\}$ and $\{Z, -W\}$ of lengths n, n, $2n-1$, $2n-1$ have zero non-periodic autocorrelation function, ie $CS(n, 2n-1; 6n-2)$.

Lengths	Sums of Squares	X, Y, Z, W of lengths n, n, n, $n-1$
39,20	$8^2+6^2+3^2+3^2$	$X=(+++-++++++--++-+-++-)$,
		$Y=(----+++-+++++-+-++++)$,
		$Z=(-+-+-+--+-++++--++--)$,
		$W=(+++++---+--++-+--++)$
	$0^2+6^2+9^2+1^2$	$X=(+-+++-+-+--++-----++)$,
		$Y=(-+-++-+++-+-+++++-+-)$,
		$Z=(-------++++-+--++--+)$,
		$W=(+-+-++-+++--++++--+)$
		can be obtained by changing the signs of
		odd elements of each sequence in
		$2t=118=3^2+3^2+6^2+8^2$
43,22	$0^2+0^2+9^2+7^2$	$X=(----+++-++-+--++++-+--)$,
		$Y=(---+---+++--++++++--+-)$,
		$Z=(--++-++++++-++-+-+++-+)$,
		$W=(++-++---+-+-+--++++--)$
47,24	$2^2+4^2+11^2+1^2$	$X=(++++---+-++-++-+-+-++---)$,
		$Y=(++++-++--++++-+--+---++-)$,
		$Z=(+--+++-+++----++-++-++++)$,
		$W=(++++-++-+++-+-+-+----++)$

Table 1 (continued): $\{X\}$, $\{Y\}$, $\{Z,W\}$ and $\{Z,-W\}$ of lengths n, n, $2n-1$, $2n-1$ have zero non-periodic autocorrelation function, ie $CS(n, 2n-1; 6n-2)$.

2 Multiplication theorems

We now summarize some constructions using sequences. Not all these constructions are new but they are collected here to help construct our table.

The following lemma, which closely follows work of Turyn is to be found in [37]

Lemma 4 *If there exist*

(i) $2-CS(m_1, w_1)$ *and* $2k-DCS(m_2, w_2)$ *there exist* $2k-CS(m_1m_2, w_1w_2))$,

(ii) $2-DCS(m_1, w_1)$ *and* $2k-CS(m_2, w_2)$ *there exist* $2k-CS(m_1m_2, w_1w_2))$,

(iii) $2-DCS(m_1, w_1)$ *and* $2k-DCS(m_2, w_2)$ *there exist* $2k-DCS(m_1m_2, w_1w_2))$.

In the special case with 4 sequences where A_1, A_2 *have length* m_2 *and* A_2, A_3 *have length* m_3 *we obtain* $SS((m_1m_2, m_1m_3, w_1w_2)$. □

Example 4 Suppose A_1, A_2, A_3, A_4 are $\{\bar{1}\ 1\ 1\ 0\}$, $\{0\ 0\ 0\ 1\}$, $\{1\ 0\ 1\}$, $\{0\ 1\ 0\}$ then if X, Y are 2-CS(m, w) then

$$\{\bar{X}XXY\}, \{\bar{X}YY\bar{Y}\}, \{XYX\}, \{Y\bar{X}Y\}$$

are 4-CS$(4m, 7w)$.

Suppose A_1, A_2, A_3, A_4 are $\{1\ 1\ \bar{1}\ 0\ 0\}$, $\{0\ 0\ 0\ 1\ 1\}$, $\{1\ 0\ 1\ 0\ 0\}$, $\{0\ 0\ 0\ 1\ \bar{1}\}$ then if X, Y are 2-CS(m, w) then

$$\{XX\bar{X}YY\}, \{\bar{X}\bar{X}\bar{Y}YY\}, \{X0XY\bar{Y}\}, \{X\bar{X}Y0Y\}$$

are 4-CS$(5m, 9w)$. So 2-CS(3,5) gives 4-CS(12,35) and 4-CS(15,45), 2-CS(6,10) gives 4-CS(24,70) and 4-CS(30,90), and 2-CS(11,13) gives 4-CS(44,91) and 4-CS(55,107).

Example 5 We give some useful examples of the last lemma

(i) $2-CS(3,5)$ and $2-DCS(6,5)$ give $2-CS(18,25)$;

(ii) $2-CS(3,5)$ and $2-DCS(14,13)$ give $2-CS(42,65)$;

(iii) $2-CS(3,5)$ and $2-DCS(g,g)$ give $2-CS(3g,5g)$, g a Golay number;

(iv) $2-CS(11,13)$ and $2-DCS(6,5)$ give $2-CS(66,65)$;

(v) $2-CS(11,13)$ and $2-DCS(14,13)$ give $2-CS(154,169)$;

(vi) $2-CS(11,13)$ and $2-DCS(g,g)$ give $2-CS(11g,13g)$, g a Golay number;

(vii) $4-CS(3,11)$ and $2-DCS(6,5)$ give $4-CS(18,55)$;

(viii) $2-DCS(6,5)$ and $2-DCS(6,5)$ give $2-DCS(36,25)$ and $2-DCS(6^e, 5^e)$, $e \geq 0$;

(ix) $2-DCS(14,13)$ and $2-DCS(14,13)$ give $2-DCS(196,169)$ and $2-DCS(14^f, 13^f)$, $f \geq 0$;

(x) $2-DCS(6,5)$, $2-DCS(14,13)$ and $2-DCS(g,g)$ give $2-DCS(6^e 14^f g, 5^e 13^f g)$, $e, f \geq 0$, g a Golay number. □

Lemma 5 *If there exist*

(i) $2k-CS(m_1, w_1)\ A_1, A_2, \ldots, A_{2k}$ *and* $2-CS(m_2, w_2)$, X, Y, *then there are* $2k-CS(2m_1m_2, w_1w_2)$;

(ii) $2k-CS(m_1, w_1)\ A_1, A_2, \ldots, A_{2k}$ *and* $2-DCS(m_2, w_2)$, X, Y, *then there are* $2k-DCS(2m_1m_2, w_1w_2)$;

(iii) $2k-DCS(m_1, w_1)\ A_1, A_2, \ldots, A_{2k}$ *and* $2-CS(m_2, w_2)$, X, Y, *then there are* $2k-DCS(2m_1m_2, w_1w_2)$.

