[135] K Jennifer Seberry and Mieko Yamada, Products of Hadamard, Williamson and other orthogonal matrices using M-structures, *Proceedings of Combinatorics Conference*, Kobe, Japan, November 1989.

(see tables)

Products of Hadamard Matrices, Williamson Matrices and Other Orthogonal Matrices using M-Structures

Jennifer Seberry* and Mieko Yamada†

Abstract

The new concept of M-structures is used to unify and generalize a number of concepts in Hadamard matrices including Williamson matrices, Goethals-Seidel matrices, Wallis-Whiteman matrices and generalized quaternion matrices. The concept is used to find many new symmetric Williamson-type matrices, both in sets of four and eight, and many new Hadamard matrices. We give as corollaries "that the existence of Hadamard matrices of orders $4g$ and $4h$ implies the existence of an Hadamard matrix of order $8gh$" and "the existence of Williamson type matrices of orders u and v implies the existence of Williamson type matrices of order $2uv$". This work generalizes and utilizes the work of Masahiko Miyamoto and Mieko Yamada. Lists of odd orders < 1000 for which Hadamard and Williamson type matrices are known are given.

1 Definitions and Introduction

For the definitions used in this paper, and for detailed proofs, we refer the reader to [47].

2 M-structures

An orthogonal matrix of order $4t$ can be divided into sixteen (16) $t \times t$ blocks M_{ij}. This partitioned matrix is said to be an M-structure. If the orthogonal matrix can be partitioned into sixty-four (64) $s \times s$ blocks M_{ij} it will be called a 64 block M-structure.

An Hadamard matrix made from (symmetric) Williamson matrices W_1, W_2, W_3, W_4 is an M-structure with

$$\begin{aligned}W_1 = M_{11} = M_{22} = M_{33} = M_{44},\\ W_2 = M_{12} = -M_{21} = M_{34} = -M_{43},\\ W_3 = M_{13} = -M_{31} = -M_{24} = M_{42}, \quad \text{and}\\ W_4 = M_{14} = -M_{41} = M_{23} = -M_{32}.\end{aligned}$$

An Hadamard matrix made from four (4) circulant (or type 1) matrices A_1, A_2, A_3, A_4 of order n, where R is the matrix which makes all the A_iR back-circulant (or type 2), is an M-structure with

$$\begin{aligned}A_1 = M_{11} = M_{22} = M_{33} = M_{44},\\ A_2 = M_{12}R = -M_{21}R = RM_{34}^T = -RM_{43}^T,\\ A_3 = M_{13}R = -M_{31}R = -RM_{24}^T = RM_{42}^T, \quad \text{and}\\ A_4 = M_{14}R = -M_{41}R = RM_{23}^T = -RM_{32}^T.\end{aligned}$$

The next theorem and corollary are easy to prove using M-structures.

Theorem 1 *Suppose there are T-matrices of order t. Further suppose there is an $OD(4s; u_1, \ldots, u_n)$ constructed of sixteen circulant (or type 1) $s \times s$ blocks on the variables $x_1, \ldots, x_n$. Then there is an $OD(4st; tu_1, \ldots, tu_n)$. In particular if there is an $OD(4s; s, s, s, s)$ constructed of sixteen circulant (or type 1) $s \times s$ blocks then there is an $OD(4st; st, st, st, st)$.*

Corollary 2 *Suppose the T-matrices are of order t. Then there are orthogonal designs $OD(20t; 5t, 5t, 5t, 5t)$ and $OD(36t; 9t, 9t, 9t, 9t)$.*

Conjecture 3 *There exists an $OD(4t; t, t, t, t)$ for every positive integer t.*

We also conjecture

Conjecture 4 *There exists an M-structure $OD(4t; t, t, t, t)$ for every $t \equiv 1 \pmod 4$ comprising sixteen circulant or type 1 blocks.*

3 Some properties of certain amicable orthogonal matrices

We use the following three lemmas proved in [47].

Lemma 5 *Suppose there exist two amicable $(0,+1,-1)$ matrices U, V of order u satisfying $UU^T + VV^T = (2u-1)I$. Then there exist matrices A, B, D of order u satisfying*

$$AA^T + BB^T = B^TB + D^TD = (2u-1)I$$
$$A^T = (-1)^{\frac{1}{2}(u-1)}A, D^T = (-1)^{\frac{1}{2}(u-1)}D,$$

where A and D have zero diagonal.

Lemma 6 *Let $q+1$ be the order of a conference matrix. Then there exist four matrices C_1, C_2, C_3, C_4, of order $\frac{1}{2}(q-1)$ satisfying*

$$C_1C_1^T + C_2C_2^T = C_3C_3^T + C_4C_4^T = qI - 2J,$$
$$eC_1^T = eC_4^T = e, \quad eC_2^T = eC_3^T = 0,$$
$$C_1C_3^T - C_2C_4^T = 0, \quad C_1^T = C_1, \quad C_4^T = C_4, \quad C_3^T = C_2,$$

where e is the $1 \times \frac{1}{2}(q-1)$ matrix of ones, C_1 and C_4 have zero diagonal elements ±1, C_2 and C_4 have elements ±1.

Lemma 7 *Suppose there exist two amicable $(0,+1,-1)$ matrices U, V of order u satisfying $UU^T + VV^T = (2u-1)I$. Further suppose U has zero diagonal and U, V have other elements $+1$ or -1. Then there exist matrices A, B of order $u-1$ satisfying*

$$AA^T + BB^T = (2u-1)I_{u-1} - 2J_{u-1},$$
$$eA^T = e, \quad eB^T = 0, \quad AB^T = BA^T,$$

where A has one zero element per row and column and the other entries of A and B are ±1. Further if U and V are symmetric (or skew-type respectively) then A and B are symmetric (or skew-type respectively).

Furthermore if U and V satisfy $UU^T + VV^T = 2uI$ (U, V are $(1,-1)$ matrices), u even, then there exist matrices A, B of order $u-1$, with entries ±1, satisfying

$$AA^T + BB^T = 2uI_{u-1} - 2J_{u-1},$$
$$eA^T = e, \quad eB^T = e, \quad AB^T = BA^T, \ .$$

and if U and V are symmetric (or skew-type respectively) then A and B are symmetric (or skew-type respectively).

4 A multiplication Theorem using M-structures

Theorem 8 *Let $N = (N_{ij})$, $i,j = 1,2,3,4$ be an Hadamard matrix of order $4n$ of M-structure. Further let T_{ij}, $i,j = 1,2,3,4$ be 16 $(0,+1,-1)$ type 1 or circulant matrices of order t which satisfy*

(i) $T_{ij} * T_{ik} = 0$, $T_{ji} * T_{ki} = 0$, $j \neq k$, ($*$ the Hadamard product)

(ii) $\sum_{k=1}^{4} T_{ik}$ is a $(1,-1)$ matrix, (1)

(iii) $\sum_{k=1}^{4} T_{ik}T_{ik}^T = tI_t = \sum_{k=1}^{4} T_{ki}T_{ki}^T$,

(iv) $\sum_{k=1}^{4} T_{ik}T_{jk}^T = 0 = \sum_{k=1}^{4} T_{ki}T_{kj}^T$, $i \neq j$.

Then there is an M-structure Hadamard matrix of order $4nt$.

Corollary 9 *If there exists an Hadamard matrix of order $4h$ and an orthogonal design $OD(4u; u_1, u_2, u_3, u_4)$, then an $OD(8hu; 2hu_1, 2hu_2, 2hu_3, 2hu_4)$ exists.*

Corollary 10 *If there exists an Hadamard matrix of order $4h$ and an orthogonal design $OD(4u; u, u, u, u)$, then there exists an $OD(8hu; 2hu, 2hu, 2hu, 2hu)$.*

This gives the theorem of Agayan and Sarukhanyan [2] as a corollary by setting all variables equal to one:

Corollary 11 *If there exists Hadamard matrices of orders $4h$ and $4u$ then there exists an Hadamard matrix of order $8hu$.*

We now give as a corollary a result, motivated by, and a little stronger than that of Agayan and Sarukhanyan [2]:

Corollary 12 *Suppose there are Williamson or Williamson type matrices of orders u and v. Then there are Williamson type matrices of order $2uv$.*

If the matrices of orders u and v are symmetric the matrices of order $2uv$ are also symmetric.

If the matrices of orders u and v are circulant and/or type 1 the matrices of order $2uv$ are type 1.

5 Miyamoto's Theorem and Corollaries via M-structures

We reformulate Miyamoto's results so that symmetric Williamson-type matrices can be obtained.

Lemma 13 (Miyamoto's Lemma Reformulated) *Let U_i, V_j, $i, j = 1, 2, 3, 4$ be $(0, +1, -1)$ matrices of order n which satisfy*

(i) U_i, U_j, $i \neq j$ are pairwise amicable,

(ii) V_i, V_j, $i \neq j$ are pairwise amicable, *(iii)* $U_i \pm V_i$, $(+1,-1)$ *matrices,* $i = 1,2,3,4$,

(iv) the row sum of U_1 *is 1, and the row sum of* U_j, $i = 2,3,4$ *is zero,*

(v) $\sum_{i=1}^{4} U_iU_i^T = (2n+1)I - 2J$, $\sum_{i=1}^{4} V_iV_i^T = (2n+1)I$.

Then there are 4 Williamson type matrices of order $2n+1$*. If* U_i *and* V_i *are symmetric,* $i = 1,2,3,4$ *then the Williamson-type matrices are symmetric. Hence there is a Williamson type Hadamard matrix of order* $4(2n+1)$.

Corollary 14 *Let* $q \equiv 1 \pmod 4$ *be a prime power then there are symmetric Williamson type matrices of order* $q+2$ *whenever* $\frac{1}{2}(q+1)$ *is a prime power or* $\frac{1}{2}(q+3)$ *is the order of a symmetric conference matrix. Also there exists an Hadamard matrix of Williamson type of order* $4(q+2)$.

Remark 15 Some of the results in Corollary 14 are also due to A.L. Whiteman [35]. This gives symmetric Williamson-type matrices of orders

7	11	15	19	27	39	51	55	63	75
83	91	99	123	159	195	243	279	315	339
363	399	423	451	459	543	579	615	627	663
675	735	759	843	879	883	999	1095	1155	1203
1215	1239	1251	1323	1383	1455	1623	1659	1683	1755
1875	1935	1995							

(since Mathon found conference matrices of orders 46 and 442). Almost all these, with symmetry, are new though Miyamoto [12] has found Williamson-type matrices for these orders and hence Hadamard matrices for four times these orders.

Koukouvinos and Kounias [10] have shown there are no circulant symmetric Williamson matrices of order 39 but here a symmetric but not circulant Williamson matrix of order 39 is given.

