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# Some (1, −1 )Matrices

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## ABSTRACT

We define an $n$-type $(1, -1)$ matrix $N = I + R$ of order $n \equiv 2 \pmod 4$ to have $R$ symmetric and $R^2 = (n-1)I_n$. These matrices are analogous to skew-type matrices $M = I + W$ which have $W$ skew-symmetric.

If $n$ is the order of an $n$-type matrix, $h_1$ and $h_2$ the orders of Hadamard matrices, $h$ the order of a skew-Hadamard matrix, and $p^r \equiv 1 \pmod 4$ is a prime power then we show there are:

$n$-type matrices of orders $p^r + 1$, $(h-1)^2 + 1$, $(n-1)^2 + 1$, $(n-1)^3 + 1$;
symmetric Hadamard matrices of orders $2n$, $2n(n-1)$, $2p^r(p^r+1)$;
Hadamard matrices of orders $h_1 n$, $h_1 h_2 n(n-1)$, $h_1 h_2 n(n-3)$

(this latter with $n+4$ also the order of an $n$-type matrix);

Hadamard matrices of orders 452, 612, 2452 and 3044, all "new."

We also give existence conditions for many other classes of Hadamard matrices and another formulation for Goldberg's skew-Hadamard matrix of order $(h-1)^3 + 1$.

## INTRODUCTION

An *Hadamard matrix* $H$ is a matrix of order $n$, all of whose elements are $+1$ or $-1$ and which satisfies $HH^T = nI_n$. It is conjectured that an Hadamard matrix exists for $n = 2$ and for $n = 4t$, where $t$ is any positive integer. Many classes of Hadamard matrices are known; most of these can be found by reference to [2], [3], and [4]. Hadamard matrices are known for all orders less than 188.

An Hadamard matrix $H = S + I$ is called *skew-Hadamard* if $S^T = -S$. It is conjectured that, whenever there exists an Hadamard matrix of order $n$, there exists a skew-Hadamard matrix of the same order. As the existence of skew-Hadamard matrices is needed for some of my results I list the classes of orders for which skew-Hadamard matrices are known to exist:

| | | |
|---|---|---|
| I. | $2^t \Pi k_i$ | $t$, $r_i$ all positive integers, $k_i = p_i^{r_i} + 1 \equiv 0 \pmod 4$ $p_i$ a prime; from [5]. |
| II. | $(p-1)^3 + 1$ | $p$ the order of a skew Hadamard matrix; from [1]. |
| III. | $2^t(q+1)$ | $t \geqslant 1$ an integer, $q$ (prime power) $\equiv 5 \pmod 8$; from [7]. |
| IV. | 52 | From [6]. |
| V. | 36 | From [8]. |
| VI. | $p^r(p^r+1)(m-1)$ | $(m-1)(p^r+1)/m$ the order of a skew-Hadamard matrix, $m$ of class I and $p^r \equiv 3 \pmod 4$ a prime power; from [4]. |
| VII. | $p^r(p^r-3)(m-1)$ | $(m-1)(p^r+3)/m$ the order of a skew-Hadamard matrix, $m$ and $p^r$ as in class VI; from [4]. |
| VIII. | $h(p^r+1)$ | $h$ the order of a skew-Hadamard matrix, $p^r \equiv 3 \pmod 4$ a prime power; from [4]. |
| IX. | $2h$ | $h$ the order of a skew-Hadamard matrix. |

The only orders up to 200 for which skew-Hadamard matrices have not yet been discovered are 92, 100, 116, 148, 156, 172, 184, 188, and 196.

A $(v, k, \lambda)$-*configuration* is an arrangement of $v$ elements $x_1, x_2, \ldots, x_v$ into $v$ sets $S_1, S_2, \ldots, S_v$ such that every set contains exactly $k$ elements, every pair of sets has exactly $\lambda$ elements in common. A $(v, k, \lambda)$-configuration can be characterized by its *incidence matrix* $A = (a_{ij})$ defined by $a_{ij} = 1$ if $x_j \in S_i$ and $a_{ij} = -1$ if $x_j \notin S_i$. This matrix $A$, of order $v$, consists entirely of 1's and $-1$'s, and it can be seen that $A$ satisfies the *incidence equation*

$$AA^T = 4(k-\lambda)I + v - 4(k-\lambda)J$$

where $I$ is the identity matrix of order $v$ and $J$ is the matrix of order $v$ with every element $+1$.

