

**Further results on base sequences, disjoint complementary sequences,  
OD(4t; t, t, t, t) and the excess of Hadamard matrices**

**Christos Koukouvinos, Stratis Kounias**

Department of Mathematics  
University of Thessaloniki  
Thessaloniki, 54006  
Greece

**Jennifer Seberry**

Department of Computer Science  
University College, University of New South Wales  
Australian Defence Force Academy  
Canberra, ACT, 2600  
Australia

**Abstract.** We obtain new base sequences, that is four sequences of lengths  $m + p$ ,  $m + p$ ,  $m$ ,  $m$ , with  $p$  odd, which have zero auto correlation function which can be used with Yang numbers and four disjoint complementary sequences (and matrices) with zero non-periodic (periodic) auto correlation function to form longer sequences.

We give an alternate construction for  $T$ -sequences of length  $(4n + 3)(2m + p)$  where  $n$  is the length of a Yang nice sequence.

These results are then used in the Goethals-Seidel or (Seberry) Wallis-Whiteman construction to determine eight possible decompositions into squares of  $(4n + 3)(2m + p)$  in terms of the decomposition into squares of  $2m + 1$  when there are four suitable sequences of lengths  $m + 1$ ,  $m + 1$ ,  $m$ ,  $m$  and  $m$ , the order of four Williamson type matrices. The new results thus obtained are tabulated giving OD(4t; t, t, t, t) for the new orders  $t \in \{121, 135, 217, 221, 225, 231, 243, 245, 247, 253, 255, 259, 261, 265, 273, 275, 279, 285, 287, 289, 295, 297, 299\}$ .

The Hadamard matrix with greatest known excess for these new  $t$  is then listed.

## **1. Definitions and Introduction**

We follow closely the work of Cohen [1], and Jenkins [4] and refer the reader to those papers for most definitions and constructions.

Four sequences of elements  $+1, -1$  of lengths  $m + p$ ,  $m + p$ ,  $m$ ,  $m$ , with  $p$  odd, which have zero non-periodic auto correlation function are called *base sequences*. In [1] base sequences are displayed for lengths  $m + 1$ ,  $m + 1$ ,  $m$ ,  $m$ , for  $m + 1 \in \{2, 3, \dots, 18, 21, 24, 30\}$ .

In Table 1 we give new base sequences for  $m = 12, 13, 15, 16, 17, 18, 25$  and  $p = 1$ . Some of these are new in the sense that they correspond to new decompositions into squares of  $2m + 1$ .

If  $X$  and  $Y$  are Golay sequences of length  $m$ ,  $\{1, X\}, \{1, -X\}, \{Y\}, \{Y\}$  are base sequences of lengths  $m + 1$ ,  $m + 1$ ,  $m$ ,  $m$ . So base sequences exist for all  $m = 2^a 10^b 26^c$ ,  $a, b, c$  non-negative integers,  $p = 1$  and  $m \in \{1, \dots, 18, 20, 23, 25, 29\}$ .

Base sequences are crucial to Yang's [10], [11], [12] constructions for finding longer  $T$ -sequences of odd length.

Four  $(1, -1)$  sequences  $A = (X, U, Y, V)$  where

$$\begin{aligned} X &= \{x_1 = 1, x_2, x_3, \dots, x_m, -x_m, \dots, -x_3, -x_2, -x_1 = -1\}, \\ U &= \{u_1 = 1, u_2, u_3, \dots, u_m, -u_m, \dots, -u_3, -u_2, 1\}, \\ Y &= \{y_1, y_2, \dots, y_{m-1}, y_m, y_{m-1}, \dots, y_2, y_1\}, \\ V &= \{v_1, v_2, \dots, v_{m-1}, v_m, v_{m-1}, \dots, v_2, v_1\}, \end{aligned}$$

which have zero non periodic auto correlation function and  $8m - 6$  the sum of two squares, or four  $(1, -1)$  sequences  $A = (X, U, Y, V)$ , where

$$\begin{aligned} X &= \{x_1 = 1, x_2, x_3, \dots, x_m, x_{m+1}, x_m, \dots, x_3, x_2, x_1 = 1\}, \\ U &= \{u_1 = 1, u_2, u_3, \dots, u_m, u_{m+1}, u_m, \dots, u_3, u_2, -1\}, \\ Y &= \{y_1, y_2, \dots, y_m, -y_m, \dots, -y_2, -y_1\}, \\ V &= \{v_1, v_2, \dots, v_m, -v_m, \dots, -v_2, -v_1\}, \end{aligned}$$

which have zero non periodic auto correlation function and  $8m + 2$  the sum of two squares will be called *Turyn sequences* of lengths  $n + 1, n + 1, n, n$ .

Known Turyn sequences are given in [1] where the longer sequence is of length 2, 3, 4, 5, 6, 7, 8, 13, 15. They cannot exist for 11, 12, 17 or 18 and a complete machine search showed they do not exist for lengths 9, 10, 14 or 16. So the first unresolved cases (and unresolved since 1976) are lengths 19, 20 and 21.

Length	Sum of Squares	Sequences
$m$	$2m + 1$	
12	$25 = 5^2$	(i) $\{-++++-+---+---+\}, \{+-+---+-----+\}$ $\{---++---+-----+\}, \{---++---+-----+\}$ (ii) $\{-+++++---+---+\}, \{+-+---+-----+\}$ $\{+-----+---+---+\}, \{+-----+---+---+\}$
13	$27 = 4^2 + 3^2 + 1^2 + 1^2$	(i) $\{----+---+---+---+\}, \{----+---+---+---+\}$ $\{+---+---+---+---+\}, \{+---+---+---+---+\}$ (ii) $\{----+---+---+---+\}, \{+---+---+---+---+\}$ $\{----+---+---+---+\}, \{+---+---+---+---+\}$ (iii) $\{----+---+---+---+\}, \{+---+---+---+---+\}$ $\{----+---+---+---+\}, \{+---+---+---+---+\}$ (iv) $\{----+---+---+---+\}, \{+---+---+---+---+\}$ $\{----+---+---+---+\}, \{+---+---+---+---+\}$