Proof. Consider, where i = 1,2, ⋯, 2k,

$$\{A_{2i} \times X, A_{2i+1} \times Y^*\} \text{ and } \{A_{2i} \times Y, A_{2i+1} \times -X^*\}$$

or

$$\{A_{2i} \times X, A_{2i+1} \times Y\} \text{ and } \{A_{2i} \times Y^*, A_{2i+1} \times -X^*\}$$

□

Example 6 2-CS(3,5) and 2-CS(11,13) give 2-CS(66,65). □

Lemma 6 *If X, Y are $2-CS(m,w)$ (or $2-DCS(m,w)$) then*

$$\{cX,0_t\},\{cY,0_t\},\{a,0_{t+m+1}\},\{b,0_{t+m+1}\}$$

are $4-CS(m+t,w+2)$ when $a=b=c=1$ (or $4-DCS(m+t,w+2)$) and when a,b,c are commuting variables these four sequences can be used to generate circulant matrices to use in the Goethals-Seidel array to form $OD(4(m+t);1,1,w)$ for all $t \geq 0$.

Example 7 2-CS(3,5) and 2-DCS(10,10) give 2-CS(30,50), by lemma 4, which can be used to give an OD(4(30+t);1,1,50), and 2-CS(18,25) can be used to give an OD(4(18+t);1,1,25) .

Lemma 7 *If X,Y,Z,W are $4-CS(m,w)$ (or $4-DCS(m,w)$) then*

$$\{aX,bY,0_t\},\{bX,a\bar{Y},0_t\},\{aZ,bW,0_t\},\{bZ,a\bar{W},0_t\}$$

are $4-CS(2m+t,2w)$ when $a=b=1$ (or $4-DCS(2m+t,2w)$) and when a,b are commuting variables these four sequences can be used to generate circulant matrices to use in the Goethals-Seidel array to form $OD(4(2m+t);w,w)$ for all $t \geq 0$.

Lemma 8 *Suppose there are 2-$CS(m,w)$, X,Y. Let a,b,c be commuting variables. Then $\{cX,a,c\bar{X}^*,0_t\},\{cY,b,c\bar{Y}^*,0_t\},\{cX,0,cX^*,0_t\},\{cY,0,cY^*,0_t\}$ are the 4-$CS(4w+2,2m+t+1)$ which can be used to form the first rows of four circulant matrices which then used in the Goethals-Seidel array to form an $OD(4(2m+t+1);1,1,4w)$ for all $t \geq 0$.*

Example 8 There exist 2-DCS(n,u) and 2-CS$(n,2u)$ where $n = 2^a10^b26^d6^e14^b$ and $u = 2^a10^b26^d5^e13^b$ or $2u$. By the lemma $OD(4(2n+t+1);1,1,4u)$ exist for all $t \geq 0$. Hence we have

$$OD(4(17+t);1,1,64), OD(4(13+t);1,1,40), OD(4(23+t);1,1,52)$$

$$OD(4(29+t);1,1,104), OD(4(21+t);1,1,80).$$

Giving the entries for weights 65 and 81 in the table. There exist 2-CS(11,13) and hence $OD(4(21+t);1,1,52)$. □

From [17, p406] Table H.4 there are 4-sequences with zero periodic autocorrelation function which give useful results.

Lemma 9 *$OD(4n;1,1,50)$ exist for all $n \geq 13$. $OD(4n;1,52)$ and $OD(4n;1,1,58)$ exist for all $n \geq 15$.*

3 Some ad hoc multiplication lemmas

Lemma 10 *Suppose A, B, C are $NS(n)$ of weight w then there are $SS(3n,3w)$. If $B \pm C$ are $(0,1,-1)$ sequences then there are $TW(6n,3w)$ and 4-$CS(2n,3w)$.*

Proof. We use the sequences

$$\{\ A,\ B,\ C\},\ \{\ B,\ -C\},\ \{\ A,\ 0_n,\ -C\},\ \{\ A,\ -B\},$$

$$\{\ A,\ B^*+C^*\},\ \{\ 0_{2n},\ A,\ -B^*\},\ \{\ 0_{3n},\ C,-A^*\},\ \{\ 0_{5n},\ B-C\}$$

and

$$\{\ A,\ B+C\},\ \{\ A,\ -B\},\ \{\ A,\ -C\},\ \{\ B,\ -C\}$$

respectively. □

Lemma 11 *Suppose A, B, C and D are $4-CS(n,w)$ then there are*

(i) $4-CS(4n,3w)$

(ii) $4-CS(6n,5w)$

(iii) $4-DCS(n,w)$ *give* $4-DCS(6n,5w)$.

Proof. We use the sequences

$$\{\ A,\ B,\ C\},\ \{\ A,\ -B,\ O_n,\ D\},\ \{\ A,\ 0_n,\ -C,\ -D\},\ \{\ B,\ -C,\ D\},$$

and

$$\{A,A,-A,C,0_n,C\},\ \{B,B,-B,D,0_n,D\},\ \{A,0_n,A,C,-C,-C\},$$

$$\{B,0_n,B,D,-D,-D\},$$

for (ii) and (iii) respectively. □

Example 9 The following 4-$CS(3,11)$ $\{111\},\{1-1\},\{11-\},\{10-\}$ have length 3 and total weight 11 so there are 4-$CS(18,55)$ of length 18 and total weight 55.

Using sequences of lengths 5 and weights 17 and 19 gives the sequences of lengths 30 and weights 85 and 95 respectively.