Corollary 16 *Let* $q \equiv 1 \pmod 4$ *be a prime power. Then*

(i) if there are Williamson type matrices of order $(q-1)/4$ *or an Hadamard matrix of order* $\frac{1}{2}(q-1)$ *there exist Williamson type matrices of order* q*;*

(ii) if there exist symmetric conference matrices of order $\frac{1}{2}(q-1)$ *or a symmetric Hadamard matrix of order* $\frac{1}{2}(q-1)$ *then there exist symmetric Williamson type matrices of order* q.

Hence there exists an Hadamard matrix of Williamson type of order $4q$.

Remark 17 Part (i) of Corollary 16 for Williamson matrices of order $(q-1)/4$ was found by Miyamoto [12]. Part (i) with Hadamard matrices of order $\frac{1}{2}(q-1)$ is new. Part (ii) with symmetry is new.

Corollary 16 (ii) gives symmetric Williamson-type matrices of order q when $q \equiv 1 \pmod 4$ is a prime power and $\frac{1}{2}(q-1)$ is the order of a symmetric conference matrix. This gives symmetric Williamson-type matrices for the following orders:

13	29	37	53	61	101	109	125	149	181
197	229	277	317	349	389	397	461	541	557
677	701	709	797	821	1021	1061	1117	1229	1237
1549	1597	1621	1709	1861	1877	1997			

Corollary 16 part (ii) gives symmetric Williamson-type matrices of order q when $q \equiv 1 \pmod 4$ is a prime power and $\frac{1}{2}(q-1)$ is the order of a symmetric Hadamard matrix. Rembering that symmetric Hadamard matrices exist for orders $p+1$ when $p \equiv 3 \pmod 4$ is a prime power we have symmetric Williamson-type matrices for the following orders:

5	9	17	25	41	49	73	81	89	97
113	121	169	193	241	257	281	289	337	353
361	401	409	433	449	457	529	569	577	593
601	617	625	641	673	729	761	769	841	881
929	937	961	977	1009	1033	1049	1097	1129	1153
1201	1217	1249	1289	1297	1321	1361	1369	1409	1481
1489	1553	1601	1609	1657	1681	1697	1721	1777	1801
1849	1873								

Corollary 16 part (i) gives Williamson-type matrices of order q when $q \equiv 1 \pmod 4$ is a prime power and $\frac{1}{2}(q-1)$ is the order of an Hadamard matrix. This gives Williamson-type matrices for the following orders not given above:

137	233	313	521	809	953	1193	1753	1889	1993

Corollary 16 part (i) gives Williamson-type matrices of order q when $q \equiv 1 \pmod 4$ is a prime power and $(q-1)/4$ is the order of Williamson-type matrices. This result is also due to Miyamoto [12]. This gives Williamson-type matrices for the following orders:

157	173	293	373	613	757	757	773	1109	1301
1453	1493	1637	1693	1733	1741				

Corollary 18 *Let $q \equiv 1 \pmod 4$ be a prime power or $q+1$ the order of a symmetric conference matrix. Let $2q-1$ be a prime power. Then there exist*

symmetric Williamson type matrices of order $2q+1$ and an Hadamard matrix of Williamson type of order $4(2q+1)$.

Remark 19 Corollary 18 is satisfied for the appropriate primes or conference matrix orders to give symmetric Williamson-type matrices for the following orders:

11	19	27	51	75	83	91	99	123	195
243	315	339	363	451	459	579	627	675	843
883	1155	1203	1251	1323	1659	1683	1755	1875	1995
2019	2139	2403	2475	2595	2859	3043	3219	3315	3363
3483	3699	3723							

Note this last corollary is a modified version of Miyamoto's Corollary 5 (original manuscript). A new proof of Miyamoto's result, preserving symmetry, gives:

Corollary 20 *Let $q \equiv 5 \pmod 8$ be a prime power. Further let $\frac{1}{2}(q-3)$ be a prime power or $\frac{1}{2}(q-1)$ be the order of a symmetric conference matrix then there exist symmetric Williamson type matrices of order q and an Hadamard matrix of Williamson type of order $4q$.*

Theorem 21 (Miyamoto's Theorem Reformulated) *Let U_{ij}, V_{ij}, $i,j = 1,2,3,4$ be $(0,+1,-1)$ matrices of order n which satisfy*

(i) U_{ki}, U_{kj}, $i \neq j$ *are pairwise amicable,* $k = 1,2,3,4$,

(ii) V_{ki}, V_{kj}, $i \neq j$ *are pairwise amicable,* $k = 1,2,3,4$,

(iii) $U_{ki} \pm V_{ki}$, $(+1,-1)$ *matrices,* $i,k = 1,2,3,4$,

(iv) the row sum of U_{ii} is 1, and the row sum of U_ij is zero, $i \neq j$, $i,j = 1,2,3,4$,

(v) $\sum_{i=1}^{4} U_{ji}U_{ji}^T = (2n+1)I - 2J$, $\sum_{i=1}^{4} V_{ji}V_{ji}^T = (2n+1)I$, $j = 1,2,3,4$,

(vi) $\sum_{i=1}^{4} U_{ji}U_{ki}^T = 0$, $\sum_{i=1}^{4} V_{ji}V_{ki}^T = 0$, $j \neq k$, $j,k = 1,2,3,4$.

If conditions (i) to (v) hold, there are four Williamson matrices type of order $2n+1$ and thus a Williamson type Hadamard matrix of order $4(2n+1)$. Furthermore if the matrices U_{ki} and V_{ki} are symmetric for all $i,j = 1,2,3,4$ the Williamson matrices obtained of order $2n+1$ are also symmetric.

If conditons (iii) to (vi) hold, there is an M-structure Hadamard matrix of order $4(2n+1)$.

Proof: Use

$$X_{11} = \begin{bmatrix} -1 & -e \\ -e^T & S_{11} \end{bmatrix} \quad X_{12} = \begin{bmatrix} 1 & e \\ e^T & S_{12} \end{bmatrix} \quad X_{13} = \begin{bmatrix} 1 & e \\ e^T & S_{13} \end{bmatrix} \quad X_{14} = \begin{bmatrix} -1 & e \\ e^T & S_{14} \end{bmatrix}$$

$$X_{21} = \begin{bmatrix} 1 & e \\ e^T & S_{21} \end{bmatrix} \quad X_{22} = \begin{bmatrix} -1 & -e \\ -e^T & S_{22} \end{bmatrix} \quad X_{23} = \begin{bmatrix} 1 & e \\ e^T & S_{23} \end{bmatrix} \quad X_{24} = \begin{bmatrix} -1 & e \\ e^T & S_{24} \end{bmatrix}$$

$$X_{31} = \begin{bmatrix} 1 & e \\ e^T & S_{31} \end{bmatrix} \quad X_{32} = \begin{bmatrix} 1 & e \\ e^T & S_{32} \end{bmatrix} \quad X_{33} = \begin{bmatrix} -1 & -e \\ -e^T & S_{33} \end{bmatrix} \quad X_{34} = \begin{bmatrix} -1 & e \\ e^T & S_{34} \end{bmatrix}$$

$$X_{41} = \begin{bmatrix} -1 & e \\ e^T & -S_{41} \end{bmatrix} \quad X_{42} = \begin{bmatrix} 1 & e \\ e^T & -S_{42} \end{bmatrix} \quad X_{43} = \begin{bmatrix} -1 & e \\ e^T & -S_{43} \end{bmatrix} \quad X_{44} = \begin{bmatrix} -1 & -e \\ -e^T & -S_{44} \end{bmatrix}$$

We note that the following always holds as it is just a case of Miyamoto's Lemma Reformulated:

$$\sum_{i=1}^{4} S_{ji} S_{ji}^T = 4(2n+1)I_{2n} - 4J_{2n}. \tag{2}$$

In all cases though assumption (vi) assures us that

$$\sum_{i=1}^{4} S_{ki} S_{ji}^T = 0, \quad j \neq k. \tag{3}$$

Note that if we write our M-structure from the theorem as

-1	1	1	-1	-e	e	e	e
1	-1	1	-1	e	-e	e	e
1	1	-1	-1	e	e	-e	e
1	1	1	1	-e	-e	-e	e
$-e^T$	e^T	e^T	e^T	S_{11}	S_{12}	S_{13}	S_{14}
e^T	$-e^T$	e^T	e^T	S_{21}	S_{22}	S_{23}	S_{24}
e^T	e^T	$-e^T$	e^T	S_{31}	S_{32}	S_{33}	S_{34}
$-e^T$	$-e^T$	$-e^T$	e^T	S_{41}	S_{42}	S_{43}	S_{44}

and we can see Yamada's matrix with trimming [46] or the J. Wallis-Whiteman [30] matrix with a border embodied in the construction.

Corollary 22 *Suppose there exists a symmetric conference matrix of order $q+1 = 4t+2$ and an Hadamard matrix of order $4t = q-1$. Then there is an Hadamard matrix with M-structure of order $4(4t+1) = 4q$. Further if the Hadamard matrix is symmetric the Hadamard matrix of order $4q$ is of the form*

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix},$$

where X, Y are amicable and symmetric.

We note that complex Hadamard matrices of order $n \equiv 2 \pmod 4$ do exist when symmetric conference matrices cannot exist (see [22, Chapter VI]). These complex Hadamard matrices may be written as $K = X + iY$ where $KK* = kI_k$ ($*$ the Hermitian conjugate).

Hence we have

Corollary 23 *Let $q \equiv 4f + 1$ be a prime power. Suppose there is a complex Hadamard matrix of order $2f$. Then there is an Hadamard matrix of order $4(4f+1)$.*

Note complex Hadamard matrices exist for orders 22, 34, 58, 86, 306, 650, 870, 1046, 2450, 3782, ..., for which either a symmetric conference matrix cannot exist or is not known. None of these orders give new Hadamard matrices.

6 Using 64 Block M-structures

In a similar fashion, we consider the following lemma so symmetric 8-Williamson-type matrices can be obtained.

Lemma 24 *Let U_i, V_j, $i,j = 1,\ldots,8$ be $(0,+1,-1)$ matrices of order n which satisfy*

(i) U_i, U_j, $i \neq j$ are pairwise amicable,

(ii) V_i, V_j, $i \neq j$ are pairwise amicable,

(iii) $U_i \pm V_i$, $(+1,-1)$ matrices, $i = 1,\ldots,8$,

(iv) the row(column) sums of U_1 and U_2 are both 1, and the row sum of U_j, $i = 3,\ldots,8$ is zero,

(v) $\sum_{i=1}^{8} U_iU_i^T = 2(2n+1)I - 4J$, $\sum_{i=1}^{8} V_iV_i^T = 2(2n+1)I$.