Table 1: Base sequences of lengths  $m + 1, m + 1, m, m$

Length	Sum of Squares	Sequences
$m$	$2m + 1$	
13	$27 = 3^2 + 3^2 + 3^2$	(i) {-----+--++++++--+}, {++++-+-+-----+--} {++++-----+--+}, {+-----+-----+--+} (ii) {+---+-----+--+}, {-+---+-----+--+} {-+-----+-----+--}, {+-----+-----+--+} (iii) {----+---+---+---+}, {+-----+-----+--+} {++++-+-+-----+--+}, {+-----+-----+--+} (iv) {-+---+---+---+---+}, {++++-+-+-----+--+} {++++-+-+-----+--+}, {+-----+-----+--+} (v) {-+---+---+---+---+}, {+-----+-----+--+} {-+-----+-----+--}, {+-----+-----+--+} (vi) {----+---+---+---+}, {+-----+-----+--+} {-+---+---+---+---+}, {+-----+-----+--+}
15	$31 = 5^2 + 2^2 + 1^2 + 1^2$	see [1]
15	$31 = 3^2 + 3^2 + 3^2 + 2^2$	(i) {-----+--+---+---+---+} {+-----+-----+-----+} {++-----+---+---+---+} {+---+---+---+---+---+} (ii) {-----+--+---+---+---+} {+-----+-----+-----+} {++-----+---+---+---+} {+---+---+---+---+---+}
16	$33 = 5^2 + 2^2 + 2^2$	(i) {+---+---+---+---+---+} {++++-+-+-----+---+} {+-----+-----+---+---+} {+---+---+---+---+---+} (ii) {+---+---+---+---+---+} {++-----+---+---+---+---+} {-+---+---+---+---+---+} {+---+---+---+---+---+}
16	$33 = 4^2 + 4^2 + 1^2$	see [1], or (i) {-++++-----+---+---+---+} {-+---+---+---+---+---+} {-+---+---+---+---+---+} {++++-+-+-----+---+}
16	$33 = 4^2 + 3^2 + 2^2 + 2^2$	(i) {-----+---+---+---+---+} {-++++-+-+-----+---+} {++-----+---+---+---+} {-++++-+-+-----+---+}

Table 1(cont.): Base sequences of lengths  $m + 1, m + 1, m, m$

Length	Sum of Squares	Sequences
$m$	$2m + 1$	
17	$35 = 5^2 + 3^2 + 1^2$	(ii) { - + - + + - + + - + - - - } { - - + + + - + + - + - + + } { + + - - - + + - + - + + - } { + + + - + - + - + + + - - } (iii) { - - + + - + - + - + - + + } { + - - + + - + - + - + + + + } { - + + - - - + + + - + + - } { + - + - - - + + + - + + + + } see [1],[7], or (i) { - + + - + + - - - + - + - + + + - } { + - + + + + - - + + + - - - - } { + + + + - - - + - + + - + - + } { + - + - + - - - + + - + + + + } (ii) { + + + - + - + - + - + - - + } { - - + + + - + - - + + + + - + } { + + + + - - - + - + + - + - + } { + - + - + - - - + + - + + + + }
17	$35 = 4^2 + 3^2 + 3^2 + 1^2$	(i) { - - - + + + - + + - - + - + + - } { - - + + + - + - - + + - + - + } { + + + + - - + + - - + - - + } { + - + - + - - - + + - + + + + }
18	$37 = 6^2 + 1^2$	(i) { + + + - - - - + - + - + - - + + } { + - + - - + - - + + - + + + - - } { + - - + + + - + - + + + + - + - } { + + + + - + + - + + - - + + - - }
18	$37 = 5^2 + 2^2 + 2^2 + 2^2$	(i) { - - + - + + - - - + - + - + + + + } { + + + - - + + - + - + + + - + - } { - + + + - - - + - + - + + - + - } { + + - - + - - + + - - + + + + }
18	$37 = 4^2 + 4^2 + 2^2 + 1^2$	(i) { + + + + - + + - - - - + - + - - } { + - + - - - + - + - + - + + + + } { - + + - + - - + + - - + + - + - } { + + + + - - - + + - + + - + + }
25	$51 = 7^2 + 1^2 + 1^2$	(i) { + + + + + + - - + + - - + - + - + - } { - + + + + + - - + + - - + - + - + } { + + + + + - - - + + - + + - + - - } { + - + - + - - + - + - + - - + + + - }

Table 1(cont.): Base sequences of lengths  $m + 1, m + 1, m, m$

## 2. Constructions and decompositions into squares

The results that we now give arise from a remarkable composition theorem due to C.H. Yang [13] which has been announced.

Let  $X, Y, U, W$  be Turyn sequences of lengths  $n+1, n+1, n, n$ . The sequences defined by

$$F = X/U, G = Y/0, H = W/0, E = (1) \quad (1)$$

will be called *Yang nice sequences* (or *Turyn normal sequences*). Otherwise we use the notation and definitions of [1].

**Theorem 1.** (Based on a Theorem of C.H. Yang): Let  $F, G, H, E$  be Yang nice sequences of length  $4n+3$  constructed from Turyn sequences as in (1), and  $A, B, C, D$  be base sequences of lengths  $m+p, m+p, m, m, (t = 2m+p)$ . Then the sequences  $Q, R, S, T$ , given below, become 4-complementary sequences (that is the sum of the non-periodic auto-correlation functions is 0) of lengths  $(4n+3)t$ .

$$\begin{aligned} Q &= (-B'e + Af' + Cg + Dh) \\ R &= (A'e + Bf' + Dg' - Ch') \\ S &= (-D'e - Cf' + Ag' - Bh) \\ T &= (C'e - Df + Bg + Ah') \end{aligned}$$

Furthermore if we define sequences

$$P_1 = \frac{1}{2}(Q + R), P_2 = \frac{1}{2}(Q - R), P_3 = \frac{1}{2}(S + T), P_4 = \frac{1}{2}(S - T),$$

then these sequences become  $T$ -sequences of length  $(4n+3)t, t = 2m+p$ .

**Theorem 2.** Under the conditions stated in Theorem 1 the following sequences can be used, as in Theorem 1, to form disjoint  $T$ -sequences of lengths  $(4n+3)(2m+p)$ .

$$\begin{aligned} Q &= (A'e + Bf' + Cg' + Dh') \\ R &= (-B'e + Af' + Dg - Ch) \\ S &= (-C'e - Df + Ag' + Bh) \\ T &= (-D'e - Cf - Bg + Ah') \end{aligned}$$

Turyn sequences of lengths  $n+1, n+1, n, n$ , exist for  $n \leq 7, n = 12$  or  $n = 14$ , hence we can construct Yang nice sequences for lengths  $(4n+3)$  and so, from Theorems 1 and 2, 4-disjoint  $T$ -sequences of lengths  $(4n+3)t$  where  $t = 2m+p$  is the length of base sequences can be constructed.

