

A skew-Hadamard matrix of order 92

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There is a skew-Hadamard matrix of order 92 .

Previously the smallest order for which a skew-Hadamard matrix was not known was 92 . We construct such a matrix below. The orders < 200 which are now undecided are 100, 116, 148, 156, 172, 188, 196; see [2], [3]. The existence of any Hadamard matrix of order 92 was unknown until 1962 [1].

We construct a skew-Hadamard matrix of Williamson-type by using the matrix

$$W = \begin{pmatrix} A & B & C & D \\ -B & A & D & -C \\ -C & -D & A & B \\ -D & C & -B & A \end{pmatrix} .$$

Then if A is a $(1, -1)$ skew-type cyclic matrix of order 23 (that is $a_{i+1, j+1} = a_{i, j}$ where the subscripts are taken modulo 23), B, C, D are $(1, -1)$ anticyclic matrices of order 23 having symmetrical first rows (that is $b_{i, j} = b_{i+1, j-1}$, $b_{11} = 1$, $b_{1j} = b_{1, 25-j}$ and so on, subscripts modulo 23) and

$$AA^T + BB^T + CC^T + DD^T = 92I_{23} ,$$

W is a skew-Hadamard matrix of order 92 .

Suitable first rows for the blocks A, B, C, D are

$A : 1 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \ 1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1$
 $B : 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1$
 $C : 1 \ 1 \ -1 \ -1 \ -1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1 \ 1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1 \ -1 \ -1 \ 1$
 $D : 1 \ -1 \ -1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ -1 \ -1$

If $W = U + I$ is a skew-Hadamard matrix of order 92 where I is the identity matrix then

$$U+I \quad U+I$$

$$U-I \quad -U+I$$

is a skew-Hadamard matrix of order 184 .

References

- [1] Leonard Baumert, S.W. Golomb and Marshall Hall, Jr, "Discovery of an Hadamard matrix of order 92", *Bull. Amer. Math. Soc.* 68 (1962), 237-238.
- [2] Jennifer Wallis, " (v, k, λ) configurations and Hadamard matrices", *J. Austral. Math. Soc.* 11 (1970), 297-309.
- [3] Albert Leon Whiteman, "An infinite family of skew Hadamard matrices", (to appear).

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