Then there are 8-Williamson type matrices of order $2n+1$. Furthermore, if the U_i and V_i are symmetric, $i = 1,\ldots,8$, then the 8-Williamson type matrices are symmetric. Hence there is a block type Hadamard matrix of order $8(2n+1)$.

Corollary 25 *Let $q+1$ be the order of amicable Hadamard matrices $I + S$ and P. Suppose there exist 4 Williamson type matrices of order q. Then there exist Williamson type matrices of order $2q+1$. Furthermore there exists an Hadamard matrix of block type of order $8(2q+1)$.*

Using the amicable Hadamard matrices given in [22] and [16, Table 1] we get 8 Williamson type matrices for the following orders for which 4 Williamson matrices are not known:

47, 111, 127, 167, 319, 487, 655, 831, ...

This gives new constructions for Hadamard matrices of orders 8.167 and 8.487.

Corollary 26 *Let q be a prime power and $(q-1)/2$ be the order of four (symmetric) Williamson type matrices. Then there exist (symmetric) 8-Williamson type matrices of order q and an Hadamard matrix of block structure of order $8q$.*

In particular we have 8-Williamson matrices for the following orders for which no Williamson type matrices are known:
59, 67, 103, 107, 151, 163, 179, 227, 251, 283, 347, 463, 467, 523, 563, 571, 587, 631, 643, 823, 859, 919, 947, ...

This gives new Hadamard matrices or new constructions for Hadamard matrices of orders 8.107, 8.163, 8.179, 8.251, 8.283, 8.347, 8.463, 8.523, 8.571, 8.631, 8.643, 8.823, 8.859, 8.919, 8.947, ...

Corollary 27 *Let $q \equiv 1 \pmod 4$ be a prime power or $q+1$ the order of a symmetric conference matrix. Suppose there exist four (symmetric) Williamson type matrices of order q. Then there exist (symmetric) 8-Williamson type matrices of order $2q+1$ and an Hadamard matrix of block structure of order $8(2q+1)$.*

This corollary gives 8 Williamson type matrices for the following new orders: 219, 275, 299, 395, 483, 515, 579, 635, 699, 707, 723, 779, 795, 803, 899, 915, 923, ...

It does not give new Hadamard matrices for these orders.

Corollary 28 *Let $q = 9^t$, $t > 0$. Now there exist four (symmetric) Williamson type matrices of order 9^t, $t > 0$. Hence there exist (symmetric) 8-Williamson type matrices of order $2\cdot 9^t + 1$, $t > 0$, and an Hadamard matrix of block structure of order $8(2\cdot 9^t + 1)$.*

This gives symmetric 8-Williamson type matrices for the new order 163, 13123, ...

Also we have the following theorem:

Theorem 29 *Let U_{ij}, V_{ij}, $i,j = 1,\ldots,8$ be $(0,+1,-1)$ matrices of order n which satisfy*

(i) U_{ki}, U_{kj}, $i \neq j$ are pairwise amicable, $k = 1,\ldots,8$,

(ii) V_{ki}, V_{kj}, $i \neq j$ are pairwise amicable, $k = 1,\ldots,8$,

(iii) $U_{ki} \pm V_{ki}$, $(+1,-1)$ *matrices,* $i,k = 1,\ldots,8$,

(iv) the row(column) sum of U_{ab} *is 1 for* $(a,b)\epsilon\{(i,i),(i,i+1),(i+1,i)\}$, $i = 1,3,5,7$, *the row(column) sum of* U_{aa} *is -1 for* $(a,a) = 2,4,6,8$ *and otherwise, and the row(column) sum of* U_{ij}, $i \neq j$ *is zero,*

(v) $\sum_{i=1}^{8} U_{ji}U_{ji}^T = 2(2n+1)I - 4J$, $\sum_{i=1}^{8} V_{ji}V_{ji}^T = 2(2n+1)I$, $j = 1,\ldots,8$,

(vi) $\sum_{i=1}^{8} U_{ji}U_{ki}^T = 0$, $\sum_{i=1}^{8} V_{ji}V_{ki}^T = 0$, $j \neq k$, $j,k = 1,\ldots,8$.

If (iii) to (vi) hold, there is a 64 block M-structure Hadamard matrix of order $8(2n+1)$.

Proof: Use

$$X_{11} = \begin{bmatrix} -1 & -e \\ -e^T & S_{11} \end{bmatrix}, \quad X_{12} = \begin{bmatrix} -1 & -e \\ -e^T & S_{12} \end{bmatrix}, \quad X_{13} = \begin{bmatrix} 1 & e \\ e^T & S_{13} \end{bmatrix}, \quad X_{14} = \begin{bmatrix} 1 & e \\ e^T & S_{14} \end{bmatrix},$$

$$X_{15} = \begin{bmatrix} 1 & e \\ e^T & S_{15} \end{bmatrix}, \quad X_{16} = \begin{bmatrix} 1 & e \\ e^T & S_{16} \end{bmatrix}, \quad X_{17} = \begin{bmatrix} -1 & e \\ e^T & S_{17} \end{bmatrix}, \quad X_{18} = \begin{bmatrix} -1 & e \\ e^T & S_{18} \end{bmatrix},$$

$$X_{21} = \begin{bmatrix} -1 & -e \\ -e^T & S_{21} \end{bmatrix}, \quad X_{22} = \begin{bmatrix} 1 & e \\ e^T & S_{22} \end{bmatrix}, \quad X_{23} = \begin{bmatrix} 1 & e \\ e^T & S_{23} \end{bmatrix}, \quad X_{24} = \begin{bmatrix} -1 & -e \\ -e^T & S_{24} \end{bmatrix},$$

$$X_{25} = \begin{bmatrix} 1 & e \\ e^T & S_{25} \end{bmatrix}, \quad X_{26} = \begin{bmatrix} -1 & -e \\ -e^T & S_{26} \end{bmatrix}, \quad X_{27} = \begin{bmatrix} -1 & e \\ e^T & S_{27} \end{bmatrix}, \quad X_{28} = \begin{bmatrix} 1 & -e \\ -e^T & S_{28} \end{bmatrix},$$

$$X_{31} = \begin{bmatrix} 1 & e \\ e^T & S_{31} \end{bmatrix}, \quad X_{32} = \begin{bmatrix} 1 & e \\ e^T & S_{32} \end{bmatrix}, \quad X_{33} = \begin{bmatrix} -1 & -e \\ -e^T & S_{33} \end{bmatrix}, \quad X_{34} = \begin{bmatrix} -1 & -e \\ -e^T & S_{34} \end{bmatrix},$$

$$X_{35} = \begin{bmatrix} 1 & e \\ e^T & S_{35} \end{bmatrix}, \quad X_{36} = \begin{bmatrix} 1 & e \\ e^T & S_{36} \end{bmatrix}, \quad X_{37} = \begin{bmatrix} -1 & e \\ e^T & S_{37} \end{bmatrix}, \quad X_{38} = \begin{bmatrix} -1 & e \\ e^T & S_{38} \end{bmatrix},$$

$$X_{41} = \begin{bmatrix} 1 & e \\ e^T & S_{41} \end{bmatrix}, \quad X_{42} = \begin{bmatrix} -1 & -e \\ -e^T & S_{42} \end{bmatrix}, \quad X_{43} = \begin{bmatrix} -1 & -e \\ -e^T & S_{43} \end{bmatrix}, \quad X_{44} = \begin{bmatrix} 1 & e \\ e^T & S_{44} \end{bmatrix},$$

$$X_{45} = \begin{bmatrix} 1 & e \\ e^T & S_{45} \end{bmatrix}, \quad X_{46} = \begin{bmatrix} -1 & -e \\ -e^T & S_{46} \end{bmatrix}, \quad X_{47} = \begin{bmatrix} -1 & e \\ e^T & S_{47} \end{bmatrix}, \quad X_{48} = \begin{bmatrix} 1 & -e \\ -e^T & S_{48} \end{bmatrix},$$

$$X_{51} = \begin{bmatrix} 1 & e \\ e^T & S_{51} \end{bmatrix}, \quad X_{52} = \begin{bmatrix} 1 & e \\ e^T & S_{52} \end{bmatrix}, \quad X_{53} = \begin{bmatrix} 1 & e \\ e^T & S_{53} \end{bmatrix}, \quad X_{54} = \begin{bmatrix} 1 & e \\ e^T & S_{54} \end{bmatrix},$$

$$X_{55} = \begin{bmatrix} -1 & -e \\ -e^T & S_{55} \end{bmatrix}, \quad X_{56} = \begin{bmatrix} -1 & -e \\ -e^T & S_{56} \end{bmatrix}, \quad X_{57} = \begin{bmatrix} -1 & e \\ e^T & S_{57} \end{bmatrix}, \quad X_{58} = \begin{bmatrix} -1 & e \\ e^T & S_{58} \end{bmatrix},$$

$$X_{61} = \begin{bmatrix} 1 & e \\ e^T & S_{61} \end{bmatrix}, \quad X_{62} = \begin{bmatrix} -1 & -e \\ -e^T & S_{62} \end{bmatrix}, \quad X_{63} = \begin{bmatrix} 1 & e \\ e^T & S_{63} \end{bmatrix}, \quad X_{64} = \begin{bmatrix} -1 & -e \\ -e^T & S_{64} \end{bmatrix},$$

$$X_{65} = \begin{bmatrix} -1 & -e \\ -e^T & S_{65} \end{bmatrix}, \quad X_{66} = \begin{bmatrix} 1 & e \\ e^T & S_{66} \end{bmatrix}, \quad X_{67} = \begin{bmatrix} -1 & e \\ e^T & S_{67} \end{bmatrix}, \quad X_{68} = \begin{bmatrix} 1 & -e \\ -e^T & S_{68} \end{bmatrix},$$

$$X_{71}=\begin{bmatrix} 1 & -e \\ -e^T & S_{71} \end{bmatrix},\ X_{72}=\begin{bmatrix} 1 & -e \\ -e^T & S_{72} \end{bmatrix},\ X_{73}=\begin{bmatrix} 1 & -e \\ -e^T & S_{73} \end{bmatrix},\ X_{74}=\begin{bmatrix} -1 & -e \\ -e^T & S_{74} \end{bmatrix},$$

$$X_{75}=\begin{bmatrix} 1 & -e \\ -e^T & S_{75} \end{bmatrix},\ X_{76}=\begin{bmatrix} 1 & -e \\ -e^T & S_{76} \end{bmatrix},\ X_{77}=\begin{bmatrix} 1 & e \\ e^T & S_{77} \end{bmatrix},\ X_{78}=\begin{bmatrix} 1 & e \\ e^T & S_{78} \end{bmatrix},$$

$$X_{81}=\begin{bmatrix} 1 & -e \\ -e^T & S_{81} \end{bmatrix},\ X_{82}=\begin{bmatrix} -1 & e \\ e^T & S_{82} \end{bmatrix},\ X_{83}=\begin{bmatrix} 1 & -e \\ -e^T & S_{83} \end{bmatrix},\ X_{84}=\begin{bmatrix} -1 & e \\ e^T & S_{84} \end{bmatrix},$$

$$X_{85}=\begin{bmatrix} 1 & -e \\ -e^T & S_{85} \end{bmatrix},\ X_{86}=\begin{bmatrix} -1 & e \\ e^T & S_{86} \end{bmatrix},\ X_{87}=\begin{bmatrix} 1 & e \\ e^T & S_{87} \end{bmatrix},\ X_{88}=\begin{bmatrix} -1 & -e \\ -e^T & S_{88} \end{bmatrix},$$

Then provided conditions (i) to (v) hold and $S_{7i}^T = S_{7i}$, $i = 1, \ldots, 8$ are symmetric, X_{7i}, $i = 1, \ldots, 8$ are symmetric 8-Williamson type matrices. Otherwise X_{7i}, $i = 1, \ldots, 8$ are 8-Williamson type matrices. This can be verified by straightforward checking. Hence there is an Hadamard matrix of block structure of order $8(2n+1)$.

