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SELECTED PAPERS IN COMBINATORICS
A VOLUME DEDICATED TO R.G. STANTON

Guest Editors:
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J.R. SEBERRY
S.A. VANSTONE

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Ralph Gordon Stanton

Professor Stanton has had a very illustrious career. His contributions to mathematics are varied and numerous. He has not only contributed to the mathematical literature as a prominent researcher but has fostered mathematics through his teaching and guidance of young people, his organizational skills and his publishing expertise. The following briefly addresses some of the areas where Ralph Stanton has made major contributions.

Mathematics at the University of Waterloo

In 1957, Professor Stanton took the courageous step of leaving the University of Toronto and joining the embryonic University of Waterloo. At that time, he was the entire Mathematics Department. When he left Waterloo nine years later, in 1966, he had formed a Faculty of Mathematics which is now recognized as one of the major research centres in Canada. He was able to do this because of his own stature, which allowed him to attract excellent colleagues; his tireless energy in teaching and research; and his general dedication to excellence. In addition to his work in Mathematics, he served as Dean of Graduate Studies in the University and helped in building up many other areas besides Mathematics. Since his departure from Waterloo, he has undertaken the task of building up Computer Science at the University of Manitoba, and he has succeeded in forming a strong research group there.

Contributions to specific disciplines

Professor Stanton's first love was *Statistics,* an area in which there was a great shortage twenty-five years ago; he trained many statisticians personally, and built up the nucleus of the present Waterloo department, which is now among the strongest statistical research groups in Canada. He also recognized, very early, the coming importance of *Computer Science*; at the very beginning of the University of Waterloo, he introduced courses in that discipline, teaching many of them himself. His guidance was instrumental in forming the present world-class Computer Science group at the University of Waterloo. Finally, he has worked for twenty years in *Combinatorics,* a relatively new area that dates only from the 1930's (and mainly from 1950'); in this area, he started a group at Waterloo that has made Waterloo one of the major world centres for combinatorial research. He has continued this initiative at the University of Manitoba, where he has built up a fine group of young researchers in Combinatorics.

Professor Stanton's contributions to Combinatorics have not been restricted to the national scene. He has of course, spoken in many centres, such as Budapest, Southampton, Aberdeen, Rome, and many USA, Japanese and Australian centres. Undoubtedly, his greatest impact outside Canada has been in Australia. He first accepted an invitation to lecture on Combinatorics there in 1973, and since then has been very energetic in promoting Combinatorics in that country. He has arranged for visits in both directions, and has encouraged the work of many Australian scholars, expecially the younger ones. His work in promoting mathematical collaboration between Canada and Australia was recognized by the award of an honorary D.Sc. degree by the University of Newcastle in 1979 and the University of Queensland in 1989.

Training of mathematicians

Professor Stanton has always been noted as an outstanding teacher; in the early days at Waterloo, he would teach anywhere from three to five courses per year himself. He believes in getting students involved in research early, and has frequently succeeded in this by taking undergraduates to conferences and getting them involved in co-authoring research papers. Many of the students whom he has encouraged or supervised have become fine researchers themselves. Professor Stanton's interest for fostering mathematics among young people has stretched right back into the high schools. His liaison with the high schools, when he was at Waterloo, was very energetic, and he led in developing the Canadian Junior Mathematics Contest and the Descartes Senior Mathematics Competition, which are written by thousands of students all across Canada every year. It was the use of these contests as a model, plus Professor Stanton's contacts in Australia, that led to the development of the Australian High School Mathematics Contest which, with its involvement of over 80,000 students each year, is now the largest mathematics contest in the world.

Dissemination of mathematical knowledge

We can think of no single person who has contributed more to the spread of combinatorial knowledge than Professor Stanton. In 1970, he helped organize the first Southeastern Conference on Combinatorics, Graph Theory, and Computing; there were thirty attendees. The twentieth conference was held at Boca Raton, Florida in February 1989 and had 500 participants. This is now the largest annual combinatorial meeting in the world. Professor Stanton remains as one of the organizers.

He has also run an annual conference at the University of Manitoba on the subject of Numerical Mathematics in general (including Combinatorics). This conference has been held regularly since 1971 and, although not nearly as large as the Southeastern Conference, it attracts very prominent international researchers.

Conferences are important in spreading mathematical knowledge, but speedy publication of results in new areas is also essential. When he was at the University of Waterloo, Professor Stanton started the journal *Aequationes Mathematicae.* At the University of Manitoba, he initiated the journal *Utilitas Mathematica,* of which he still serves as editor-in-chief; it is now in its thirty-fifth volume. The breadth of his interests, and his friendships, can be illustrated by the fact that it was *Utilitas Mathematica* which published, in 1982, two special volumes in his honour of the 80th birthday of Frank Yates.