**Theorem 3.** Let  $a, b, c, d$  be the row sums of four suitable sequences,  $A, B, C, D$  of length  $m+1, m+1, m, m$ , so that  $2m+1 = a^2 + b^2 + c^2 + d^2$ . Let  $f, g, h$ , and  $i$ , be the row sums of four Turyn sequences of length  $4n+3$  so that  $4n+3 = f^2 + g^2 + h^2 + i^2$ . Then using Yang's method to multiply by  $4n+3$ ,

as in Theorem 1, we get four disjoint  $T$ -sequences of lengths  $(4n+3)(2m+1)$  corresponding to one of the decompositions:

$$\begin{aligned}
 (4n+3)(2m+1) &= (af - bg + ch + di)^2 + (ag + bf - ci + dh)^2 \\
 &\quad + (ah - bi - cf - dg)^2 + (ai + bh + cg - df)^2 \\
 (4n+3)(2m+1) &= (bf - ag + ch + di)^2 + (bg + af - ci + dh)^2 \\
 &\quad + (bh - ai - cf - dg)^2 + (bi + ah + cg - df)^2 \\
 (4n+3)(2m+1) &= (af - bg + dh + ci)^2 + (ag + bf - di + ch)^2 \\
 &\quad + (ah - bi - df - cg)^2 + (ai + bh + dg - cf)^2 \\
 (4n+3)(2m+1) &= (bf - ag + dh + ci)^2 + (bg + af - di + ch)^2 \\
 &\quad + (bh - ai - df - cg)^2 + (bi + ah + dg - cf)^2
 \end{aligned}$$

**Corollary 4.** Let  $a, b, c, d$  be the row sums of suitable sequences of length  $m+1, m+1, m, m$ , so that  $2m+1 = a^2 + b^2 + c^2 + d^2$ . Then using Yang's method to multiply by  $4n+3$ , as in Theorem 1, we get four disjoint  $T$ -sequences of lengths  $(4n+3)(2m+1)$  corresponding to one of four decompositions indicated in Table 2.

**Theorem 5.** Let  $a, b, c, d$  be the row sums of four suitable sequences,  $A, B, C, D$  of length  $m+1, m+1, m, m$ , so that  $2m+1 = a^2 + b^2 + c^2 + d^2$ . Let  $f, g, h,$  and  $i,$  be the row sums of four Turyn sequences of length  $4n+3$  so that  $4n+3 = f^2 + g^2 + h^2 + i^2$ . Then using Yang's method to multiply by  $4n+3$ , as in theorem 2, we get four disjoint  $T$ -sequences of lengths  $(4n+3)(2m+1)$  corresponding to one of the decompositions:

$$\begin{aligned}
 (4n+3)(2m+1) &= (ag + bf + ch + di)^2 + (af - bg - ci + dh)^2 \\
 &\quad + (ah + bi - cg - df)^2 + (ai - bh + cf - dg)^2 \\
 (4n+3)(2m+1) &= (bg + af + ch + di)^2 + (bf - ag - ci + dh)^2 \\
 &\quad + (bh + ai - cg - df)^2 + (bi - ah + cf - dg)^2 \\
 (4n+3)(2m+1) &= (ag + bf + dh + ci)^2 + (af - bg - di + ch)^2 \\
 &\quad + (ah + bi - dg - cf)^2 + (ai - bh + df - cg)^2 \\
 (4n+3)(2m+1) &= (bg + af + dh + ci)^2 + (bf - ag - di + ch)^2 \\
 &\quad + (bh + ai - dg - cf)^2 + (bi - ah + df - cg)^2
 \end{aligned}$$

**Corollary 6.** Let  $a, b, c, d$  be the row sums of suitable sequences of length  $m+1, m+1, m, m$ , so that  $2m+1 = a^2 + b^2 + c^2 + d^2$ . Then using Yang's method to multiply by  $4n+3$ , as in Theorem 2, we get four disjoint  $T$ -sequences of lengths

$(4n + 3)(2m + 1)$  corresponding to one of four decompositions indicated in Table 3.

Example: There are suitable sequences for  $m = 5$  corresponding to  $a = 1, b = 1, c = 3$  and  $d = 0$ . These give sequences of lengths 121 corresponding to:

$$121 = 8^2 + 7^2 + 2^2 + 2^2 \text{ giving excess } \sigma(H_{484}^1) = 9196.$$

$$121 = 8^2 + 5^2 + 4^2 + 4^2 \text{ giving excess } \sigma(H_{484}^2) = 10164.$$

They also give sequences of lengths 165 corresponding to:

$$165 = 8^2 + 8^2 + 6^2 + 1^2 \text{ giving excess } \sigma(H_{660}) = 15180.$$

$$165 = 10^2 + 6^2 + 5^2 + 2^2 \text{ giving excess } \sigma(H_{660}) = 15180.$$

These are examples of the method: the new sequences that arise from the decompositions and which lead to Hadamard matrices with maximum known excess are given in Table 2 and 3 are listed in Tables 6 and 7. Full details are given in a Technical Report (University College, ADFA, TR CS88/18) available from the authors.

Remark: We note that it is only the numbers  $4n + 3$  which are the sum of four (and not fewer) squares which actually lead to an inequivalent decomposition between Yang's construction and our modified construction (Theorem 2). However our modification does prove to be important for obtaining maximum known excess.

We have exhibited the new  $T$ -sequences in Tables 4 and 5 as they are possibly inequivalent to previously known sequences.

$t$		Sum of Squares	Comment	KF Upper Bound	Excess
165	$15 \times 11$	$8^2 + 7^2 + 6^2 + 4^2$	$y = 15, m = 5$	16936	16500
287	$7 \times 41$	$11^2 + 9^2 + 9^2 + 2^2$	$y = 7, m = 20$	38888	35588†
299	$23 \times 13$	$17^2 + 3^2 + 1^2$	$y = 23, m = 6$	41328	40664†

**Table 7:** T-disjoint T-sequences giving Hadamard matrices with maximum known excess

† new maximum known excess

### 3. New excess results

We now use the new  $T$ -sequences, with the results of the previous section, as in Hammer, Levingston and Seberry [3] or Jenkins *et. al.* [4] to obtain Hadamard matrices of order  $4t$  with maximum known excess.

These are given in two tables: Table 6 corresponds to Yang's construction and Table 7 to our new construction. We have more complete results for this section in the Technical report available from the authors referred to above.

#### 4. Summary

The sequences constructed here can be combined with those of [1] and used to obtain orthogonal designs  $OD(4t; t, t, t, t)$  for  $t = 1, 3, \dots, 41, 45, \dots, 65, 67, 69, 75, 77, 81, 85, 87, 91, 93, 95, 99, 101, 105, 111, 115, \dots, 125, 129, 133, 135, 141, \dots, 147, 153, 155, 159, 161, 165, 169, 171, 175, 177, 183, 185, 187, 189, 195, 201, 203, 205, 207, 209, 217, 221, 225, 231, 235, 243, 245, 247, 253, 255, 259, 261, 265, 273, 275, 279, 285, 287, 289, 295, 297, 299$ .