The sequences of lengths 5, 6, 7, 7, 8, 9 and 9 and weights 19, 21, 23, 25, 29, 31 and 33 give the results for weights 57, 63, 69, 75, 87, 93 and 99 and lengths 20, 24, 28, 28, 32, 36 and 36 respectively. □

We restate a theorem of Yang in the language used in [27] at the same time deleting the zeros from the beginnings or the ends of the constructed sequences:

4 Multiplication by y a Yang multiplier

Lemma 12 *Let A, B, C, D be $SS(p,n;w)$ and X,Y,G,H of lengths m, m, $2m$, $2n$ respectively be obtained from $NN(4m+1)$, then the following sequences Q,R,S,T are TW-sequences, $TW(y(p+n);yw)$, $y=4m+1$, where $X=\{x_1,x_2,\cdots,x_{m-1},x_m\}$, $Y=\{y_1,y_2,\cdots,y_{m-1},y_m\}$, $G=\{g_1,g_2,\cdots,g_{2m-1},g_{2m}\}$, and $H=\{h_1,h_2,\cdots,h_{2m-1},h_{2m}\}$.*

$$Q=\{Ag_{2m}-Bh_1,-Cy_1;Ag_{2m-1}-Bh_2,-D^*x_1;\cdots;Ag_2-Bh_{2m-1},-Cy_m;$$

$$Ag_1-Bh_{2m},-D^*x_m;0',-D^*;0',0;0',0;\cdots;0',0;0',0\}$$

$$R = \{Bg_1 + Ah_{2m}, -Dy_1; Bg_2 + Ah_{2m-1}, C^*x_1; \cdots; Bg_{2m-1} + Ah_2, -Dy_m;$$
$$Bg_{2m} + Ah_1, C^*x_m; 0', C^*; 0', 0; 0', 0; \cdots; 0', 0; 0', 0\}$$
$$S = \{0', 0; 0', 0; \cdots; 0', 0; 0', 0; -B, 0; -Bx_m, C^*g_{2m} + D^*h_{2m}; A^*y_1,$$
$$C^*g_{2m-1} + D^*h_{2m-1}; \cdots; -Bx_1, C^*g_2 + D^*h_2; A^*y_m, C^*g_1 + D^*h_1\}$$
$$T = \{0', 0; 0', 0; \cdots; 0', 0; 0', 0; A, 0; Ax_m, D^*g_1 - C^*h_1; B^*y_1, D^*g_2 - C^*h_2; \cdots;$$
$$\cdots; Ax_1, D^*g_{2m-1} - C^*h_{2m-1}; B^*y_m, D^*g_{2m} - C^*h_{2m}\}$$

where $0'$, 0 are sequences of zeros of length p and n respectively. If A, B, C, D are $SS(p, n; 2p + 2n)$ then Q,R,S,T are T-sequences.

$$Q_1 = \{Ag_{2m} - Bh_1, -Cy_1; Ag_{2m-1} - Bh_2, -D^*x_1; \cdots; Ag_2 - Bh_{2m-1}, -Cy_m;$$
$$Ag_1 - Bh_{2m}, -D^*x_m; 0', -D^*\}$$
$$R_1 = \{Bg_1 + Ah_{2m}, -Dy_1; Bg_2 + Ah_{2m-1}, C^*x_1; \cdots; Bg_{2m-1} + Ah_2, -Dy_m;$$
$$Bg_{2m} + Ah_1, C^*x_m; 0', C^*\}$$
$$S_1 = \{-B, 0; -Bx_m, C^*g_{2m} + D^*h_{2m}; A^*y_1, C^*g_{2m-1} + D^*h_{2m-1}; \cdots; -Bx_1,$$
$$C^*g_2 + D^*h_2; A^*y_m, C^*g_1 + D^*h_1\}$$
$$T_1 = \{A, 0; Ax_m, D^*g_1 - C^*h_1; B^*y_1, D^*g_2 - C^*h_2; \cdots; Ax_1, D^*g_{2m-1} - C^*h_{2m-1};$$
$$B^*y_{2m}, D^*g_{2m} - C^*h_{2m}\}$$

of lengths $(y+1)(p+n)/2$ and total weight yw ie $SS((y+1)(p+n)/2; yw)$.

Now since Q, R, S, T are $TW(y(m+n); yw)$

$$\{Q, R^*\}, \{Q, \bar{R}^*\}, \{S, T^*\}, \{T, \bar{S}^*\}$$

are $TW(2y(p+n); 2yw)$. These TW-sequences can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4s; yw, yw, yw, yw)$ for $s \geq y(p+n)$ and $OD(4t; 2yw, 2yw, 2yw, 2yw)$ for $t \geq 2y(p+n)$.

Let a and b be commuting variables. Then $aQ_1 + bR_1$, $bQ_1 - aR_1$, $aS_1 + bT_1$, $aS_1 - bT_1$ can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4u; yw, yw)$ for $u \geq (y+1)(p+n)/2$.

Lemma 13 *If there exist $SS(p, n; w)$ and $NN(y)$ then there exist $SS((y+1)(p+n)/2, yw$.*

Example 10 Let E = {1 1}, F= {1}, G = { 1 -}, H = {0 0}.

Then $x_1 = 1$, $y_1 = 1$, $g_1 = 1$, $g_2 = -1$, $h_1 = 0$ and $h_2 = 0$ giving

$$Q_1 = \{-A, -C; A, -D^*; 0, -D^*\}$$
$$R_1 = \{B, -D; -B, C^*; 0, C^*\}$$
$$S_1 = \{-B, 0 : -B, -C^*; A^*, C^*\}$$
$$T_1 = \{A, 0; A, D^*; B^*, -D^*\}$$

show that 4-CS(n;w) and NN(5) give 4-CS($6n$;$5w$) and 4-CS(m,n;w) and NN(5) give 4-CS(3(m+n;$5w$). □

Example 11 4-CS(5;17) and 4-CS(5;19) can be used with NN(5) to obtain 4-CS(30;85) and 4-CS(30;95) respectively.