If conditions (iii) to (vi) hold then straightforward verification shows the 64 block M-structure X_{ij} is an Hadamard matrix of order $8(2n+1)$. □

Corollary 30 *Let q be an odd prime power and suppose there exist Williamson-type matrices of order $\frac{1}{2}(q-1)$. Then there exists an M-structure Hadamard matrix of order $8q$.*

Remark 31 This corollary gives new Hadamard matrices of order $8q$ for $q =$ 179, 1087, 1283, 1327, 1619, 1907, 2099, 2459, 2579, 2647, ...

Corollary 32 *Let $q = 2m+1 \equiv 9 \pmod{16}$ be a prime power. Suppose there are Williamson-type matrices of order q. Then there is a M-structure Hadamard matrix of order $8(2q+1)$.*

The analogous Yamada-J. Wallis-Whiteman structure to Theorem 29 is:

$$\begin{matrix}
-1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -e & -e & e & e & e & e & e & e \\
-1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -e & e & e & -e & e & -e & e & -e \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & e & e & -e & -e & e & e & e & e \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & e & -e & -e & e & e & -e & e & -e \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & e & e & e & e & -e & -e & e & e \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & e & -e & e & -e & -e & e & e & -e \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -e & -e & -e & -e & -e & -e & e & e \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -e & e & -e & -e & e & e & -e & e \\
-e^T & -e^T & e^T & e^T & e^T & e^T & e^T & e^T & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} \\
-e^T & e^T & e^T & -e^T & e^T & -e^T & e^T & -e^T & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} \\
e^T & e^T & -e^T & -e^T & e^T & e^T & e^T & e^T & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} \\
e^T & -e^T & -e^T & e^T & e^T & -e^T & e^T & -e^T & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} \\
e^T & e^T & e^T & e^T & -e^T & -e^T & e^T & e^T & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} \\
e^T & -e^T & e^T & -e^T & -e^T & e^T & e^T & -e^T & S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} \\
-e^T & -e^T & -e^T & -e^T & -e^T & -e^T & e^T & e^T & S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} \\
-e^T & e^T & -e^T & e^T & -e^T & e^T & e^T & -e^T & S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88}
\end{matrix}$$

We can see Yamada's matrix with trimming [46] or the J. Wallis-Whiteman [30] matrix with a border embodied in the construction.

References

[1] S.S. Agaian, *Hadamard Matrices and their Applications*, Lecture Notes in Mathematics, Vol 1168, Springer-Verlag, Berlin-Heidelberg-New York-Tokyo, 1985.

[2] S.S. Agayan and A.G. Sarukhanyan, Recurrence formulas for the construction of Williamson-type matrices, translated from *Matemacheskie Zametki*, 30, (1981), 603–617 = *Math Notes*, 30, (1982), 796–804.

[3] L.D. Baumert and M. Hall, Jr., Hadamard matrices of Williamson type, *Math. Comp.* 19, (1965), 442–447.

[4] L.D. Baumert and M. Hall, Jr., A new construction for Hadamard matrices, *Bull. Amer. Math. Soc.*, 71, (1965), 169–170.

[5] Joan Cooper and Jennifer Wallis, A construction for Hadamard arrays, *Bull. Austral. Math. Soc.*, 7, (1972), 269–278.

[6] Gavin Cohen, David Rubie, Christos Koukouvinos, Stratis Kounias, Jennifer Seberry and Mieko Yamada, A survey of base sequences, disjoint complementary sequences and $OD(4t;t,t,t,t)$, *JCMCC*, 5, (1989), 69–104.

[7] P. Delsarte, J-M. Goethals and J.J. Seidel, Orthogonal matrices with zero diagonal, II, *Canad. J. Math.*, 23, (1971), 816–832.

[8] A.V. Geramita and Jennifer Seberry, *Orthogonal Designs: Quadratic forms and Hadamard matrices*, Marcel Dekker, New York-Basel, 1979, viii, 460 pages.

[9] C. Koukouvinos and S. Kounias, Hadamard matrices of the Williamson type of order $4m$, $m = pq$. An exhaustive search for $m = 33$, *Discrete Math.*, 68, (1988), 45–57.

[10] C. Koukouvinos and S. Kounias, There are no circulant symmetric Williamson matrices of order 39, (to appear).

[11] C. Koukouvinos, S. Kounias and Jennifer Seberry, Further results on base sequences, disjoint complementary sequences, $OD(4t;t,t,t,t)$ and the excess of Hadamard matrices, *Ars Combinatoria*, (to appear).

[12] Masahiko Miyamoto, A construction for Hadamard matrices, *J. Combinatorial Th.*, Ser. A, (to appear).

[13] Tamio Ono and Kazue Sawade, The Baumert-Hall-Welch array of order 36, in Japanese (translation by Mieko Yamada).

[14] R.E.A.C. Paley, On orthononal matrices, *J. Math. Phys.*, 12, (1933), 311–320.

[15] Kazue Sawade, Hadamard matrices of order 100 and 108, *Bull. Nagoya Inst. Technol.*, 29, (1977), 147–152.

[16] Jennifer Seberry, On skew Hadamard matrices, *Ars Combinatoria*, 6, (1978), 255–276.

[17] Jennifer Seberry, A new construction for Williamson-type matrices, *Graphs and Combinatorics*, 2, (1986), 81–87.

[18] Jennifer Seberry, Hadamard matrices of order $2^t \cdot pq$: *I*, *Ars Combinatoria*, 23B, (1987), 195–213.

[19] Jennifer Seberry Wallis, Some matrices of Williamson-type, *Utilitas Math.*, 4, (1973), 147–154.

[20] Jennifer Seberry Wallis, Williamson matrices of even order, *Combinatorial Mathematics, Proceedings of the Second Australian Conference*, editor D.A. Holton, Lecture Notes in Mathematics, Vol 403, Springer-Verlag, Berlin-Heidelberg-New York-Tokyo 1974, 132–142.

[21] Jennifer Seberry Wallis, Construction of Williamson type matrices, *Linear and Multilinear Algebra*, 3, (1975), 197–207.

[22] Jennifer Seberry Wallis, Hadamard matrices, in W.D. Wallis, Anne Penfold Street and Jennifer Seberry Wallis, *Combinatorics: Room Squares, Sum-free Sets, Hadamard matrices*, Lecture Notes in Mathematics, Vol 292, Springer-Verlag, Berlin-Heidelberg-New York, 1972, 508 pages.

[23] E. Spence, An infinite family of Williamson matrices, *J. Austral. Math. Soc. Ser A*, **24**, (1977), 252–256.

[24] G. Szekeres, Tournaments and Hadamard matrices, *Enseignment Math.*, **15**, (1969), 269–278.

[25] G. Szekeres, Cyclotomy and complementary difference sets, *Acta. Arith.*, **18**, (1971), 349–353.

[26] Jennifer (Seberry) Wallis, A skew-Hadamard matrix of order 92, *Bull. Austral. Math. Soc.*, **5**, (1971), 203–204.

[27] Jennifer (Seberry) Wallis, Amicable Hadamard matrices, *J. Combinatorial Th., Ser. A.*, **11**, (1971), 296–298.

[28] Jennifer (Seberry) Wallis, A note on amicable Hadamard matrices, *Utilitas Math.*, **3**, (1973), 119–125.

[29] Jennifer (Seberry) Wallis, Some remarks on supplementary difference sets, *Colloq. Math. Soc. Janos Bolyai*, **10**, (1973), 1503–1526.

[30] Jennifer Wallis and Albert Leon Whiteman, Some classes of Hadamard matrices with constant diagonal, *Bull. Austral. Math. Soc.*, **7**, (1972), 233–249.

[31] Albert Leon Whiteman, An infinite family of skew-Hadamard matrices, *Pacific J. Math.*, **38**, (1971), 817–822.

[32] Albert Leon Whiteman, Skew-Hadamard matrices of Goethals-Seidel type, *Discrete Math.*, **2**, (1972), 397–405.

[33] Albert Leon Whiteman, An infinite family of Hadamard matrices of Williamson type, *J. Combinatorial Th., Ser. A.*, **14**, (1973), 334–340.

[34] Albert Leon Whiteman, Hadamard matrices of Williamson type, *J. Austral. Math. Soc.*, **21**, (1976), 481–486.

[35] Albert Leon Whiteman, Hadamard matrices of order $4(2p+1)$, *J. Number Theory*, **8**, (1976), 1–11.

[36] John Williamson, Hadamard's determinant theorem and the sum of four squares, *Duke Math. J.*, **11**, (1944), 65–81.

[37] K. Yamamoto, Hadamard matrices of Williamson type and the maximal-length shift register sequences, Abstract, *Symp. Combinatorics at Osaka City University*, December 1978 (in Japanese).

[38] K. Yamamoto, On a generalized Williamson equation, *Colloq. Math. Janos Bolyai*, **37**, (1981), 839–850.

[39] K. Yamamoto and Mieko Yamada, Williamson matrices of Turyn's type and Gauss sums, *J. Math. Soc. Japan*, **37**, (1985), 703–717.

[40] Mieko Yamada, On the Williamson type j matrices of orders 4.29, 4.41 and 4.37, *J. Combinatorial Th., Ser. A.*, **27**, (1979), 378–381.