Williamson matrices are used with these orthogonal designs to form Hadamard matrices with the maximum known excess for order  $4t$ ,  $t = 141, 143, 153, 155, 165, 171, 175, 177, 185, 189, 195, 203, 205, 217, 221, 231, 235, 245, 247, 253, 255, 259, 261, 265, 273, 275, 279, 285, 287, 289, 295, 297, 299$ .

$4n+3$		Decompositions
7	$7(2m+1)$	$(a-b+2c+d)^2 + (a+b-c+2d)^2 + (2a-b-c-d)^2 + (a+2b+c-d)^2$ $(b-a+2c+d)^2 + (b+a-c+2d)^2 + (2b-a-c-d)^2 + (b+2a+c-d)^2$ $(a-b+2d+c)^2 + (a+b-d+2c)^2 + (2a-b-d-c)^2 + (a+2b+d-c)^2$ $(b-a+2d+c)^2 + (b+a-d+2c)^2 + (2b-a-d-c)^2 + (b+2a+d-c)^2$
11	$11(2m+1)$	$(3a-b+c)^2 + (a+3b+d)^2 + (a-3c-d)^2 + (b+c-3d)^2$ $(3b-a+c)^2 + (b+3a+d)^2 + (b-3c-d)^2 + (a+c-3d)^2$ $(3a-b+d)^2 + (a+3b+c)^2 + (a-3d-c)^2 + (b+d-3c)^2$ $(3b-a+d)^2 + (b+3a+c)^2 + (b-3d-c)^2 + (a+d-3c)^2$
15	$15(2m+1)$	$(3a-b+2c+d)^2 + (a+3b-c+2d)^2 + (2a-b-3c-d)^2 + (a+2b+c-3d)^2$ $(3b-a+2c+d)^2 + (b+3a-c+2d)^2 + (2b-a-3c-d)^2 + (b+2a+c-3d)^2$ $(3a-b+2d+c)^2 + (a+3b-d+2c)^2 + (2a-b-3d-c)^2 + (a+2b+d-3c)^2$ $(3b-a+2d+c)^2 + (b+3a-d+2c)^2 + (2b-a-3d-c)^2 + (b+2a+d-3c)^2$
19	$19(2m+1)$	$(3a-b+3c)^2 + (a+3b+3d)^2 + (3a-3c-d)^2 + (3b+c-3d)^2$ $(3b-a+3c)^2 + (b+3a+3d)^2 + (3b-3c-d)^2 + (3a+c-3d)^2$ $(3a-b+3d)^2 + (a+3b+3c)^2 + (3a-3d-c)^2 + (3b+d-3c)^2$ $(3b-a+3d)^2 + (b+3a+3c)^2 + (3b-3d-c)^2 + (3a+d-3c)^2$
23	$23(2m+1)$	$(3a-b+2c+3d)^2 + (a+3b-3c+2d)^2 + (2a-3b-3c-d)^2 + (3a+2b+c-3d)^2$ $(3b-a+2c+3d)^2 + (b+3a-3c+2d)^2 + (2b-3a-3c-d)^2 + (3b+2a+c-3d)^2$ $(3a-b+2d+3c)^2 + (a+3b-3d+2c)^2 + (2a-3b-3d-c)^2 + (3a+2b+d-3c)^2$ $(3b-a+2d+3c)^2 + (b+3a-3d+2c)^2 + (2b-3a-3d-c)^2 + (3b+2a+d-3c)^2$
27	$27(2m+1)$	$(5a-b+c)^2 + (a+5b+d)^2 + (a-5c-d)^2 + (b+c-5d)^2$ $(5b-a+c)^2 + (b+5a+d)^2 + (b-5c-d)^2 + (a+c-5d)^2$ $(5a-b+d)^2 + (a+5b+c)^2 + (a-5d-c)^2 + (b+d-5c)^2$ $(5b-a+d)^2 + (b+5a+c)^2 + (b-5d-c)^2 + (a+d-5c)^2$
31	$31(2m+1)$	$(5a-b+2c+d)^2 + (a+5b-c+2d)^2 + (2a-b-5c-d)^2 + (a+2b+c-5d)^2$ $(5b-a+2c+d)^2 + (b+5a-c+2d)^2 + (2b-a-5c-d)^2 + (b+2a+c-5d)^2$ $(5a-b+2d+c)^2 + (a+5b-d+2c)^2 + (2a-b-5d-c)^2 + (a+2b+d-5c)^2$ $(5b-a+2d+c)^2 + (b+5a-d+2c)^2 + (2b-a-5d-c)^2 + (b+2a+d-5c)^2$
51	$51(2m+1)$	$(7a-b+c)^2 + (a+7b+d)^2 + (a-7c-d)^2 + (b+c-7d)^2$ $(7b-a+c)^2 + (b+7a+d)^2 + (b-7c-d)^2 + (a+c-7d)^2$ $(7a-b+d)^2 + (a+7b+c)^2 + (a-7d-c)^2 + (b+d-7c)^2$ $(7b-a+d)^2 + (b+7a+c)^2 + (b-7d-c)^2 + (a+d-7c)^2$

**Table 2:** Decomposition arising from Yang's composition  $(4n+3)(2m+1)$  where  $2m+1 = a^2 + b^2 + c^2 + d^2$  using Theorem 1.