Lemma 14 *If there exist 4-CS$(n;w)$ and NN(y) then there exist 4-CS$((y+1)n;yw)$.*

Example 12 4-CS(3,7) gives 4-CS(42,91). □

5 Multiplication by 3

Lemma 15 *Let A, B, C, D be $SS(m,n;w)$. Then consider the following sets of four sequences, where 00 is the sequences of $m+n$ zeros and 0 is the sequence of n zeros,*

$$X = \{AC; 00; B^*0\}$$

$$Y = \{BD; 00; \bar{A}^*0\}$$

$$Z = \{00; A\bar{C}; 0D^*\}$$

$$W = \{00; B\bar{D}; 0\bar{C}^*\}$$

are $SS(3(m+n), 3(m+n); 3w)$. Now X, Y, Z, W are $TW(3(m+n);3w)$ and

$$\{X, Y^*\}, \{Y, \bar{X}^*\}, \{Z, W^*\}, \{W, \bar{Z}^*\}$$

are TW (6(m+n);6w). These TW-sequences can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4s; 3w, 3w, 3w, 3w)$ for $s \geq 3(m+n)$ and $OD(4t; 6w, 6w, 6w, 6w)$ for $t \geq 6(m+n)$. Further we note that zeros can be eliminated giving

$$P = \{AC; 00; B^*\}$$

$$Q = \{BD; 00; \bar{A}^*\}$$

$$R = \{A\bar{C}; 0D^*\}$$

$$S = \{B\bar{D}; 0\bar{C}^*\}$$

of lengths $3m+2n$, $3m+2n$, $2m+2n$, $2m+2n$ and total weight $3w$ ie 4-CS$(3m+2n;3w)$). These are also $SS(3m+2n, 2m+2n; 3w)$ and can be used recursively in the construction.

Let a and b be commuting variables. Then $aP+bQ$, $bP-aQ$, $aR+bS$, $aR-bS$ can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4u; 3w, 3w)$ for $u \geq 3m+2n$.

Example 13 Let $A = \{10\}$, $B = \{01\}$, $C = \{1\}$, $D = \{0\}$.

$$X = \{101; 000; 100\}$$

$$Y = \{010; 000; 0-0\}$$

$$Z = \{000; 10-; 000\}$$

$$W = \{000; 010; 00-\}$$

are $SS(9,9;9)$ and are also $TW(9;9)$ and give $OD(4t;9,9,9,9)$, for $4t \geq 36$. Further

$$\{X,Y^*\},\{Y,\bar{X}^*\},\{Z,W^*\},\{W,\bar{Z}^*\}$$

are $TW(18;18)$ and give $OD(4s;18,18,18,18)$, for $4s \geq 72$.

$$P = \{101;000;10\}$$

$$Q = \{010;000;0-\}$$

$$R = \{10-;000\}$$

$$S = \{010;00-\}$$

are 4-complementary sequences of 8, 8, 6 and 6 and total weight 9 and zero autocorrelation function. Furthermore they are $SS(8,6;9)$.

Using the construction recursively we get $TW(42;27)$ and $SS(36,18;27)$.

We note that in this case the sequences can be shortened further to give 4-complementary sequences $P_1 = \{1010001\}$, $Q_1 = \{100000-\}$, $R_1 = \{10-\}$, $S_1 = \{1000-\}$ of lengths 7, 7, 3, 5 and total weight 9. □

Lemma 16 *Suppose A, B, C, and D are 4-CS$(m,n;w)$ of lengths m, m, n, n where $m \geq n$ then*

$$\{\ 0_m,\ A,\ C,\ 0_m,\ 0_n,\ B^*\},\ \{\ A,\ 0_m,\ 0_n,\ -D^*,\ -B^*,\ 0_m\},$$

and

$$\{\ A,\ -C,\ 0_m,\ D*,\ 0_n,\ 0_m\},\ \{\ 0_m,\ 0_n,\ C,\ 0_m,\ D^*,\ -B^*\},$$

are $SS(4m+2n, 3m+3n; 3w)$, or, prefacing the last two sequences by 0_{3m+3n}, $TW(7m+5n;3w)$.

6 Multiplication by 7

Lemma 17 *Let A, B, C, D be $SS(m,n;w)$. Then consider the following sets of four sequences, where 00 is the sequence of $m+n$ zeros and 0 is the sequence of n zeros,*

$$X = \{\bar{A}C;00;AD;00;AC;00;\bar{B}^*0\}$$

$$Y = \{BD;00;BC;00;BD;00;A^*0\}$$

$$Z = \{00;A\bar{C};00;\bar{B}\bar{C};00;AC;0D^*\}$$

$$W = \{00;B\bar{D};00;A\bar{D};00;BD;0C^*\}$$

all of length $7(m+n)$ and total weight $7w$, or $SS(7(m+n),7(m+n);7w)$. Now X, Y, Z, W are $TW(7(m+n);7w)$ and

$$\{X,Y^*\},\{Y,\bar{X}^*\},\{Z,W^*\},\{W,\bar{Z}^*\}$$

are $TW(14(m+n);14w)$. These TW-sequences can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4s;7w,7w,7w,7w)$ for $s \geq 7(m+n)$ and $OD(4t;14w,14w,14w,14w)$ for $t \geq 14(m+n)$.

Further we note that zeros can be eliminated giving

$$P = \{\bar{A}C;00;AD;00;AC;00;\bar{B}^*\}$$

$$Q = \{\bar{B}D; 00; B\bar{C}; 00; BD; 00; A^*\}$$
$$R = \{A\bar{C}; 00; \bar{B}\bar{C}; 00; AC; 0\bar{D}^*\}$$
$$S = \{B\bar{D}; 00; A\bar{D}; 00; BD; 0C^*\}$$

of lengths $7m+6n$, $7m+6n$, $6m+6n$, $6m+6n$ and total weight $7w$, These are $SS(7m+6n, 6m+6n; 7w)$ and can be used recursively in the construction.