[41] Mieko Yamada, On the Williamson matrices of Turyn's type and of type j, *Comment. Math. Univ. San Pauli*, **31**, (1982), 71–73.

[42] Mieko Yamada, Hadamard matrices generated by an adaption of the generalized quaternion type array, *Graphs and Combinatorics*, **2**, (1986), 179–187.

[43] Mieko Yamada, Supplementary relative difference sets and their application to Hadamard matrices, *Ars Combinatoria*, **26A**, (1989), 223–238.

[44] Mieko Yamada, Some new series of Hadamard matrices, *J. Austral. Math. Soc., Ser. A*, **46**, (1989), 371–383.

[45] Mieko Yamada, Supplementary difference sets and Jacobi sums, *Discrete Math.*, (to appear).

[46] Mieko Yamada, Hadamard matrices of generalized quaternion type, *Discrete Math.*, (to appear).

[47] Jennifer Seberry and Mieko Yamada, On the Products of Hadamard Matrices, Williamson Matrices and Other Orthogonal Matrices using M-Structures, *JCMCC*, (to appear).

[48] L.D. Baumert, Hadamard matrices of orders 116 and 232, *Bull. Amer. Math. Soc.*, **72**, (1966), 237.

[49] L.D. Baumert, S.W. Golomb, M. Hall, Jr., Discovery of an Hadamard matrix of order 92, *Bull. Amer. Math. Soc.*, **68**, (1962), pp. 237–238.

[50] Jennifer Seberry Wallis, On Hadamard matrices, *J. Combinatorial Theory, Ser. A*, **18**, (1975), pp. 149–164.

[51] R.J. Turyn, An infinite class of Williamson matrices, *J. Combinatorial Theory, Ser. A*, **12**, (1972), pp. 319–321.

[52] R.J. Turyn, A special class of Williamson matrices and difference sets, *J. Combinatorial Th(A)*, **36**, (1984), pp. 111–115.

* Department of Computer Science
University College
University of New South Wales
ADFA
Canberra, 2600
AUSTRALIA

† Department of Mathematics
Tokyo Woman's Christian University
Zempukuji
Suginami-Ku
Tokyo, 167
JAPAN

Index of Williamson Matrices

This table contains odd integers $q < 40000$ for which Williamson matrices exist. The following legend gives the method of construction used

Key	Method	Explanation
w1	$\{1,\ldots,33,37,41,43\}$	
w2	$\frac{p+1}{2}$	$p \equiv 1(mod4)$ a prime power
w3	9^d	d a natural number
w4	$\frac{p(p+1)}{2}$	$p \equiv 1(mod4)$ a prime power
w5	$s(4s+3), s(4s-1)$	$s \in \{1,3,5,\ldots,31\}$
w6	93	
w7	$\frac{(f-1)(4f+1)}{4}$	$p = 4f+1, f$ odd, is a prime power of the form $1+4t^2$, $\frac{f-1}{8}$ is the order of a good matrix
w8	$\frac{(f+1)(4f+1)}{4}$	$p = 4f+1, f$ odd, is a prime power of the form $25+4t^2$, $\frac{f+1}{8}$ is the order of a good matrix
w9	$\frac{p(p-1)}{2}$	$p = 4f+1$ is a prime power and $\frac{p-1}{4}$ is the order of a good matrix
w0	$(p+2)(p+1)$	$p \equiv 1(mod4)$ a prime power, $p+3$ is the order of a symmetric Hadamard matrix
wa	$\frac{(f+1)(4f+1)}{2}$	$p = 4f+1, f$ odd, is a prime power of the form $9+4t^2$, $\frac{f-1}{2} \equiv 1(mod4)$ a prime power
wb	$\frac{(f-1)(4f+1)}{2}$	$p = 4f+1, f$ odd, is a prime power of the form $49+4t^2$, $\frac{f-3}{2} \equiv 1(mod4)$ a prime power
wc	$2p+1$	$q = 2p-1$ is a prime power p is a prime
wd	7.3^i	$i \geq 0$
w#e	$7^{i+1}, 11.7^i$	$i \geq 0$ (Gives 8-Williamson matrices)
wf	$\frac{q^d(q+1)}{2}$	$q \equiv 1(mod4)$ is a prime $d \geq 2$
wg	$\frac{p^2(p+1)}{2}$	$p \equiv 1(mod4)$ is a prime power
wh	$\frac{p^2(p+1)}{4}$	$p \equiv 3(mod4)$ is a prime power and $\frac{p+1}{4}$ is the order of a Williamson type matrix
wi	$q+2$	$q \equiv 1(mod4)$, is a prime power and $\frac{q+1}{2}$ is a prime power
wj	$q+2$	$q \equiv 1(mod4)$, is a prime power and $\frac{q+3}{2}$ is the order of a symmetric conference matrix

Key	Method	Explanation
wk	q	$q \equiv 1(mod4)$ is a prime power and $\frac{q-1}{2}$ is the order of a symmetric conference matrix or the order of a symmetric hadamard matrix
wl	q	$q \equiv 1(mod4)$, is a prime power and $\frac{q-1}{4}$ is the order of a williamson type matrix
wm	q	$q \equiv 1(mod4)$, is a prime power and $\frac{q-1}{2}$ is the order of a hadamard matrix
wn	wn	w is the order of a williamson type matrix n is the order of a symmetric conference matrix
wo	$2wu$	w and u are the orders of williamson type matrices
w#p	$2q+1$	$q+1$ is the order of an amicable hadamard matrix and q is the order of a willliamson type matrix
w#q	q	q is a prime power and $\frac{q-1}{2} isthe$ order of a williamson type matrix
w#r	$2q+1$	$q \equiv 1(mod4)$ is a prime power or $q+1$ is the order of a symmetric conference matrix and q is the order of a williamson type matrix
w#s	2.9^t+1	$t > 0$

$S = \{1, \ldots, 31\}$ is the set of good matrices.

$q-1$ is a Hadamard matrix of SC-form if one of the following is true
(i) $\frac{q-1}{4}$ is a Williamson matrix.
(ii) $\frac{q-1}{2}$ is a Conference matrix.
(iii) $\frac{q-1}{4}$ is a Hadamard matrix. Note: The fact that if there is a

Williamson matrix of order n then there is a Williamson matrix of order $2n$, is used in the calculation of wg.

[The references for these papers are w1 [48], [4], [49], [40], [22], w2 [51], [33], w3 [52], w4 [19], w5, w6 [21], w7, w8, w9, w0, we, wl [20], wf, wg [17], wh, wi, wj, wk [12]] will.tex

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1	0	w1	101	0	wk	201	0	w2	301	0	w2	401	0	wk
3	0	w1	103	0	w#q	203	1	w9	303	1	w7	403	1	wn
5	0	w1	105	1	wn	205	0	w2	305	1	wn	405	0	w2
7	0	w1	107	0	w#q	207	1	wn	307	0	w2	407	1	wn
9	0	w1	109	0	wk	209	1	wn	309	0	w2	409	0	wk
11	0	w1	111	1	wn	211	0	w2	311			411	0	w2
13	0	w1	113	0	wk	213			313	0	w2	413		
15	0	w1	115	0	w2	215	1	wn	315	0	w5	415	0	w2
17	0	w1	117	0	w2	217	0	w2	317	0	wk	417	1	wn
19	0	w1	119	1	wn	219	1	wn	319	1	wo	419		
21	0	w1	121	0	w2	221	1	wn	321	0	w2	421	0	w2
23	0	w1	123	0	wi	223			323	1	wn	423	0	wi
25	0	w1	125	0	wk	225	0	w2	325	0	w4	425	1	wn
27	0	w1	127	0	w#p	227	0	w#q	327	0	w2	427	0	w2
29	0	w1	129	0	w2	229	0	w2	329			429	0	w2
31	0	w1	131			231	0	w2	331	0	w2	431		
33	0	w1	133	1	wn	233	0	wl	333	1	w9	433	0	wk
35	1	wn	135	0	w2	235			335			435	0	w4
37	0	w1	137	0	wl	237	1	wn	337	0	w2	437	1	wn
39	0	wi	139	0	w2	239			339	0	w2	439	0	w2
41	0	w1	141	0	w2	241	0	wk	341	1	wn	441	0	w2
43	0	w1	143	1	wn	243	0	wj	343	1	wn	443		
45	0	w2	145	0	w2	245	1	wn	345	1	wn	445	1	wn
47	0	w#p	147	0	w2	247	1	wn	347	0	w#q	447	1	wn
49	0	w2	149	0	wk	249	1	wn	349	0	wk	449	0	wk
51	0	w2	151	0	w#q	251	0	w#q	351	0	w2	451	0	wj
53	0	wk	153	0	w4	253	1	wn	353	0	wl	453		
55	0	w2	155	1	wn	255	0	w2	355	0	w2	455	1	wn
57	0	w2	157	0	w2	257	0	wl	357	1	wn	457	0	wk
59	0	w#q	159	0	w2	259	1	wn	359			459	0	wi
61	0	w1	161	1	wn	261	0	w2	361	0	wk	461	0	wk
63	0	w2	163	0	w#q	263			363	0	wi	463	0	w#q
65	1	wn	165	1	wn	265	0	w2	365	0	w2	465	0	w2
67	0	w#q	167	0	w#p	267	1	wn	367	0	w2	467	0	w#q
69	0	w2	169	0	w2	269			369	1	wn	469	0	w2
71			171	1	wn	271	0	w2	371	1	wn	471	0	w2
73	0	wk	173	0	wl	273	1	wn	373	0	wl	473	0	w5
75	0	w2	175	0	w2	275	1	wn	375	0	wf	475	1	wn
77	1	wn	177	0	w2	277	0	wk	377	1	wn	477	0	w2
79	0	w2	179	0	w#q	279	0	w2	379	0	w2	479		
81	0	w3	181	0	w2	281	0	wl	381	0	w2	481	0	w2
83	0	wi	183	1	wn	283	0	w#q	383			483	1	wn
85	0	w2	185	1	wn	285	0	w2	385	0	w2	485	1	wn
87	0	w2	187	0	w2	287	1	wn	387	0	w2	487	0	w#p
89	0	wl	189	0	w5	289	0	w2	389	0	wk	489	0	w2
91	0	w2	191	0	w#p	291	1	wn	391	1	wn	491		
93	0	w6	193	0	wk	293	0	wl	393			493	1	wo
95	0	w5	195	0	w2	295			395	1	wn	495	1	wn
97	0	w2	197	0	wk	297	0	w2	397	0	wk	497		
99	0	w2	199	0	w2	299	1	wn	399	0	w2	499	0	w2