$4n+3$		Decompositions
59	$59(2m+1)$	$(7a-b+3c)^2 + (a+7b+3d)^2 + (3a-7c-d)^2 + (3b+c-7d)^2$ $(7b-a+3c)^2 + (b+7a+3d)^2 + (3b-7c-d)^2 + (3a+c-7d)^2$ $(7a-b+3d)^2 + (a+7b+3c)^2 + (3a-7d-c)^2 + (3b+d-7c)^2$ $(7b-a+3d)^2 + (b+7a+3c)^2 + (3b-7d-c)^2 + (3a+d-7c)^2$

Table 2 (cont'd)

$4n+3$		Decompositions
7	$7(2m+1)$	$(a+b+2c+d)^2 + (a-b-c+2d)^2 + (2a+b-c-d)^2 + (a-2b+c-d)^2$ $(b+a+2c+d)^2 + (b-a-c+2d)^2 + (2b+a-c-d)^2 + (b-2a+c-d)^2$ $(a+b+2d+c)^2 + (a-b-d+2c)^2 + (2a+b-d-c)^2 + (a-2b+d-c)^2$ $(b+a+2d+c)^2 + (b-a-d+2c)^2 + (2b+a-d-c)^2 + (b-2a+d-c)^2$
11	$11(2m+1)$	$(a+3b+c)^2 + (3a-b+d)^2 + (a-c-3d)^2 + (-b+3c-d)^2$ $(b+3a+c)^2 + (3b-a+d)^2 + (b-c-3d)^2 + (-a+3c-d)^2$ $(a+3b+d)^2 + (3a-b+c)^2 + (a-d-3c)^2 + (-b+3d-c)^2$ $(b+3a+d)^2 + (3b-a+c)^2 + (b-d-3c)^2 + (-a+3d-c)^2$
15	$15(2m+1)$	$(a+3b+2c+d)^2 + (3a-b-c+2d)^2 + (2a+b-c-3d)^2 + (a-2b+3c-d)^2$ $(b+3a+2c+d)^2 + (3b-a-c+2d)^2 + (2b+a-c-3d)^2 + (b-2a+3c-d)^2$ $(a+3b+2d+c)^2 + (3a-b-d+2c)^2 + (2a+b-d-3c)^2 + (a-2b+3d-c)^2$ $(b+3a+2d+c)^2 + (3b-a-d+2c)^2 + (2b+a-d-3c)^2 + (b-2a+3d-c)^2$
19	$19(2m+1)$	$(a+3b+3c)^2 + (3a-b+3d)^2 + (3a-c-3d)^2 + (-3b+3c-d)^2$ $(b+3a+3c)^2 + (3b-a+3d)^2 + (3b-c-3d)^2 + (-3a+3c-d)^2$ $(a+3b+3d)^2 + (3a-b+3c)^2 + (3a-d-3c)^2 + (-3b+3d-c)^2$ $(b+3a+3d)^2 + (3b-a+3c)^2 + (3b-d-3c)^2 + (-3a+3d-c)^2$
23	$23(2m+1)$	$(a+3b+2c+3d)^2 + (3a-b-3c+2d)^2 + (2a+3b-c-3d)^2 + (3a-2b+3c-d)^2$ $(b+3a+2c+3d)^2 + (3b-a-3c+2d)^2 + (2b+3a-c-3d)^2 + (3b-2a+3c-d)^2$ $(a+3b+2d+3c)^2 + (3a-b-3d+2c)^2 + (2a+3b-d-3c)^2 + (3a-2b+3d-c)^2$ $(b+3a+2d+3c)^2 + (3b-a-3d+2c)^2 + (2b+3a-d-3c)^2 + (3b-2a+3d-c)^2$
27	$27(2m+1)$	$(a+5b+c)^2 + (5a-b+d)^2 + (a-c-5d)^2 + (-b+5c-d)^2$ $(b+5a+c)^2 + (5b-a+d)^2 + (b-c-5d)^2 + (-a+5c-d)^2$ $(a+5b+d)^2 + (5a-b+c)^2 + (a-d-5c)^2 + (-b+5d-c)^2$ $(b+5a+d)^2 + (5b-a+c)^2 + (b-d-5c)^2 + (-a+5d-c)^2$
31	$31(2m+1)$	$(a+5b+2c+d)^2 + (5a-b-c+2d)^2 + (2a+b-c-5d)^2 + (a-2b+5c-d)^2$ $(b+5a+2c+d)^2 + (5b-a-c+2d)^2 + (2b+a-c-5d)^2 + (b-2a+5c-d)^2$ $(a+5b+2d+c)^2 + (5a-b-d+2c)^2 + (2a+b-d-5c)^2 + (a-2b+5d-c)^2$ $(b+5a+2d+c)^2 + (5b-a-d+2c)^2 + (2b+a-d-5c)^2 + (b-2a+5d-c)^2$
51	$51(2m+1)$	$(a+7b+c)^2 + (7a-b+d)^2 + (a-c-7d)^2 + (-b+7c-d)^2$ $(b+7a+c)^2 + (7b-a+d)^2 + (b-c-7d)^2 + (-a+7c-d)^2$ $(a+7b+d)^2 + (7a-b+c)^2 + (a-d-7c)^2 + (-b+7d-c)^2$ $(b+7a+d)^2 + (7b-a+c)^2 + (b-d-7c)^2 + (-a+7d-c)^2$
59	$59(2m+1)$	$(a+7b+3c)^2 + (7a-b+3d)^2 + (3a-c-7d)^2 + (-3b+7c-d)^2$ $(b+7a+3c)^2 + (7b-a+3d)^2 + (3b-c-7d)^2 + (-3a+7c-d)^2$ $(a+7b+3d)^2 + (7a-b+3c)^2 + (3a-d-7c)^2 + (-3b+7d-c)^2$ $(b+7a+3d)^2 + (7b-a+3c)^2 + (3b-d-7c)^2 + (-3a+7d-c)^2$

Table 3: Decomposition arising from Yang's composition  $(4n+3)(2m+1)$  where  $2m+1 = a^2 + b^2 + c^2 + d^2$  using Theorem 2.