Let a and b be commuting variables. Then $aP+bQ$, $bP-aQ$, $aR+bS$, $aR-bS$ can be used to form circulant matrices used in the Goethals-Seidel array which give $OD(4u; 7w, 7w)$ for $u \geq 7m+6n$.

If A, B, C, D are $4-CS(n;w)$ then P, Q, R, S are $4-CS(13n;7w)$.

Example 14 Let $A = \{11-0\}$, $B = \{0001\}$, $C = \{101\}$, $D = \{010\}$, then
$X = \{--10101; 0_7; 11-0010; 0_7; 11-0101; 0_7; -000000\}$
$Y = \{000-010; 0_7; 0001-0-; 0_7; 0001010; 0_7; 0-11000\}$
$Z = \{0_7; 11-0-0-; 0_7; 000--0-; 0_7; 11-0101; 00000-0\}$
$W = \{0_7; 00010-0; 0_7; 11-00-0; 0_7; 0001010; 0000101\}$
are $TW(49;49)$ and give $OD(4t; 49, 49, 49, 49)$, for $4t \geq 196$. Further

$$\{X, Y^*\}, \{Y, \bar{X}^*\}, \{Z, W^*\}, \{W, \bar{Z}^*\}$$

are $TW(98;98)$ and give $OD(4s; 98, 98, 98, 98)$, for $4s \geq 392$.

We also note that
$P = \{--10101; 0_7; 11-0010; 0_7; 11-0101; 0_7; -000\}$
$Q = \{000-010; 0_7; 0001-0-; 0_7; 0001010; 0_7; 0-11\}$
$R = \{11-0-0-; 0_7; 000--0-; 0_7; 11-0101; 00000-0\}$
$S = \{00010-0; 0_7; 11-00-0; 0_7; 0001010; 0000101\}$
give sequences of length 46 and weight 49 and hence $OD(4u; 49, 49)$ for $4u \geq 184$. Careful observation shows that P, Q, R, S can all be further shortened by removing zeros from both the beginning and the end of each sequence giving
$P_1 = \{--10101; 0_7; 11-0010; 0_7; 11-0101; 0_7; -\}$
$Q_1 = \{-010; 0_7; 0001-0-; 0_7; 0001010; 0_7; 0-11\}$
$R_1 = \{11-0-0-; 0_7; 000--0-; 0_7; 11-0101; 00000-\}$
$S_1 = \{10-0; 0_7; 11-00-0; 0_7; 0001010; 0000101\}$
give sequences of length 43 and weight 49 and hence $OD(4u; 49, 49)$ for $4u \geq 172$. □

Corollary 2 *All $OD(4s; 98, 98, 98, 98)$ exist for $4s \geq 392$. Hence all $W(4s, 392)$ exist for $4s \geq 392$.*

Example 15 Using the SS(3,4;7) $\{101\}, \{010\}, \{11-0\}, \{0001\}$ and using the NS(3) we obtain SS(45,45;91). □

Example 16 4-CS(3,7) gives 4-CS(39,49), 4-CS(3,11) gives 4-CS(39,77) and 4-CS(4,13) gives 4-CS(52,91). □

Length=13	Sequences with zero periodic autocorrelation function
1,42	{− + + − − 0 *a* 0 + + − − +}, {− + − 0 − + 0 + + + + + +}, {+ + + + − + − 0 + 0 0 + −}, {+ + − − − + 0 0 + + + − −}

Table 2: Sequences of length 13 with zero periodic autocorrelation function

Length=15	Sequences with zero non-periodic autocorrelation function
1,42	{0 + 0 + − − + *a* − + + − 0 − 0}, {+ 0 0 + + + 0 + − 0 + − 0 0 +}, {0 0 + + 0 + + + + − + − − − +}, {− 0 − + + + 0 − − + 0 + − 0 +}
1,48	{+ + + − − + − *a* + − + + − − −}, {+ 0 + − + − 0 0 + 0 − + + + 0}, {+ − + + 0 + 0 0 + + + − 0 − 0}, {+ + − − − + − 0 + + − + + + +}
1,54	{+ − + − − − − 0 + + + + − + −}, {− + − + + + + 0 + + + + − + −}, {+ + − − + + + 0 + − − + + − 0}, {+ − − + + − + 0 + + − − + + 0}
55	{− + + + − + − + + + − 0 + 0 +}, {− + − + − + + − + + + + − + +}, {− + + − − − − 0 + + + + + + −}, {+ + + − − + − − + + − 0 − 0 +}
1,56	{+ − + − − − − *a* + + + + − + −}, {− + − + + + + 0 + + + + − + −}, {+ + − − + + + + + − − + + − 0}, {+ − − + + − + − + + − − + + 0}
56	{0 − − − + − + + − − + − + + +}, {− + + + + − + + + − + − + + 0}, {+ + − + + + 0 − + − − − + + +}, {+ + + + − + 0 − + − − − + + −}
1,59	{− − + − + − − *a* + + − + − + +}, {+ + + + + − + − + − − + + − +}, {− + − − + + + + + + − − + + +}, {+ + − + + + − + + − − − − + +}

Table 3: Sequences of length 15 with zero non-periodic autocorrelation function

Length=15	Sequences with zero periodic autocorrelation function
1,48	{+ + + 0 − + − *a* + − + 0 − − −},
	{0 + − + − + − 0 0 + + − + + +},
	{− + + + + + 0 0 + + − − − + 0},
	{− + + 0 + + − 0 + + + 0 − + −}
1,56	{*a* − − − + − + + − − + − + + +},
	{− + + + + − + + + − + − + + 0},
	{+ + − + + + 0 − + − − − + + +},
	{+ + + + − + 0 − + − − − + + −}