We might mention many other contributions to publications made by Professor Stanton in reviewing or serving on editorial boards. But we should certainly note his work as one of the founding and most active editors of the journal *Ars Combinatoria, Journal of Combinatorial Mathematics and Combinatorial Computing and the Australasian Journal of Combinatorics.* Professor Stanton also edits *Congressus Numerantium,* a conference journal which is now in its 69th volume; it specializes in making research results available very quickly.

Professor Stanton is continually being asked to edit and publish important conference Proceedings; he has edited the Proceedings of the British Combinatorial Conference, the Australasian Combinatorial Conference and numerous others. He has also initiated a series of *Selected Papers* of prominent contemporary research workers in mathematics. So far, six sets have appeared (Tutte, Lehmer, Greville, Shrikhande, Neumann, and Bartlett).

In 1990 Professor Stanton became the first Registrar and driving force for the Institute for Combinatorics and Its Applications which has hundreds of members worldwide, mostly due to his efforts and influence. He edits the *Bulletin* of the Institute.

Personal research

A sketch of Professor Stanton's personal research has been left until the end. He is not merely an administrator of great ability, but a researcher notable both for the quantity and the quality of his work. One might well ask when he finds time to do research; the answer is that he is a very dedicated scholar with an enormously wide breadth of interests. (As an aside, we might mention that the late Dr. Maurice Ettinghausen, who was one of the world's greatest experts on Portuguese literature, considered Professor Stanton to be one of the leading authorities on the Portuguese epic and considered Professor Stanton's personal library of that genre to be second to none. That library is now in the University of Toronto Rare Book Library, where it has been named the 'Stanton Collection of Portuguese Literature').

In mathematical research, Professor Stanton started in the area of Statistics, mainly statistical design theory, and moved from there into combinatorial theory. His writings are noteworthy for the breadth of topics with which he deals and for the ingenuity of the methods that he employs. The following areas are representative of the major thrusts of his research work.

(a). *Design theory*

Much modern research in combinatorics has developed from the work done in the 30's and 40's on Balanced Incomplete Block Designs. Professor Stanton has made numerous contributions in this area, of which we would like to cite, as examples, two that are particularly important.

The first of these is the paper 'A Family of Difference Sets' in the Canadian Journal of Mathematics, Volume X, Number 1 (1957), 73–77. This article gives an elegant algebraic construction for a family of symmetrical balanced incomplete block designs by making use of the number-theoretic notion of twin prime powers. This construction is one of the relatively few direct constructions known for this interesting class of designs, and is a standard reference in any text on the subject.

A second major contribution to the area of design theory is in regard to the classification and listing of designs. In general, there are five basic parameters associated with a design. These are v, b, r, k, and, which refer to the number of varieties or points in the design, the number of blocks, the replication number, the block size, and the balance number, respectively. Historically, any table of valid design parameters was organized with respect to increasing values of r (as in the table of Fisher and Yates, in Statistical Tables for Biological, Agricultural, and Medical Research, published by Oliver and Boyd, Edinburgh). Stanton observed that a much more enlightening classification results if one concentrates on the parameters v and k. For example, if D is a design with parameters (v, b, r, k, λ) and the greatest common divisor of b, r and λ is unity, then any other design with the same v and k must have the remaining parameters tb, tr, and t for some integer t. The knowledge of two (v, k)-designs having t values that are relatively prime is sufficient to deduce the existence of all such designs beyond a certain t value; this idea was the first fundamental contribution to the general problem of the existence of (v, k)-designs beyond a certain lower bound. These ideas and ramifications of them are recorded in two articles: 'Classification and Embedding of BIBDs' Sankhya, Volume 30, Series A (1968), 91–100 and 'A Computer System for Research on the Family Classification of BIBDs', Proceedings of the International Congress on Combinatorial Theory (invited address), Rome (1973), Academia dei Lincei, 133–169. Both these papers make major contributions to the very difficult general existence problem for BIBDs'.