Length	Sum of Squares	Sequences
31	$5^2 + 2^2 + 1^2 + 1^2$	see [1]
31	$3^2 + 3^2 + 3^2 + 2^2$	(i) { 1 0 0 -1 0 -1 1 1 1 -1 0 1 0 0 1 0 <sub>16</sub> } { 0 <sub>16</sub> -1 0 0 -1 1 0 -1 1 0 1 1 0 0 1 1 } { 0 <sub>15</sub> 1 0 1 -1 0 0 -1 0 0 1 0 0 1 1 0 0 } { 0 -1 1 0 1 0 <sub>5</sub> -1 0 1 1 0 <sub>17</sub> } (ii) { 1 0 0 1 0 -1 1 1 1 -1 0 -1 0 0 1 0 <sub>16</sub> } { 0 <sub>16</sub> -1 0 0 -1 1 0 -1 1 0 1 1 0 0 1 1 } { 0 <sub>15</sub> 1 0 1 -1 0 0 -1 0 0 1 0 0 1 1 0 0 } { 0 -1 1 0 1 0 <sub>5</sub> -1 0 1 1 0 <sub>17</sub> }
33	$5^2 + 2^2 + 2^2$	(i) { 0 <sub>16</sub> 1 0 1 0 1 0 -1 1 0 1 1 0 -1 0 1 0 0 } { 1 -1 0 <sub>3</sub> 1 1 1 1 -1 1 0 <sub>5</sub> -1 -1 0 <sub>17</sub> } { 0 0 1 1 -1 0 <sub>6</sub> 1 1 -1 0 <sub>19</sub> } { 0 <sub>17</sub> -1 0 1 0 -1 0 0 -1 0 0 1 0 1 0 1 -1 } (ii) { 0 <sub>16</sub> 1 0 -1 0 0 1 1 0 <sub>3</sub> 1 1 0 0 1 0 0 } { 0 -1 1 -1 1 -1 -1 1 -1 -1 1 1 1 1 1 0 <sub>13</sub> } { 1 0 <sub>14</sub> 1 0 <sub>17</sub> } { 0 <sub>17</sub> -1 0 1 0 -1 0 0 1 -1 1 0 0 -1 -1 0 1 1 }
33	$4^2 + 4^2 + 1^2$	see [1] or (i) { 0 1 1 0 1 -1 1 0 0 1 1 -1 0 1 -1 0 <sub>18</sub> } { 1 0 0 1 0 <sub>3</sub> 1 1 0 <sub>3</sub> -1 0 0 1 0 <sub>17</sub> } { 0 <sub>16</sub> -1 0 0 1 0 -1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> } { 0 <sub>17</sub> 1 1 0 1 0 -1 -1 -1 1 -1 0 -1 0 1 1 -1 }
33	$4^2 + 3^2 + 2^2 + 2^2$	(i) { 0 <sub>16</sub> -1 -1 0 0 1 1 -1 1 1 1 -1 1 1 0 0 1 0 } { 0 <sub>18</sub> 1 1 0 <sub>9</sub> -1 1 0 1 } { 0 1 0 1 0 -1 0 -1 1 0 -1 0 1 0 1 0 <sub>18</sub> } { -1 0 1 0 1 0 1 0 0 -1 0 1 0 1 0 -1 0 <sub>17</sub> } (ii) { 0 <sub>16</sub> -1 0 0 1 1 1 -1 1 1 1 -1 1 1 -1 0 <sub>3</sub> } { 0 <sub>17</sub> -1 1 0 <sub>11</sub> 1 1 1 } { 1 1 0 -1 0 -1 0 <sub>4</sub> 1 0 1 0 -1 1 0 <sub>17</sub> } { 0 0 1 0 1 0 1 -1 -1 1 0 1 0 -1 0 <sub>19</sub> } (iii) { 0 <sub>17</sub> -1 0 1 0 -1 1 1 -1 1 -1 1 0 1 0 1 1 } { 0 <sub>16</sub> 1 0 -1 0 1 0 <sub>7</sub> 1 0 1 0 0 } { 0 0 1 -1 -1 -1 -1 1 1 1 1 -1 1 1 0 <sub>19</sub> } { 1 -1 0 <sub>12</sub> 1 1 0 <sub>17</sub> }

Table 4: new T-sequences of length  $t$

Length	Sum of Squares	Sequences
35	$5^2 + 3^2 + 1^2$	see [1, 7], or (i) { 1 0 1 0 1-1-1-1 1 1-1 1 1 0 1 0 1 0 <sub>18</sub> } { 0 <sub>19</sub> 1 0 1 1 0-1-1 1 1 0 1-1 0 1 0-1 } { 0 <sub>17</sub> 1-1 0 1 0 0 1 0 <sub>4</sub> 1 0 0-1 0-1 0 } { 0 1 0 1 0 <sub>9</sub> -1 0-1 0 <sub>19</sub> } (ii) { 1 0 1 0 1-1-1-1 1 1-1 1 1 0 1 0 1 0 <sub>18</sub> } { 0 <sub>19</sub> 1 0 1-1 0 1-1-1 1 0 1 1 0-1 0 1 } { 0 <sub>17</sub> -1-1 0 1 0 0-1 0 <sub>4</sub> 1 0 0 1 0 1 0 } { 0 1 0 1 0 <sub>9</sub> -1 0-1 0 <sub>19</sub> }
35	$4^2 + 3^2 + 3^2 + 1^2$	(i) { 1 0 1 0 0-1 0 1 0-1 0 1 0 0 1 0 1 0 <sub>18</sub> } { 0-1 0-1 1 0-1 0 1 0 1 0 1 1 0 1 0 <sub>19</sub> } { 0 <sub>17</sub> -1-1 0 1 1 1 0 <sub>6</sub> 1-1 1 0 1 0 } { 0 <sub>19</sub> 1 0 <sub>3</sub> -1 1-1-1 1 1 0 <sub>3</sub> -1 0 1 }
37	$6^2 + 1^2$	(i) { 1 0 0 1 1 0 0 1 0 0 1 0 0 1 1 0 0-1 0 <sub>19</sub> } { 0 <sub>18</sub> 1 0 1 0-1 0-1-1 1 0 1-1 1 0 1 0-1 0 0 } { 0 <sub>19</sub> 1 0 1 0-1 0 <sub>3</sub> -1 0 <sub>3</sub> -1 0-1 0 1 1 } { 0-1-1 0 0 1-1 0 1-1 0 1 1 0 0-1 1 0 <sub>20</sub> }
37	$5^2 + 2^2 + 2^2 + 2^2$	(i) { 0 <sub>20</sub> 1-1 0 1 0-1 0 <sub>3</sub> 1 0 1 0 1 1 0 1 } { 0 <sub>18</sub> 1 1 0 0-1 0 1 0 1-1 1 0 1 0-1 0 0-1 0 } { 0 1 0 0 1-1-1 0 1-1 0-1 1 1 0 0 1 0 <sub>20</sub> } { 1 0-1-1 0 <sub>3</sub> 1 0 0-1 0 <sub>3</sub> 1 1 0 1 0 <sub>19</sub> }
37	$4^2 + 4^2 + 2^2 + 1^2$	(i) { 0 1 1 0 1-1-1 0 1 1 0-1 1 1 1 0-1 1 0 <sub>20</sub> } { 1 0 0 1 0 <sub>3</sub> -1 0 0 1 0 <sub>3</sub> 1 0 0 1 0 <sub>19</sub> } { 0 <sub>18</sub> 1 0 1 0 0-1 1 0 0-1 0 0-1 1 0 0 1 0 0 } { 0 <sub>19</sub> -1 0-1-1 0 0-1 1 0 1 1 0 0-1 1 0 1 1 }