Existence of Williamson Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
501			601	0	w2	701	0	wk	801	0	w2	901	0	w2
503			603	1	wn	703	0	w4	803	1	wo	903	1	wn
505	0	w2	605	1	wn	705	0	w2	805	0	w2	905	1	wn
507	0	w2	607	0	w2	707	1	wn	807	0	w2	907		
509			609	0	w2	709	0	wk	809	0	wl	909	1	wn
511	0	w2	611			711	1	wn	811	0	w2	911		
513	1	wn	613	0	wl	713	1	wn	813	1	wn	913	1	wo
515	0	w#r	615	0	w2	715	0	w2	815			915	1	w9
517	0	w2	617	0	wl	717	0	w2	817	1	wn	917		
519	1	wn	619	0	w2	719			819	0	w2	919	0	w#q
521	0	wl	621	1	wn	721			821	0	wk	921	1	wn
523	0	w#q	623	1	wn	723	1	wn	823	0	w#q	923	0	w#r
525	0	w2	625	0	w2	725	1	wn	825	1	wn	925	0	w2
527	1	wn	627	0	wi	727	0	w2	827			927	1	wn
529	0	wl	629	1	wn	729	0	w3	829	0	w2	929	0	wl
531	0	w2	631	0	w#q	731	1	wo	831	1	wn	931	0	w2
533	1	wn	633	1	wn	733	0	w#q	833	1	wn	933		
535	0	w2	635	0	w#r	735	0	wi	835	0	w2	935	1	wn
537			637	1	wn	737			837	1	wn	937	0	w2
539	1	wn	639	0	w2	739			839			939	0	w2
541	0	wk	641	0	wk	741	0	w2	841	0	w2	941		
543	0	wi	643	0	w#q	743			843	0	wi	943	1	wn
545	1	wn	645	0	w2	745	0	w2	845	1	wn	945	0	w2
547	0	w2	647			747	0	w2	847	0	w2	947	0	w#q
549	0	w2	649	0	w2	749			849	0	w2	949	1	wn
551	1	wn	651	0	w2	751	0	w#q	851	1	wn	951	0	w2
553	1	wn	653			753			853			953	0	wl
555	0	w2	655	0	w#p	755			855	0	w2	955		
557	0	wk	657	1	wn	757	0	wl	857			957	0	w2
559	0	w2	659			759	0	wi	859	0	w#q	959	1	wn
561	1	wn	661	0	w2	761	0	wl	861	0	w2	961	0	wk
563	0	w#q	663	0	w5	763	1	wa	863			963	1	wn
565	0	w2	665	1	wn	765	1	wn	865	1	wn	965	1	wn
567	1	wn	667	1	wn	767			867	0	w2	967	0	w2
569	0	wm	669			769	0	wk	869	1	wn	969	1	wn
571	0	w#q	671	1	wn	771	1	wn	871	0	w2	971		
573			673	0	wk	773	0	wl	873	1	wn	973	1	wn
575	1	wn	675	0	wi	775	0	w2	875	1	wn	975	0	w2
577	0	w2	677	0	wk	777	0	w2	877	0	w2	977	0	wk
579	0	wj	679	1	wn	779	1	wn	879	0	wi	979	1	wo
581	1	wn	681	0	w2	781			881	0	wk	981	1	wn
583	1	wo	683			783	1	wn	883	0	w#q	983		
585	1	wn	685	0	w2	785	1	wn	885	0	w5	985	1	wn
587	0	w#q	687	0	w2	787			887			987	0	w2
589	1	wn	689	1	w9	789			889	0	w2	989	1	wn
591	0	w2	691	0	w2	791	1	wn	891	1	wn	991		
593	0	wk	693	1	wn	793	1	wn	893			993	1	wn
595	1	wn	695	1	wn	795	1	wn	895	0	w2	995	1	wn
597	0	w2	697	1	wn	797	0	wk	897	1	wn	997	0	w2
599			699	1	wn	799	0	w2	899	1	wn	999	0	w2

Existence of Williamson Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1001	1	wn	1101	1	wn	1201	0	w2	1301	0	wl	1401	0	w2
1003			1103			1203	0	wi	1303	0	w#q	1403	1	wn
1005	1	wn	1105	0	w2	1205	1	wn	1305	0	w2	1405	0	w2
1007	1	wn	1107	0	w2	1207	0	w5	1307	0	w#r	1407	1	wn
1009	0	w2	1109	0	wl	1209	0	w2	1309	0	w2	1409	0	wl
1011	1	wn	1111	0	w2	1211	1	wn	1311	0	w2	1411	1	wo
1013	1	wn	1113	1	wn	1213	0	w#q	1313	1	wn	1413	1	wn
1015	0	w2	1115	0	w#r	1215	0	wi	1315			1415		
1017	1	wn	1117	0	wk	1217	0	wk	1317	0	w2	1417	0	w2
1019	0	w#r	1119	0	w2	1219	0	w2	1319			1419	0	w2
1021	0	wk	1121			1221	0	w2	1321	0	wl	1421	1	wn
1023	1	wn	1123			1223			1323	0	wi	1423		
1025	1	wn	1125	1	wn	1225	0	w4	1325	1	wn	1425	0	w5
1027	0	w2	1127	1	wn	1227	1	wn	1327	0	w#p	1427		
1029	1	wn	1129	0	wk	1229	0	wk	1329	0	w2	1429	0	w2
1031			1131	1	wn	1231	0	w#p	1331	1	wn	1431	0	w2
1033	0	wk	1133			1233	1	wn	1333	1	wn	1433		
1035	0	w2	1135	0	w2	1235	1	wn	1335	1	wn	1435	1	wn
1037	1	wn	1137	0	w2	1237	0	w2	1337			1437		
1039			1139	0	w5	1239	0	w2	1339	0	w2	1439		
1041	0	w2	1141	0	w2	1241	1	wo	1341	1	wb	1441		
1043	1	wn	1143	1	wn	1243	1	wn	1343	1	wn	1443	1	wn
1045	0	w2	1145	1	wn	1245	1	wn	1345	0	w2	1445	1	wn
1047	1	wn	1147	0	w2	1247	1	wo	1347	0	w2	1447		
1049	0	wm	1149	0	w2	1249	0	wl	1349			1449	0	w2
1051	0	w#q	1151			1251	0	wi	1351	1	wn	1451		
1053	1	wn	1153	0	wk	1253			1353	1	wn	1453	0	wl
1055	1	wn	1155	0	w2	1255			1355	1	wn	1455	0	w2
1057	0	w2	1157	1	wn	1257			1357	0	w2	1457		
1059	1	wn	1159	1	wn	1259			1359			1459	0	w2
1061	0	wk	1161	1	wn	1261	0	w2	1361	0	wk	1461		
1063	0	w#q	1163			1263	1	wn	1363			1463	2	wn
1065	0	w2	1165	1	wn	1265	1	wn	1365	0	w2	1465	1	wn
1067	1	wn	1167	0	w2	1267	1	wn	1367			1467	1	wn
1069	0	w2	1169			1269	1	wn	1369	0	wl	1469	1	wn
1071	0	w2	1171	0	w2	1271	1	wn	1371	0	w2	1471	0	w#p
1073	1	wn	1173	1	wn	1273			1373	0	w#q	1473		
1075	1	wn	1175			1275	0	w2	1375	0	w2	1475		
1077	0	w2	1177			1277	0	w#q	1377	0	w2	1477	0	w2
1079	1	wn	1179	0	w2	1279	0	w2	1379	1	wn	1479	0	w2
1081	0	w2	1181			1281	1	wn	1381	0	w#q	1481	0	wl
1083	1	wn	1183	0	wf	1283	0	w#q	1383	0	wi	1483	0	w#q
1085	1	wn	1185	1	wn	1285	1	wn	1385	1	wn	1485	0	w2
1087	0	w#p	1187	0	w#q	1287	1	wn	1387	1	wn	1487		
1089	1	wn	1189	0	w2	1289	0	wl	1389	0	w2	1489	0	wl
1091			1191	0	w2	1291	0	w#q	1391			1491		
1093	0	w#q	1193	0	wl	1293			1393	1	wn	1493	0	wl
1095	0	wi	1195	0	w2	1295	2	wn	1395	0	w2	1495	1	wn
1097	0	wl	1197	0	w2	1297	0	w2	1397			1497	1	wn
1099	0	w2	1199	1	wo	1299	1	wn	1399	0	w2	1499		

Existence of Williamson Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1501	0	w2	1601	0	wk	1701	1	wn	1801	0	wk	1901	0	w#q
1503			1603	1	wn	1703			1803	1	wn	1903	1	wo
1505	1	wn	1605	0	w2	1705	1	wn	1805	0	wh	1905	1	wn
1507	1	wo	1607			1707	0	w2	1807	0	w2	1907	0	w#q
1509			1609	0	w2	1709	0	wk	1809	0	w2	1909	1	wn
1511			1611	0	w2	1711			1811			1911	0	w2
1513	1	wn	1613	0	w#q	1713			1813	1	wn	1913		
1515	1	wn	1615	0	w2	1715	0	w#r	1815	1	wn	1915		
1517	1	wn	1617	1	wn	1717	0	w2	1817	1	wn	1917	0	w2
1519	0	w2	1619	0	w#q	1719			1819	0	w2	1919	1	wn
1521	0	w2	1621	0	wk	1721	0	wl	1821	1	wn	1921	1	wn
1523	0	w#q	1623	0	wi	1723	0	w#q	1823	1	wn	1923	1	wn
1525	0	w2	1625	1	wn	1725	0	w2	1825	1	wn	1925	1	wn
1527			1627	0	w2	1727	1	wn	1827	0	w5	1927	0	w2
1529	1	wn	1629	0	w2	1729	0	w2	1829			1929		
1531	0	w2	1631	1	wn	1731	0	w2	1831			1931		
1533	1	wn	1633			1733	0	wl	1833	1	wn	1933	0	w#q
1535	1	wn	1635	1	wn	1735	0	w2	1835	1	wn	1935	0	wi
1537	1	wo	1637	0	wl	1737	1	wn	1837	0	w2	1937	1	wn
1539	1	wn	1639	1	wo	1739			1839	0	w2	1939	0	w2
1541			1641	1	wn	1741	0	w2	1841			1941	0	w2
1543			1643	1	wn	1743	0	w5	1843	1	wn	1943		
1545	0	w2	1645			1745	1	wn	1845	1	wn	1945	0	w2
1547	1	wn	1647	1	wn	1747			1847			1947	1	wn
1549	0	wk	1649	1	wn	1749	1	wn	1849	0	w2	1949		
1551	1	wn	1651	0	w2	1751			1851	0	w2	1951	0	w#p
1553	0	wk	1653	1	wn	1753	0	wl	1853	1	wo	1953	1	wn
1555	0	w2	1655	1	wn	1755	0	wi	1855	0	w2	1955	1	wn
1557	1	wn	1657	0	w2	1757			1857	1	wn	1957		
1559			1659	0	wi	1759	0	w2	1859	1	wn	1959	0	w2
1561	0	w2	1661			1761			1861	0	w2	1961	1	wn
1563	0	w2	1663			1763	1	wn	1863	1	wn	1963		
1565	1	wn	1665	0	w2	1765	0	w2	1865	1	wn	1965	0	w2
1567			1667			1767	0	w2	1867	0	w2	1967	1	wn
1569	0	w2	1669	0	w#q	1769	1	wn	1869	1	wn	1969		
1571			1671	1	wn	1771	0	w2	1871			1971	1	wn
1573	1	wn	1673			1773	1	wn	1873	0	wk	1973	0	w#q
1575	1	wn	1675			1775	1	wn	1875	0	wf	1975	1	wn
1577	1	wn	1677	1	wn	1777	0	wl	1877	0	wk	1977		
1579			1679	1	wn	1779	0	w2	1879	0	w#q	1979		
1581	1	wn	1681	0	w2	1781	1	wn	1881	0	w2	1981		
1583			1683	0	wj	1783			1883	0	w#r	1983	1	wn
1585	0	w2	1685	1	wn	1785	1	wn	1885	0	w2	1985	1	wn
1587	1	wh	1687	0	w2	1787			1887	1	wn	1987		
1589			1689			1789	0	w#q	1889	0	wm	1989	1	wn
1591	0	w2	1691	1	wn	1791	0	w2	1891	0	w4	1991	1	wn
1593	1	wn	1693	0	wl	1793			1893			1993	0	wl
1595	1	wn	1695	0	w2	1795			1895	1	wn	1995	0	w2
1597	0	wk	1697	0	wl	1797	0	w2	1897	0	w2	1997	0	wk
1599	1	wn	1699	0	w#q	1799	1	wn	1899	0	w2	1999	0	w#p