Table 4: Sequences of length 15 with zero periodic autocorrelation function

Length=17	Sequences with zero non-periodic autocorrelation function
61	{− + + − + + + + +0 − +0 − + − +},
	{+ − + + 0 + + + + + − − − + + − −},
	{− + + − + + + − 0 − + − + + + + −},
	{+ + +0 − −0 − + + − − +0 + −+}
62	{0 + + + + − + + − + + − + − + − +},
	{0 − + + + − + + + + − + 0 + − − +},
	{− − 0 + + − − + + + + + − − 0 + −},
	{− − − − 0 + + + + − + − − + + − +}
63	{+ − + − + − + + 0 + + + + + + − −},
	{+ + − + − + + − + + − + + − − + +},
	{0 − + + − 0 − − − − + + + − + ++},
	{+ − + + + − − − + + + 0 − + − − 0}

Table 5: Sequences of length 17 with zero non-periodic autocorrelation function

Length=17	Sequences with zero periodic autocorrelation function
1,42	$\{+ 0 + 0 + + - - a + + - - 0 - 0 -\}$,
	$\{- + 0 + 0 + 0 0 + + 0 + 0 + 0 - 0\}$,
	$\{0 + 0 + 0 + + - - + 0 - 0 + - + +\}$,
	$\{+ 0 0 + 0 + 0 - - + 0 + 0 - 0 - 0\}$
1,49	$\{- - 0 - - + 0 + a - 0 - + + 0 + +\}$,
	$\{- + 0 + - + + 0 0 + 0 + + + - 0 +\}$,
	$\{+ + 0 - + + + + 0 + 0 - - + - 0 -\}$,
	$\{- + + 0 + 0 - 0 0 + - - - + + 0 +\}$
1,57	$\{+ + + - + - 0 + a - 0 + - + - - -\}$,
	$\{- + + + - + - - 0 + + + 0 + + + -\}$,
	$\{+ 0 - + - 0 - - + + - + + + + 0 +\}$,
	$\{+ + - + - - + - 0 + + + 0 + - 0 +\}$
1,61	$\{- - - 0 + + - + a - + - - 0 + + +\}$,
	$\{- + - + + + + + + + + + - + - - 0\}$,
	$\{+ + - + - + + + - + + - + + - - 0\}$,
	$\{+ + - - 0 + - + + + - 0 - + + - +\}$
1,62	$\{- + + + - - + - a + - + + - - - +\}$,
	$\{0 + + + + + - + + + - + - + + - -\}$,
	$\{0 + + + + + + 0 + + - + - + - - -\}$,
	$\{0 + + - - + - 0 + + - - - + + - +\}$
1,1,64	$\{+ + + - + + - + a - + - - + - - -\}$,
	$\{+ + + - - - + - b + - + + + - - -\}$,
	$\{+ + + - + + - + 0 + - + + - + + +\}$,
	$\{+ + + - - - + - 0 - + - - - + + +\}$
1,67	$\{+ + - + + + + + a - - - - - + - -\}$,
	$\{- - - + - - + + - - + - - - + + +\}$,
	$\{- - - + - + - - + - + - - - + + +\}$,
	$\{+ - + - - - - - - + - + - - + - -\}$

Table 6: Sequences of length 17 with zero periodic autocorrelation function

Length=$2t$+15, $t \geq 0$	Sequences with zero non-periodic autocorrelation function
1,56	$\{a\ 0_t - - - + - + + - - + - + + + 0_t\}$,
	$\{- + + + + - + + + - + - + + 0\ 0_{2t}\}$,
	$\{+ + - + + + 0 - + - - - + + + 0_{2t}\}$,
	$\{+ + + + - + 0 - + - - - + + - 0_{2t}\}$

Table 7: Sequences of length $2t$+15, $t \geq 0$ with zero non-periodic autocorrelation function

Length	Weight	Comment
3	7	[17, Table H.1 and H.2]
5	15	[17, Table H.1 and H.2]
$\lceil w/4 \rceil$	$w \leq 20$	$w \neq 7,\ 15$ [17, Table H.1 and H.2]
6	21	[17]
6	22	base sequences
7	23	[17]
6	24	double sequences of length 3 [17, p401]
7	25	[17]
7	26	[17]
7	27	[17]
7	28	[17]
8	29	[17]
8	30	[17]
9	31	[17]
8	32	[17]
9	33	[17]
9	34	[17]
9	35	[17]
9	36	[17]
10	37	[17]
10	38	base sequences: remark 1
10	39	[17]
10	40	Golay sequences [17]
13	41	[17]
11	42	base sequences: remark 1
11	43	[17]
11	44	[17]
12	45	[17]
12	46	base sequences: remark 1
13	47	[17]
12	48	double sequences of length 6 weight 24
15	49	Table 3
13	50	base sequences: remark 1
13	51	[17]
13	52	[17]
15	53	[29]
14	54	base sequences: remark 1
15	55	Table 3
14	56	double sequences of length 7 weight 28
15	57	[29]
15	58	base sequences: remark 1
15	59	Table 3
15	60	base sequences of lengths 8, 8, 7, 7: remark 1

Table 8: The smallest length for which there are four sequences with zero non-periodic autocorrelation function.

Length	Weight	Comment
17	61	Table 5
16	62	base sequences: remark 1
17	63	Table 5
16	64	[17]
17	65	Golay: example 8
17	66	base sequences: remark 1
18	67	[17]
18	68	[17]
18	69	[17]
18	70	base sequences: remark 1
24	71	base sequences: remark 1
18	72	double sequences of length 9 weight 36
	73	
19	74	base sequences: remark 1
25	75	example 3
20	76	double sequences of length 10 weight 38
39	77	example 16
20	78	base sequences: remark 1
	79	
20	80	Golay sequences: [17]
21	81	Golay: example 8
21	82	base sequences: remark 1
	83	
21	84	base sequences of lengths 11,11,10,10; remark 1
30	85	example 9
22	86	base sequences: remark 1
29	87	example 3
22	88	double sequences of length 11 weight 44
	89	
23	90	base sequences: remark 1
44	91	example 4
23	92	from base sequences of lengths 12,12,11,11: remark 1
31	93	example 3
24	94	base sequences: remark 1
30	95	example 9
24	96	double sequences of length 12 weight 48
	97	
25	98	base sequences: remark 1
33	99	example 3
25	100	base sequences of lengths 13, 13, 12, 12: remark 1

Table 8 (continued):The smallest length for which there are four sequences with zero non-periodic autocorrelation function.