Table 4(cont.): new T-sequences of length  $t$

$4n+3$	$t$	$(4n+3)t$ Sum of Squares	Sequences
11	3	$33 = 5^2 + 2^2 + 2^2 + 0^2$	{1 0 <sub>5</sub> -1 0 1 0 0 0 1 0 1 0 0 0 1 0-1 0 0 0 1 0 <sub>5</sub> 1 0 0} {0 1 1 0 <sub>4</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1-1 0 <sub>4</sub> -1 0} {0 <sub>4</sub> -1-1 0 0 0-1 0-1 0 0 0 1 0-1 0 0 0 1 0 1 0 <sub>4</sub> 1-1 0 0 0} {0 0 0-1 0 <sub>4</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>4</sub> 1 0 <sub>4</sub> -1}
11	5	$55 = 7^2 + 2^2 + 1^2 + 1^2$	{1 1 0 <sub>4</sub> -1-1 0 1-1 0 <sub>5</sub> 1 1 0 1-1 0 <sub>5</sub> 1 1 0-1 1 0 <sub>5</sub> 1 1 0 <sub>4</sub> 1 0 <sub>4</sub> } {0 0 1 1-1 0 <sub>7</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1-1 1 0 <sub>7</sub> -1-1 0 0} {0 <sub>7</sub> -1-1 1 0 <sub>5</sub> -1-1 0-1 1 0 <sub>5</sub> 1 1 0-1 1 0 <sub>5</sub> 1 1 0 1-1 0 <sub>7</sub> 1-1 1 0 <sub>5</sub> } {0 <sub>5</sub> -1-1 0 <sub>10</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>7</sub> 1 1 0 <sub>8</sub> 1-1}
11	7	$77 = 6^2 + 4^2 + 4^2 + 3^2$	{1 1-1 0 0 1 0 <sub>6</sub> -1-1 1 0 1 0 1 0 <sub>7</sub> 1 1-1 0 1 0 1 0 <sub>7</sub> 1 1-1 0-1 0 -1 0 <sub>7</sub> 1 1-1 0 0-1 0 <sub>6</sub> 1 0 <sub>6</sub> } {0 0 0 1 1 0 1 0 <sub>10</sub> -1 0-1 0 <sub>11</sub> 1 0 1 0 <sub>11</sub> 1 0 1 0 <sub>11</sub> 1-1 0-1 0 <sub>8</sub> 1 -1-1 0 0 0} {0 <sub>10</sub> -1-1 0-1 0 <sub>7</sub> -1-1 1 0-1 0-1 0 <sub>7</sub> 1 1-1 0-1 0-1 0 <sub>7</sub> 1 1-1 0 1 0 1 0 <sub>10</sub> 1-1 0-1 0 <sub>5</sub> 1 0} {0 <sub>7</sub> -1-1 1 0 0-1 0 <sub>11</sub> 1 0-1 0 <sub>11</sub> 1 0-1 0 <sub>11</sub> -1 0 1 0 <sub>8</sub> 1 1-1 0 0-1 0 <sub>5</sub> -1 0-1}
15	3	$45 = 6^2 + 3^2 + 0^2 + 0^2$	{-1 0 1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> 1 0 0} {0-1 0 <sub>5</sub> 1 1 0 <sub>4</sub> -1 1 0 <sub>4</sub> 1-1 0 <sub>4</sub> 1-1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1 0 <sub>5</sub> -1 0} {0 <sub>5</sub> 1 0-1 0 <sub>4</sub> 1-1 0 <sub>4</sub> -1-1 0 <sub>4</sub> -1-1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1-1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> } {0 <sub>4</sub> 1 0 <sub>4</sub> -1 0 <sub>5</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>6</sub> 1 0 <sub>5</sub> -1}
15	5	$75 = 7^2 + 5^2 + 1^2 + 0^2$	{-1-1 0 1-1 0 <sub>5</sub> 1 1 0 <sub>4</sub> -1-1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1 1 0 1-1 0 <sub>5</sub> 1 0 <sub>4</sub> } {0 0-1 0 <sub>5</sub> 1 1-1 0 <sub>7</sub> -1 1-1 0 <sub>7</sub> 1-1 1 0 <sub>7</sub> 1-1 1 0 <sub>7</sub> 1 1-1 0 <sub>7</sub> 1 0 <sub>5</sub> -1 -1 0 0} {0 <sub>5</sub> 1 1 0-1 1 0 <sub>7</sub> 1-1 1 0 <sub>7</sub> -1-1 1 0 <sub>7</sub> -1-1 1 0 <sub>7</sub> 1 1-1 0 <sub>7</sub> 1-1 1 0 <sub>5</sub> 1 1 0 1-1 0 <sub>5</sub> } {0 <sub>7</sub> 1 0 <sub>7</sub> -1-1 0 <sub>4</sub> -1-1 0 <sub>4</sub> 1 1 0 <sub>4</sub> 1 1 0 <sub>4</sub> -1-1 0 <sub>10</sub> 1 0 <sub>5</sub> 1-1}
19	3	$57 = 7^2 + 2^2 + 2^2 + 0^2$	{1 0 <sub>5</sub> -1 0 1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> -1 0-1 0 <sub>5</sub> -1 0 1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> 1 0-1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 0} {0 1 1 0 <sub>4</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>5</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1-1 0 <sub>4</sub> -1 0} {0 <sub>4</sub> -1-1 0 <sub>5</sub> -1 0-1 0 <sub>5</sub> 1 0-1 0 <sub>5</sub> 1 0-1 0 <sub>5</sub> 1 0 1 0 <sub>5</sub> -1 0 1 0 <sub>5</sub> 1 0-1 0 <sub>5</sub> 1 0 1 0 <sub>4</sub> 1-1 0 <sub>5</sub> } {0 <sub>5</sub> -1 0 <sub>6</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> 1 0 <sub>5</sub> -1 0 <sub>4</sub> 1 0 <sub>4</sub> -1}