Existence of Williamson Matrices

Index of Hadamard Matrices

This table contains odd integers $q < 40000$ for which Hadamard matrices of the form $2^t q$ exist. The key for the methods of construction follows.

Amicable Hadamard Matrices.

Key	Method	Explanation
a1	$p^r + 1$	$p^r \equiv 3(mod4)$, is a prime power
a2	$2(q+1)$	$2q+1$ is a prime power, $q \equiv 1(mod4)$, is a prime
a5	nh	n, h, are amicable hadamard matrices

Skew Hadamard Matrices.

Key	Method	Explanation
s1	$2^t \prod k_i$	t all positive integers, $k_i - 1 \equiv 3(mod4)$ a prime power
s2	$(p-1)^u + 1$	p is a skew Hadamard matrix, $u > 0$ is an odd integer
s3	$2(q+1)$	$q \equiv 5(mod8)$ is a prime power
s4	$2(q+1)$	$q = p^t$ is a prime power where $p \equiv 5(mod8)$ and $t \equiv 2(mod4)$
s5	$4m$	$3 \leq m \leq 25$
s6	$4(q+1)$	$q \equiv 9(mod16)$ is a prime power
s7	$(\lvert t \rvert + 1)(q+1)$	$q = s^2 + 4t^2 \equiv 5(mod8)$ is a prime power and $\lvert t \rvert + 1$ is a skew Hadamard matrix
s8	$4(q^2+q+1)$	q is a prime power, $q^2+q+1 \equiv 3, 5, 7(mod8)$ a prime or $2(q^2+q+1)+1$ is a prime power
s0	hm	h is a skew hadamard matrix and m is an amicable hadamard matrix

Spence Hadamard Matrices.

Key	Method	Explanation
p1	$4(q^2+q+1)$	$q^2+q+1 \equiv 1(mod8)$ is a prime
p2	$4n$ or $8n$	n, $n-2$ are prime powers, if $n \equiv 1(mod4)$ there exists a Hadamard matrix of order $4n$, if $n \equiv 3(mod4)$ there exists a Hadamard matrix of order $8n$
p3	$4m$	m is an odd prime power for which an integer $s \geq 0$ such that $\frac{(m-(2^{s+1}+1))}{2^{s+1}}$ is an odd prime power, exists

Symmetric Hadamard Matrices.

If there exists a Conference matrix of order n then there is symmetric Hadamard matrix of order $2n$, for this reason symmetric hadamard matrices indexed according to the method used to derive the order of a conference matrix with the exception of c6 wich produces a symmetric Hadamard matrix.

Key	Method	Explanation
c1	$p^r + 1$	$p^r \equiv 1(mod 4)$ is a prime power
c2	$(h-1)^2 + 1$	h is a skew Hadamard matrix
c3	$q^2(q-2)+1$	$q \equiv 3(mod 4)$ is a prime power
		$q-2$ is a prime power
c4	$5.9^{2t+1}+1$	$t \geq 0$
c5	$(n-1)^s + 1$	n is a conference matrix
		$s \geq 2$
c6	nh	n is a conference matrix
		h is a Hadamard matrix

Note: a conference matrix of order n exists only if $n-1$ is the sum of two squares.

Hadamard Matrices Obtained From Williamson Matrices.

If a Williamson matrix of order $2^t q$ exists then there is a Hadamard matrix of order $2^{t+2}q$, the same key as in the Index of Williamson Matrices is used to index the Hadamard matrices produced from them.

OD Hadamard Matrices.

Key	Method	Explanation
o1		If a T-matrix of order $2^t q$ exists then
		there is a hadamard matrix of order $2^{t+2}q$
o2	ow	o is an OD-hadamard matrix and
		w is a Williamson matrix
o3	8pw	prus an OD(8p; p, p, p, p, p, p, p, p) exists (true for p= and 8-Williamson matrices w exist (true for w#.)

Yamada Hadamard Matrices.

Key	Method	Explanation
y1	$4q$	$q \equiv 1(mod 8)$ is a prime power
		$\frac{q-1}{2}$ is a Hadamard matrix
y2	$4(q+2)$	$q \equiv 5(mod 8)$ is a prime power
		$\frac{q+3}{2}$ is a skew Hadamard matrix
y3	$4(q+2)$	$q \equiv 1(mod 8)$ is a prime power
		$\frac{q+3}{2}$ is a conference matrix

Miyamoto Hadamard Matrices.

Key	Method	Explanation
m1	$4q$	$q \equiv 1(mod4)$ is a prime power
		$q-1$ is a Hadamard matrix
m2	$8q$	$q \equiv 3(mod4)$ is a prime power
		$2q-3$ is a prime power

Seberry.

Key	Method	Explanation
se	$2^t q$	where t is the smallest integer such that
		for given odd q, $a(q+1)+b(q-3)=2^t$
		has a solution for a, b non-negative integers

Koukouvinos and Kounias

Key	Method
k1	$2^t q = g1 + g2$ where $g1$ and $g2$ are Golay sequences.
d1	multiplication.

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1		a2	101		wk	201		o2	301		o2	401		wk
3		a2	103		y2	203		a2	303		o2	403		o2
5		a2	105		a2	205		o2	305		o2	405		a2
7		a2	107	3	w#q	207		a2	307		s3	407		a2
9		o2	109		wk	209		o2	309		c1	409		wk
11		a2	111		a2	211		s3	311	3	m2	411		o2
13		s4	113		c2	213	3	c6	313		c1	413		o2
15		a2	115		o2	215		a2	315		a2	415		o2
17		a2	117		a2	217		o2	317		wk	417		a2
19		s3	119		o2	219		o2	319		o2	419	4	a2
21		a2	121		o2	221		a2	321		a2	421		s4
23		s5	123		a2	223	3	a2	323		a2	423		o2
25		o2	125		a2	225		o2	325		o2	425		a2
27		a2	127		y2	227		a2	327		a2	427		o2
29		wl	129		o2	229		c1	329		o2	429		o2
31		s3	131		a2	231		o2	331		s3	431		a2
33		a2	133		o2	233		wl	333		a2	433		wk
35		a2	135		o2	235		o2	335		o2	435		o2
37		c1	137		a2	237		a2	337		c1	437		a2
39		o2	139		s3	239	4	a2	339		o2	439		s3
41		a2	141		a2	241		wk	341		o2	441		o2
43		wl	143		a2	243		a2	343		o2	443	3	m2
45		a2	145		o2	245		o2	345		o2	445		o2
47		o1	147		a2	247		o2	347	3	w#q	447		a2
49		o2	149		wk	249		o2	349		wk	449		wk
51		o2	151		y2	251	3	w#q	351		o2	451		o2
53		a2	153		o2	253		o2	353		wl	453		a2
55		o2	155		a2	255		a2	355		s3	455		o2
57		a2	157		c1	257		wl	357		a2	457		wk
59		o1	159		o2	259		o2	359	4	a2	459		o2
61		a2	161		a2	261		o2	361		o2	461		wk
63		a2	163	3	a2	263		a2	363		a2	463	3	w#q
65		o2	165		a2	265		o2	365		a2	465		o2
67		o1	167	3	w#p	267		o2	367		s3	467		a2
69		o2	169		o2	269		m1	369		o2	469		o2
71		a2	171		a2	271		s3	371		a2	471		o2
73		wk	173		a2	273		a2	373		wl	473		o2
75		o2	175		o2	275		o2	375		a2	475		o2
77		a2	177		o2	277		wk	377		o2	477		a2
79		s3	179	3	w#q	279		o2	379		s3	479	16	se
81		o2	181		c1	281		a2	381		a2	481		o2
83		a2	183		o2	283	3	w#q	383		a2	483		a2
85		o2	185		a2	285		o2	385		o2	485		o2
87		a2	187		o2	287		o2	387		o2	487	3	w#p
89		wl	189		o2	289		o2	389		wk	489		c1
91		o2	191	3	w#p	291		a2	391		o2	491	15	se
93		o2	193		wk	293		a2	393		a2	493		o2
95		a2	195		o2	295		o2	395		a2	495		a2
97		c1	197		a2	297		a2	397		wk	497		a2
99		o2	199		s3	299		o2	399		o2	499		s3