In concluding this section, we would like to discuss briefly another major contribution to design theory which illustrates Professor Stanton's ingenuity in making a non-obvious application of the theory of designs to a statistical problem, usually called the Doehlert–Klee Problem (or the DK Problem, for short). Consider the d-dimensional cuboctahedron (or the difference body of a d-dimensional simplex), and adjoin its central point s. We seek to co-ordinatize the figure, that is, to choose an orthonormal basis B in such a manner that the set of inner products $I_b = \langle v, b \rangle : v \in C_d \ s, b \in B$, has restricted cardinality. For each $b \in B^b$, the cardinality of I_b, known as the level of the vector b, is odd, and it was previously known that B can be chosen so that this cardinality $\leqslant 7$ for all $b \in B$. The interesting problem is to maximize the number of level 3 and level 5 vectors in B. Stanton reformulated this problem in terms of constructing a certain regular pairwise balanced design with special properties; this reformulation led to numerous advancements in the problem, including the discovery of optimal configurations in a number of cases of practical interest. He has a series of papers in this area, but probably the most important is his invited address in Canberra 'The Doehlert–Klee Problem: Part I, Statistical Background; Part II, A Reinterpretation', Proceedings of the International Conference on Combinatorial Theory (Australian Academy of Sciences), Springer-Verlag 1977, 89–100.

(b). *Room Squares*

Recent literature searches have shown that Room Squares have a longer history than previously believed; the concept in emboyonic form dates back to about 1850 and is associated with the names of Cayley, Sylvester, and Kirkman. However, no real theory was developed, and the subject remained dormant for many years. R. H. Bruck, in his brief account of what was known on the subject, attributed them to the geometer T. G. Room. Indeed, when Bruck wrote, only two Room Squares were known to exist. This fact interested Stanton, and the first major attack on the existence of these arrays is in the 1968 paper 'Construction of Room Squares', Annals of Mathematical Statistics, Volume 39, Number 5 (1968), 1540–1548. This paper is noteworthy not only for the construction of a great many Room Squares but also for its introduction of the exceptionally powerful 'starter-adder' technique. Later papers by Stanton and others led to the ultimate solution of the complete existence problem for Room Squares. However, we feel that Stanton's most important contribution is not his construction of many new squares, nor his discovery of the fundamental exceptional role played in Room Square Theory by the Fermat primes, but his contribution of the 'starter-adder' method. This concept has since been further generalized and used by many other researchers in various branches of combinatorial design theory. It is the main tool for the direct construction of designs that possess orthogonal resolutions.

Before concluding this section, we ought to add that Stanton was the first to establish that the existence of Room Squares of orders r and s implied the existence of a Room Square of order rs. This result 'composition of Room Squares' in Proceedings of the International Conference at Budapest, Bolyai-Janos Mathematical Society (1969), 1013–1021, corrected a fundamental misconception about Room quasigroups that had been extant for a number of years; it also led to the introduction of a much more general composition rule.

(c). *Near coverings*

The near-covering problem arose in the 50's in connection with the football pool problem. This problem can be easily stated in the parlance of coding theory: in the vector space of dimension n over GF (p), find the smallest set of vectors C such that each vector in the space is at distance at most one from some vector of C. If C is a linear code, then we want the covering radius of C to be one. Early investigators of this problem studied these coverings in terms of abelian groups. Professor Stanton, with his strong background in group theory (his Ph.D. Dissertation under Richard Brauer was on the Mathieu Groups) and his increasing involvment in combinatorial theory, realized the coding-theory nature of the problem, and his paper 'Covering Theorems in Groups', appearing in Recent Progress in Combinatorics, Academic Press (1968), 21–36, gave both clarity and direction to a combinatorial attack on the problem. Stanton and others have since made major advances in this area; perhaps Stanton and others have since made major advances in this area; perhaps Stanton's most effective contribution is the outstanding paper 'Covering Theorems for Vectors with Special Reference to the Case of Four and Five Components' in Journal of the London Mathematical Society (2), (1969), 493–499. This near-covering problem is still an area of major interest because of its close connection with the theory of perfect codes and especially with nonbinary Hamming codes.

(d). *Covering and packing designs*

Coverings and packings form a very large class of designs. The construction of coverings with a minimum number of blocks and packings with a maximum number of blocks and possessing t-wise balance, is, in general, a very important and difficult problem. These designs find very interesting applications in the area of error-control coding; for example, a minimum cover can result in an efficient decoding procedure for an error-correcting code. Stanton has worked extensively in this area over the past 15 years; the move into this area is a natural move from the areas discussed earlier, but it is a very challenging one. His first major paper in this area is now a standard reference 'Covering and Packing Designs', invited address in Proceedings of the 2nd Chapel Hill Conference on Combinatorial Mathematics (with Mullin and Kalbfleisch), University of North Carolina (1970), 428–450.