Table 5: Possibly inequivalent T-sequences of length  $(4n + 3)t$



t		Sum of Squares	Comment	KF ** Upper Bound	Excess
141	3x47	$11^2 + 4^2 + 2^2$	y = 3, m = 23	13380	12408†
143	11x13	$7^2 + 7^2 + 6^2 + 3^2$	y = 11, m = 6	13676	13156†
153	9x17	$12^2 + 2^2 + 2^2 + 1^2$	y = 9, m = 8	15108	14688†
155	5x31	$12^2 + 3^2 + 1^2 + 1^2$	y = 5, m = 15	15404	14880†
165	11x15	$8^2 + 7^2 + 6^2 + 4^2$	y = 11, m = 7	16936	16500
171	9x19	$13^2 + 1^2 + 1^2$	y = 9, m = 9	17880	17784
171	19x9	$13^2 + 1^2 + 1^2$	y = 19, m = 4	17880	17784
171	19x9	$11^2 + 5^2 + 4^2 + 3^2$	y = 19, m = 4	17880	17784
175	5x35	$13^2 + 2^2 + 1^2 + 1^2$	y = 5, m = 17	18496	18200†
177	59x3	$9^2 + 8^2 + 4^2 + 4^2$	y = 59, m = 1	18808	17700
185	5x37	$13^2 + 4^2$	y = 5, m = 18	20092	19240†
189	21x9	$9^2 + 6^2 + 6^2 + 6^2$	y = 21, m = 4	20756	20412
195	15x13	$9^2 + 7^2 + 7^2 + 4^2$	y = 15, m = 6	21780	21060
203	7x29	$9^2 + 9^2 + 5^2 + 4^2$	y = 7, m = 14	23108	21924
205	5x41	$14^2 + 2^2 + 2^2 + 1^2$	y = 5, m = 20	23452	22960†
217	7x31	$9^2 + 8^2 + 6^2 + 6^2$	y = 7, m = 15	25544	25172
221	17x13	$10^2 + 7^2 + 6^2 + 6^2$	y = 17, m = 6	26264	25636
231	11x21	$15^2 + 2^2 + 1^2 + 1^2$	y = 11, m = 10	28064	27720
235	5x47	$9^2 + 9^2 + 8^2 + 3^2$	y = 5, m = 23	28784	27260†
245	7x35	$9^2 + 8^2 + 8^2 + 6^2$	y = 7, m = 17	30636	30380†
247	19x13	$9^2 + 9^2 + 7^2 + 6^2$	y = 19, m = 6	31020	30628
253	11x23	$10^2 + 9^2 + 6^2 + 6^2$	y = 11, m = 11	32180	31372†
253	23x11	$10^2 + 8^2 + 8^2 + 5^2$	y = 23, m = 5	32180	31372†
255	15x17	$11^2 + 7^2 + 7^2 + 6^2$	y = 15, m = 8	32572	31620†
255	15x17	$10^2 + 9^2 + 7^2 + 5^2$	y = 15, m = 8	32572	31620†
259	7x37	$11^2 + 8^2 + 7^2 + 5^2$	y = 7, m = 18	33332	32116†
259	7x37	$9^2 + 9^2 + 9^2 + 4^2$	y = 7, m = 18	33332	32116†
261	9x29	$10^2 + 10^2 + 6^2 + 5^2$	y = 9, m = 14	33708	32364†
261	9x29	$10^2 + 9^2 + 8^2 + 4^2$	y = 9, m = 14	33708	32364†
265	53x5	$12^2 + 7^2 + 6^2 + 6^2$	y = 53, m = 2	34476	32860†
273	13x21	$16^2 + 3^2 + 2^2 + 2^2$	y = 13, m = 10	36036	34944†
275	11x25	$9^2 + 9^2 + 8^2 + 7^2$	y = 11, m = 12	36432	36300†
279	31x9	$13^2 + 7^2 + 6^2 + 5^2$	y = 31, m = 4	37240	34596†
279	31x9	$11^2 + 10^2 + 7^2 + 3^2$	y = 31, m = 4	37240	34596†
285	15x19	$11^2 + 8^2 + 8^2 + 6^2$	y = 15, m = 9	38472	37620†
285	15x19	$10^2 + 10^2 + 7^2 + 6^2$	y = 15, m = 9	38472	37620†
287	7x41	$11^2 + 11^2 + 6^2 + 3^2$	y = 7, m = 20	38888	35588†
289	17x17	$17^2$	y = 17, m = 8	39304	39304*†
295	5x59	$17^2 + 2^2 + 1^2 + 1^2$	y = 5, m = 29	40512	40120†
297	11x27	$17^2 + 2^2 + 2^2$	y = 11, m = 13	40920	40392†
297	33x9	$17^2 + 2^2 + 2^2$	y = 33, m = 4	40920	40392†

Table 6: 4-disjoint T-sequences giving Hadamard matrices with maximum known excess

\* means the matrix constructed has maximal excess

† new maximum known excess

\*\* see Kounias and Farmakis [6]

## References

1. G. Cohen, D. Rubie, J. Seberry, C. Koukouvinos, S. Kounias, and M. Yamada, *A survey of base sequences, disjoint complementary sequences and  $OD(4t; t, t, t, t)$* , JCMCC 5 (1989), 69–104.
2. A.V. Geramita and J. Seberry, *Orthogonal Designs: Quadratic forms and Hadamard matrices*, Marcel Dekker, New York—Basel, viii (1979), 460 pages.
3. J. Hammer, R. Levingston and J. Seberry, *A remark on the excess of Hadamard matrices orthogonal designs*, Ars Combinatoria 5 (1978), 237–254.
4. B.A. Jenkins, C. Koukouvinos, S. Kounias, J. Seberry, R. Seberry, *Some results on the excesses of Hadamard matrices*, JCMCC and University College (ADFA), TR CS88/8 4 (1988), 155–188.
5. C. Koukouvinos and S. Kounias, *An infinite class of Hadamard matrices*, J. Austral. Math. Soc. Ser A (to appear).
6. S. Kounias and N. Farmakis, *On the excess of Hadamard matrices*, Discrete Math. 68 (1988), 59–69.
7. J. Seberry, *Constructing Hadamard matrices from orthogonal designs*, University College (ADFA), TR CS88/5, (submitted).
8. J. Seberry Wallis, *Hadamard matrices*, in “Combinatorics: Room Squares, Sum-free Sets, Hadamard Matrices”, ed. W.D. Wallis, Anne Penfold Street and Jennifer Seberry Wallis, Vol. 292, Lecture Notes in Mathematics, Springer-Verlag, Berlin-Heidelberg-New York (1972), 508 pages.
9. R.J. Turyn, *Hadamard matrices, Baumert-Hall units, four symbol sequences, pulse compression and surface wave encodings*, J. Combinatorial Th., Ser. A 16 (1974), 313–333.
10. C.H. Yang, *Hadamard matrices and  $\delta$ -codes of length  $3n$* , Proc. Amer. Math. Soc. 85 (1982), 480–482.
11. C.H. Yang, *A composition theorem for  $\delta$ -codes*, Proc. Amer. Math. Soc. 89 (1983), 375–378.
12. C.H. Yang, *Lagrange identity for polynomials and  $\delta$ -codes of lengths  $7t$  and  $13t$* , Proc. Amer. Math. Soc. 88 (1983), 746–750.
13. C.H. Yang, *On composition of four symbol  $\delta$ -codes of length  $(4n + 3)t$  and Hadamard matrices*, Proc. Amer. Math. Soc. (to appear).