7 Numerical consequences

We use the tables of Appendix H of Geramita and Seberry [17] with non-periodic autocorrelation function zero and note that sequences with periodic autocorrelation function zero exist for $W(4t, 4t-1)$, $W(4t, 4t)$ for all $t \in \{1, \ldots, 31\}$ [37].

We extend Theorem 4.149 of Geramita and Seberry [17] using the results of [29] and those given in Tables 2 and 3:

Theorem 2 *There exists an orthogonal design $OD(4n; 1, k)$ when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9$ *with* $k \in \{x : x \leq 4t-1,\ x = a^2 + b^2 + c^2\}$;

(ii) *for* $n \geq 11$, *with* $k \in \{x : x \leq 43,\ x = a^2 + b^2 + c^2,\ x \neq 42\}$;

(iii) *for* $n \geq 13$, *with* $k \in \{x : x \leq 51,\ x = a^2 + b^2 + c^2,\ x \neq 46, 49\}$;

(iv) *for* $n \geq 15$, *with* $k \in \{x : x \leq 59,\ x = a^2 + b^2 + c^2,\ x \neq 46, 49, 57\}$;

(v) *for* $n \geq 17$, *with* $k \in \{x : x \leq 67,\ x = a^2 + b^2 + c^2,\ x \neq 46, 49, 57, 60, 61, 62, 66, 67\}$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. The sequences for $OD(4n; 1, 34)$, $n \geq 13$ are given in [29]. The $OD(36; 1, 34)$ is given in [17, Lemma 8.31] and the $OD(44; 1, 34)$ is given in [17, Lemma 8.36]. This completes the case for $t = 9$.

The sequences for $OD(4n; 1, 37)$, $n \geq 13$ are given in [29]. The $OD(44; 1, 37)$ is given in [17, Lemma 8.36]. This gives case (ii).

The sequences for $OD(52; 1, 42)$, are given in Table 2, while the sequences for $OD(4n; 1, 42)$, $n \geq 15$ are given in Table 3. The sequences for $OD(4n; 1, 45)$, $n \geq 13$, $OD(4n; 1, 53)$ and $OD(4n; 1, 56)$, $n \geq 15$ are given in [29]. The sequences for $OD(4n; 1, 48)$ and $OD(4n; 1, 54)$, $n \geq 15$, are given in Table 3. Hence noting $OD(52; 1, 48)$ is given in [29] we have cases (ii), (iii) and (iv).

Now there are Golay sequences of length 8 and total weight 16 so by [17, Lemma 4.118] we have $OD(4n; 1, 1, 64)$, $n \geq 17$.

$OD(4n; 1, 66)$ and $OD(4n; 1, 68)$, $n \geq 18$ are given in [17, Table II.2]. □

Theorem 3 *There exists a $W(4n; k)$ when*

(i) *for* $n \geq t$, $t = 3, 5, 7, 9, 11$ *with* $k \in \{x : x \leq 4t\}$;

(ii) *for* $n \geq 13$, *with* $k = 1, \ldots, 52$;

(iii) *for* $n \geq 15$, *with* $k = 1, \ldots, 60$;

(iv) *for* $n \geq 17$, *with* $k = 1, \ldots, 68$;

(v) *for* $n \geq 19$, *with* $k = 1, \ldots, 70, 72, 74, 76$.

All are constructed by using four circulant matrices in the Goethals-Seidel array.

Proof. Case (i) follows from the previous theorem by setting variables equal to each other or to zero.

The previous theorem gives us all values for case (ii) except 47 which may be found in [17, Table II.2].

Given the results in case (ii) we need to consider $k = 53, 54, 55, 56, 57, 58, 59$ and 60. The result for 58, 59 and 60 can be found in [17, Table H.4], the result for 53, 54, 56, 57 can be found in [29]. We give the result for 55 in Table 3.

For case (iv) we consider 61, 62, 63, 64, 65, 66, 67 and 68. The $OD(4n;1,1,64)$, $n \geq 17$, mentioned in the proof of the previous Theorem gives 64, 65 and 66. There is an $OD(68;1,67)$ (from good matrices [37]) and $OD(4n;1,2,66)$, $n \geq 18$ are given in [17, Table H.2] giving the result for 67 and 68. The sequences for 61, 62 and 63 are given in Table 5 for $n \geq 17$. This completes case (iv) and gives 69 for $n \geq 18$.

We note that if $\{A\}$, $\{B\}$, $\{C\}$ and $\{D\}$ are sequences of lengths $m+1$, m, $m+1$ and m and weight w then $\{A,B\}$, $\{A,-B\}$, $\{C,D\}$ and $\{C,-D\}$ are sequences of length $2m+1$ and weight $2w$. Now [17, Table H.2] gives us sequences of this type for $m = 9$ with weights $w = 35, 36, 37$ and 38 giving sequences of length ≥ 19 for 70, 72, 74 and 76. □

Lemma 18 *There exists a $W(52,k)$ for $k \in \{x : 0 \leq x \leq 52\}$. All may be constructed from four circulant matrices using the Goethals-Seidel array.*

Proof. Geramita and Seberry [17] give a $W(52,k)$ for all $k \neq 46$. Table 8 shows that $W(4t,46)$ exist for all $t \geq 12$ and may be constructed using sequences to give first rows of circulant matrices to use in the Goethals-Seidel array giving the result. □

Lemma 19 *There exists a $W(60,k)$ for $k \in \{x : 0 \leq x \leq 60\}$. All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Use Theorem 3. □

Lemma 20 *There exists a $W(68,k)$ for $k \in \{x : 0 \leq x \leq 68\}$. All are constructed from four circulant matrices in the Goethals-Seidel array.*

Proof. Use Theorem 3. □

Lemma 21 *There exists a $W(76,k)$ for for all k except possibly 71, 73, which are undecided. All are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof. Theorem 3 gives all values except 71, 73 and 75. The result for $W(76,75)$ is obtained from [37]. □

Lemma 22 *There exists a $W(84,k)$ for all k except possibly 71, 79, which are undecided. All but $W(84,77)$ are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof.