$N(t, k, v)$ is used in covering theory to mean the minimum number of k-subsets (blocks) that is required to cover all the t-subsets of a v-set. Stanton made early contributions concerning this number, especially in the case of $N(2, 4, v)$. A much more difficult problem, and one that is still not completely solved, is the determination of $N(3, 4, v)$. For many years, the particularly difficult case when $v = 12n + 7$, defied attack. An important step forward in this case was made by

Stanton and Mullin in 'Some New Results on the Covering Number $N(t, k, v)$', in Combinatorial Mathematic VII, Springer-Verlag (1980), 51–58. This paper first established the fact that, for infinitely many values of v, $N(3, 4, v)$ either attained its theoretical lower bound or was at most one unit distant from that bound. This appears to have been a major breakthrough in the problem; much oher work has followed, and it appears likely that the complete question may be settled in the near future.

Coverings and Packings represent a very active area of current combinatorial research interest, and Professor Stanton is recognized as a leading research worker in this area. We mention only one other of his many contributions in this area. The paper 'Coverings of Pairs by k-sets' was the first to discuss the behaviour of v for which $N(2, k, v)$ assumed certain fixed small values; this paper appeared in Proceedings of the New York Academy of Sciences 175, Article 1 (1970), 336–339, and led to the later outstanding work of Mills on the extension of the set of small values considered.

(e). *Current research on exact coverings and packings*

The various subjects outlined above lead in a natural progression to Professor Stanton's current research. In order to cast more light on the previous areas, Stanton felt the need to study a more general class of objects called 'exact coverings (packings) with t-wise balance'. In exact coverings, it is required that every t-subset of a v-set be covered by exactly subsets (blocks) where the blocks can now be of various sizes.

The notation $g(\lambda, t, v)$ is used to denote the minimum number of blocks in such a cover. Some early work of Erdős and de Bruijn showed that $g(1, 2, v) = v$. Then there were almost no results for 20 years until Woodall proved a strong inequality concerning the function g. For many values of v, an even more powerful inequality was developed by Stanton and Kalbfleisch in 'The $\lambda - \mu$ Problem: $\lambda = 1$ and $\mu = 3$', Proceedings 2nd Chapel Hill Conference on Combinatorics, University of North Carolina (1970), 451–662. This has since become known as the SK Inequality and is a major tool in dealing with the function $g(\lambda, t, v)$; in particular, the Erdős –de Bruijn result is a simple corollary to the SK inequality, as pointed out first by Stanton in his Adelaide paper 'Computation of $g(1, 3, 12)$' in 1975.

Stanton's work on $g(\lambda, t, v)$ has since led him to another important advance in this area. This is the realization that a different function is much more fundamental. This is the function

$$g^{(k)}(\lambda, t, v)$$

which represents the covering number under the condition that there be a block of size k but no block of larger size. This function was introduced in 'Pair-Coverings with Restricted Largest Block Length', Ars Combinatoria 11 (1981), 85–98. Consideration of this function was a major step forward; Stanton was able to determine $g^{(k)}$ for a large range of values of k, and this indicated that the behaviour of g was more or less accidental. The function $g^{(k)}$ possesses two minima, and the value of g merely depends on which of these two minima is lesser.

Conclusion

We could speak of many other research areas in combinatorics that owe much to Professor Stanton. We have not mentioned his work on extermal computer networks, his work on weighing matrices, his work on isonemal matrices, his work on Davenport–Schinzel sequences, or many other endeavours where he has made important contributions himself and has stimulated the work of others. But we would like to point out his influence on combinatorial computing over the years. He very early realized the potential and power of computers for attacking various problems in combinatorial designs. It seems that many of the smaller cases are best handled in this fashion but it is also well known that having a computer is not, in general, enough to produce combinatorial results. The so-called combinatorial explosion of cases takes over very quickly, and so it requires much ingenuity to produce feasible methods. Stanton has contributed numerous algorithms to attack design problems and in doing so has displayed a blend of both theoretical and computational skills.

We have tried to outline in the previous pages the contributions to mathematics of a very remarkable man. We have summarized his work in developing the Faculty of Mathematics at Waterloo, his impact on Combinatorics in Canada and Australia, his contribution to developing other research mathematicians, his broad involvement in publications and the dissemination of knowledge, and his own combinatorial research, where his publications (ignoring review articles or expository articles) reach to over 200 research papers, many of great importance for current and future research. It was for his many distinguished contributions to mathematics that Professor Stanton was awarded a Killam Prize of the Canada Council in 1985; he is currently one of only two dozen Killam Laureates in all Canada.

It is with profound pleasure that we dedicate this volume to Professor Ralph Gordon Stanton.

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