Existence of Hadamard Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
501		a2	601		c1	701		a2	801		a2	901		o2
503		a2	603		a2	703		o2	803		o2	903		o2
505		o2	605		o2	705		a2	805		o2	905		o2
507		a2	607		s3	707		o2	807		s3	907	3	m2
509		m1	609		o2	709		wk	809		wl	909		o2
511		o2	611		o2	711		a2	811		s3	911		a2
513		o2	613		c2	713		a2	813		a2	913		o2
515	3	w#r	615		a2	715		o2	815		a2	915		a2
517		o2	617		a2	717		c1	817		o2	917	4	a5
519		o2	619		s3	719	4	a2	819		o2	919	3	a2
521		a2	621		o2	721	4	o2	821		wk	921		o2
523	3	w#q	623		o2	723		o2	823	3	s7	923		a2
525		a2	625		o2	725		o2	825		a2	925		o2
527		o2	627		o2	727		s3	827		a2	927		o2
529		o2	629		o2	729		o2	829		c1	929		wl
531		o2	631	3	w#q	731		o2	831		a2	931		o2
533		a2	633		a2	733		m1	833		a2	933	4	c6
535		s3	635		a2	735		a2	835		s3	935		a2
537	4	c6	637		o2	737		o2	837		a2	937		c1
539		o2	639		s3	739	16	se	839	18	se	939		o2
541		wk	641		wk	741		a2	841		o2	941		m1
543		o2	643	3	w#q	743		a2	843		a2	943		o2
545		a2	645		a2	745		o2	845		o2	945		a2
547		a2	647	3	m2	747		o2	847		o2	947	3	w#q
549		o2	649		o2	749	5	o2	849		c1	949		o2
551		a2	651		o2	751	3	a2	851		o2	951		a2
553		o2	653	4	o1	753		a2	853	3	a2	953		wl
555		o2	655		y2	755		a2	855		o2	955	3	a2
557		wk	657		o2	757		s8	857		m1	957		o2
559		o2	659	17	se	759		o2	859	3	a2	959		o2
561		a2	661		c1	761		c2	861		o2	961		o2
563		a2	663		o2	763		o2	863	3	m2	963		a2
565		o2	665		a2	765		o2	865		o2	965		o2
567		a2	667		o2	767		a2	867		a2	967		s3
569		p3	669	3	a2	769		wk	869		o2	969		o2
571	3	a2	671		a2	771		a2	871		o2	971	6	a2
573	3	a2	673		wk	773		wl	873		a2	973		o2
575		o2	675		a2	775		o2	875		a2	975		o2
577		c1	677		a2	777		o2	877		c1	977		a2
579		o2	679		o2	779		o2	879		o2	979		o2
581		o2	681		c1	781	3	a2	881		wk	981		a2
583		o2	683		a2	783		o2	883	3	w#q	983		a2
585		a2	685		o2	785		o2	885		a2	985		o2
587		a2	687		o2	787	3	m2	887		a2	987		a2
589		o2	689		o2	789	3	a2	889		c1	989		o2
591		o2	691		s3	791		a2	891		o2	991	3	a2
593		a2	693		o2	793		o2	893		a2	993		o2
595		o2	695		o2	795		o2	895		s3	995		o2
597		o2	697		o2	797		a2	897		o2	997		c1
599	17	se	699		o2	799		o2	899		o2	999		o2

Existence of Hadamard Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1001		a2	1101		o2	1201		c1	1301		c2	1401		c1
1003		o2	1103	3	m2	1203		o2	1303	3	w#q	1403		o2
1005		a2	1105		o2	1205		o2	1305		o2	1405		o2
1007		a2	1107		o2	1207		w5	1307		a2	1407		o2
1009		c1	1109		wl	1209		o2	1309		o2	1409		wl
1011		o2	1111		o2	1211		o2	1311		o2	1411		o2
1013		a2	1113		a2	1213		m1	1313		o2	1413		a2
1015		o2	1115	3	w#r	1215		o2	1315	4	a5	1415		a2
1017		o2	1117		wk	1217		wk	1317		o2	1417		o2
1019	3	w#r	1119		o2	1219		o2	1319	18	se	1419		o2
1021		wk	1121		a2	1221		o2	1321		wl	1421		a2
1023		a2	1123	3	m2	1223	19	se	1323		o2	1423	3	a2
1025		a2	1125		o2	1225		o2	1325		o2	1425		o2
1027		o2	1127		a2	1227		o2	1327	3	w#p	1427	3	m2
1029		o2	1129		wk	1229		wk	1329		c1	1429		c1
1031	6	a2	1131		a2	1231		y2	1331		a2	1431		o2
1033		wk	1133	4	a2	1233		a2	1333		o2	1433		m1
1035		a2	1135		s3	1235		o2	1335		o2	1435		o2
1037		o2	1137		a2	1237		c1	1337		a2	1437	4	o1
1039	3	a2	1139		o2	1239		o2	1339		s3	1439	19	se
1041		c1	1141		c1	1241		o2	1341		o2	1441	3	a2
1043		o2	1143		o2	1243		o2	1343		o2	1443		o2
1045		o2	1145		o2	1245		o2	1345		c1	1445		a2
1047		o2	1147		o2	1247		a2	1347		a2	1447	19	se
1049		p3	1149		c1	1249		wl	1349	4	o2	1449		o2
1051	3	w#q	1151		a2	1251		a2	1351		o2	1451	6	a2
1053		a2	1153		wk	1253		a2	1353		o2	1453		wl
1055		a2	1155		o2	1255	3	a2	1355		a2	1455		o2
1057		c1	1157		o2	1257	5	c6	1357		o2	1457		a2
1059		o2	1159		o2	1259	4	a2	1359	3	c6	1459		s3
1061		a2	1161		a2	1261		o2	1361		a2	1461		a2
1063	3	w#q	1163		a2	1263		a2	1363		o2	1463		a2
1065		a2	1165		o2	1265		a2	1365		o2	1465		o2
1067		o2	1167		o2	1267		o2	1367	3	m2	1467		a2
1069		c1	1169	5	o2	1269		o2	1369		o2	1469		o2
1071		a2	1171		s3	1271		o2	1371		a2	1471	3	w#p
1073		o2	1173		a2	1273		o2	1373		m1	1473	3	a2
1075		o2	1175		o2	1275		a2	1375		o2	1475		o2
1077		c1	1177	5	a2	1277		a2	1377		a2	1477		o2
1079		o2	1179		s3	1279		s3	1379		o2	1479		o2
1081		o2	1181		a2	1281		o2	1381		m1	1481		a2
1083		o2	1183		o2	1283	3	w#q	1383		a2	1483	3	a2
1085		a2	1185		o2	1285		o2	1385		o2	1485		a2
1087	3	w#p	1187	3	w#q	1287		a2	1387		o2	1487	3	m2
1089		o2	1189		o2	1289		wl	1389		c1	1489		wl
1091		a2	1191		o2	1291	3	w#q	1391		a2	1491	3	a2
1093		p2	1193		wl	1293		a2	1393		o2	1493		wl
1095		o2	1195		s3	1295		a2	1395		o2	1495		o2
1097		wl	1197		a2	1297		c1	1397	4	o2	1497		a2
1099		o2	1199		o2	1299		o2	1399		s3	1499	18	se

Existence of Hadamard Matrices

q	t	Method	q	t	Method	q	t	Method	q	t	Method	q	t	Method
1501		o2	1601		wk	1701		a2	1801		wk	1901		a2
1503		a2	1603		o2	1703	4	o2	1803		a2	1903		o2
1505		o2	1605		o2	1705		o2	1805		a2	1905		o2
1507		o2	1607		a2	1707		a2	1807		o2	1907	3	w#q
1509	3	a2	1609		c1	1709		wk	1809		o2	1909		o2
1511		a2	1611		s3	1711		o2	1811		a2	1911		a2
1513		o2	1613		a2	1713	4	a2	1813		o2	1913	4	o1
1515		o2	1615		o2	1715		a2	1815		o2	1915	3	a2
1517		a2	1617		o2	1717		o2	1817		o2	1917		o2
1519		o2	1619	3	w#q	1719	3	a2	1819		s3	1919		o2
1521		o2	1621		wk	1721		a2	1821		a2	1921		o2
1523		a2	1623		a2	1723		s8	1823		c4	1923		a2
1525		o2	1625		o2	1725		a2	1825		o2	1925		a2
1527	3	c6	1627		s3	1727		a2	1827		a2	1927		o2
1529		o2	1629		o2	1729		o2	1829		o2	1929	4	c6
1531		s3	1631		o2	1731		o2	1831	3	m2	1931		a2
1533		a2	1633	3	a2	1733		wl	1833		a2	1933		p2
1535		o2	1635		o2	1735		s3	1835		o2	1935		o2
1537		o2	1637		a2	1737		a2	1837		c1	1937		o2
1539		o2	1639		o2	1739		o2	1839		o2	1939		o2
1541		a2	1641		a2	1741		c1	1841	4	a5	1941		c1
1543	3	a2	1643		a2	1743		a2	1843		o2	1943		o2
1545		o2	1645		o2	1745		o2	1845		o2	1945		o2
1547		o2	1647		o2	1747	3	m2	1847	3	m2	1947		o2
1549		wk	1649		o2	1749		o2	1849		o2	1949	4	a2
1551		a2	1651		s3	1751	4	o2	1851		o2	1951		y2
1553		a2	1653		o2	1753		wl	1853		a2	1953		o2
1555		s3	1655		a2	1755		a2	1855		o2	1955		o2
1557		o2	1657		c1	1757		a2	1857		o2	1957	4	o2
1559	4	a2	1659		o2	1759		s3	1859		o2	1959		s3
1561		c1	1661	4	o2	1761		a2	1861		s4	1961		o2
1563		o2	1663	3	m2	1763		o2	1863		a2	1963		s7
1565		o2	1665		a2	1765		o2	1865		a2	1965		c1
1567	19	se	1667	3	m2	1767		o2	1867		s3	1967		a2
1569		c1	1669		p2	1769		o2	1869		o2	1969	10	a5
1571	18	se	1671		o2	1771		o2	1871	3	m2	1971		a2
1573		o2	1673		a2	1773		o2	1873		wk	1973		m1
1575		a2	1675		o2	1775		o2	1875		a2	1975		o2
1577		o2	1677		o2	1777		wl	1877		a2	1977		a2
1579	5	a2	1679		o2	1779		o2	1879	3	a2	1979	4	a2
1581		a2	1681		o2	1781		o2	1881		a2	1981	5	a2
1583	3	m2	1683		o2	1783	18	se	1883	3	w#r	1983		o2
1585		o2	1685		o2	1785		o2	1885		o2	1985		o2
1587		o2	1687		o2	1787	3	m2	1887		a2	1987	16	se
1589	4	a2	1689	3	c6	1789		p2	1889		m1	1989		o2
1591		o2	1691		a2	1791		o2	1891		o2	1991		a2
1593		o2	1693		wl	1793	4	a2	1893	4	c6	1993		wl
1595		a2	1695		a2	1795	6	a5	1895		o2	1995		o2
1597		wk	1697		wl	1797		a2	1897		o2	1997		wk
1599		o2	1699	3	a2	1799		o2	1899		o2	1999		y2

Existence of Hadamard Matrices