This result appears in [35] except for the circulant property.

The Theorem 3 gives all values except 71, 73, 75, 77, 78, 79, 80, 81, 82, 83, 84. From [17, p335] proof of Lemma 8.20, we see $W(42,k)$ constructed from two circulants exist for $k \in \{0,1,2,4,5,8,10,13,16,17,20,26,32,34,40,41\}$. Hence we have $W(84,k)$ constructed from four circulants for $k \in \{61,73,75,80,81,82\}$. There are sequences of lengths 11, 10, 11, 10 and weight 39 from [17, Table H.2] and so the construction mentioned at the end of the proof of Theorem 2 gives sequences for the $W(84, 78)$. In addition, [37, 17, p345] gives a $W(84,83)$ and a $W(84,84)$ constructed using four circulants.

There is an $OD(28;7,7,7,7)$ and so replacing the variables by the back-circulant matrix with first row $\{0\ +\ -\}$, and the circulant matrices with first rows $\{-\ +\ +\}$, $\{-\ +\ +\}$ and $\{+\ +\ +\}$ respectively gives a $W(84, 77)$. □

Lemma 23 *There exists a $W(92,k)$ for all k except possibly* 71, 73, 75, 77, 79, 83, 85, 87, 89 *which are undecided. All are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof. We have the result from our Table 8 and Theorem 3 $W(92,k)$ $k \in \{1,\dots,70,72,74,76,78,80,\dots,82,84,86,88,90,92\}$. The skew-Hadamard matrix in [37] gives the required $W(92,91)$.

Lemma 24 *There exists a $W(100,k)$ for all k except possibly* 71, 77, 83, 87, 89, 91, 93, 95, 97 *which are undecided. All are constructed by using four circulant matrices in the Goethals-Seidel array.*

Proof. The proof is as for Lemma 23 but now the appropriate skew-Hadamard matrix gives a $W(100,99)$.

8 A new construction for weighing matrices

Seberry and Whiteman [36, 37] showed that for many orders there are n+1 sets of regular matrices $A_1, A_2, \dots, A_{n+1}$, with entries 1,−1 and order n^2, which pairwise satisfy $A_iA_j^T = aJ$, $i \neq j$, a constant, and $\Sigma A_iA_i^T = n^2(n+1)I_{n^2}$. Using the bipartite graph technique of Rodger, Sarvate and Seberry [33] we can ensure any weighing matrix $W(m,k)$ can be coloured with k colours so that each colour is attached to one and only one non-zero element in each row and column. Thus, we have, replacing each element coloured $i(-i)$ by $A_i(-A_i)$ and each zero by a zero matrix of order n^2

Lemma 25 *Let $n \equiv 3 (mod\ 4)$ be a prime power. Suppose there exists a $W(m, n+1)$. Then there is a $W(mn^2, n^2(n+1))$.*

Proof. Since the +1 and the −1 cancel each other when the inner product is taken of any two rows or columns of a weighing matrix the aJ contributions from the matrices will also cancel. □

We note that R. Craigen [5, 7] has proved related results using *isoclinic k-sets* and systems of distinct representatives.

Corollary 3 *Since a $W(m,4)$ exists for all $m \geq 4$, $m \neq 5,9$, there exists a $W(9m,36)$ for all $m \geq 4$, $m \neq 5,9$.*

Corollary 4 *Since a $W(2m,5)$ exists for all $m \geq 3$, there exists a $W(18m,45)$ for all $m \geq 3$.*

Corollary 5 *Since a $W(2m,8)$ exists for all $m \geq 5$, there exists a $W(98m,392)$ for all $m \geq 5$.*

Now corollary 4 gives $W(m,392)$ for $m = 490$, 686 and 892. The circulant $W(57,49)$ can be used to give a $W(114,98)$ and then taking the Kronecker product with a $W(7,4)$ we obtain a $W(798,392)$. Combining these results with those of corollary 2 gives:

Corollary 6 *All $W(n,392)$, $n \equiv 0 (mod\ 2)$, $n \geq 880$ exist and they also exist for $n = 490$, 686 and 798.*

Note. We note the circulant $W(57,49)$ also gives a $W(114,100)$.

9 New results on the $W(n,45)$

Since a $W(n,k)$ can only exist for n odd when k is a square we know a $W(n,45)$ can only exist for even k.

Lemma 26 *There exists a $W(n,45)$ for*

(i) *all $m \equiv 0 (mod\ 4)$, $m \geq 48$,*

(ii) *all $m \equiv 2 (mod\ 4)$, $m \geq 88$ and $m = 46$, 54 and 78.*

The unresolved cases are $m \in \{50, 58, 62, 66, 70, 74, 82, 86\}$.

Proof. Geramita and Seberry [17, pp346-7] give the existence of $W(n,45)$ for $n \equiv 0 (\mathrm{mod}\ 8)$, $n \geq 11$, and $n \in \{52,60,68,76,84\}$. This combined with the $W(46,45)$ found by Mathon [31] gives (1) of the enunciation and all $W(2m,45)$, $m \geq 46$. Corollary 2 gives $W(n,45)$, $n \in \{54,90,126\}$ and the Kronecker product of a $W(6,5)$ and a $W(13,9)$ gives a $W(78,45)$. This completes the proof. □

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