A set of elements $D = \{x_1, x_2, \ldots, x_k\}$ will be said to generate a *circulant* $(1, -1)$ matrix $A = (a_{ij})$ if $a_{ij} = a_{1,j-i+1} = 1$ when $j - i + 1 \in D$ (all numbers modulo $v$) and $-1$ otherwise. A *back-circulant* matrix $A = (a_{ij})$ of order $v$ has $a_{1i} = a_{1+j,i-j}$ where $1 + j$ and $i - j$ are reduced to modulo $v$.

DEFINITION. A (1, −1) matrix $N = I + R$ of order $n \equiv 2 \pmod 4$ will be called *n-type* if $N$ can be written as

$$\begin{bmatrix} 0 & 1 \cdots 1 \\ 1 & \\ \vdots & D \\ 1 & \end{bmatrix} + I_n , \tag{1}$$

where $D^T = D$ and $D^2 = (n-1) I_{n-1} - J_{n-1}$ or equivalently if $R^T = R$ and $R^2 = (n-1) I_n$.

We now investigate the existence of $n$-type matrices and we will show three classes of these matrices exist:

I. $p^r + 1$ $p^r \equiv 1 \pmod 4$ is a prime power.

II. $(n-1)^2 + 1$ Where $n$ is either the order of a skew-Hadamard matrix or the order of an $n$-type matrix.

III. $(n-1)^3 + 1$ Where $n$ is the order of an $n$-type matrix.

PROOF OF CLASS I. The observation that matrices of order $p^r + 1 \equiv 2 \pmod 4$, where $q = p^r$ is a prime power, can be found satisfying the requirements for $n$-type matrices is due to Williamson [5, p. 66]. As in [5] we define $D = (d_{ij})$ of order $q$ by

$$d_{ii} = 0,$$
$$d_{ij} = \chi(a_i - a_j),\ i \neq j,$$

where $a_1, a_2, \ldots, a_q$ are the elements of a Galois field in some fixed order and $\chi(a) = 1$ or $-1$ according as $a$ is or is not the square of an element of the $GF(p^r)$. Since $q \equiv 1 \pmod 4$, $\chi(x) = \chi(-x)$, so $D$ is symmetric and $D^2 = p^r I - J$.

Then

$$N = \begin{bmatrix} 0 & 1 \cdots 1 \\ 1 & \\ \vdots & D \\ 1 & \end{bmatrix} + I_{p^r+1} ,$$

is an $n$-type matrix of order $p^r + 1$.

PROOF OF CLASS II. If $N$ is an $n$-type matrix of order $n$ then $N$ may be written as

$$\begin{bmatrix} 0 & 1 \cdots 1 \\ 1 & \\ \vdots & D \\ 1 & \end{bmatrix} + I_{n-1} \tag{3}$$

where $D^T = D$, $DD^T = (n-1) I_{n-1} - J_{n-1}$ and $DJ_{n-1} = 0$.

If $V$ is a skew-Hadamard matrix of order $n$ then $V$ may be written as

$$\begin{bmatrix} 0 & 1 \cdots 1 \\ -1 & \\ \vdots & W \\ -1 & \end{bmatrix} + I_{n-1}, \tag{4}$$

where $W^T = -W$, $WW^T = (n-1)I_{n-1} - J_{n-1}$ and $WJ_{n-1} = 0$.

Now where $X$ is either $W$ or $D$ define $A$ by

$$A = J_{n-1} \times -I_{n-1} + I_{n-1} \times J_{n-1} + X \times X;$$

then

$$\begin{aligned} AA^T &= (n-1)J_{n-1} \times I_{n-1} + I_{n-1} \times (n-1)J_{n-1} \\ &\quad + [(n-1)I_{n-1} - J_{n-1}] \times [(n-1)I_{n-1} - J_{n-1}] \\ &\quad - 2J_{n-1} \times J_{n-1} \\ &= (n-1)^2 I_{(n-1)^2} - J_{(n-1)^2}. \end{aligned}$$

Now $A^T = A$ if $X \times X = X^T \times X^T$, but this is true for both $W$ and $D$ and so

$$M = \begin{bmatrix} 0 & 1 \cdots 1 \\ 1 & \\ \vdots & A \\ 1 & \end{bmatrix} + I_{(n-1)^2+1}$$

is an $n$-type matrix of order $(n-1)^2 + 1$.

PROOF OF CLASS III. With $D$ as in (3) and $D$, $J$, and $I$ all of order $n-1$ we define

$$A = I \times D \times J + J \times I \times D + D \times J \times I + D \times D \times D.$$

This is similar to Goldberg's construction for Hadamard matrices as his matrix may be written as

$$\begin{bmatrix} 1 & 1 \cdots 1 \\ -1 & \\ \vdots & B \\ -1 & \end{bmatrix},$$

where $B$ is given by

$$B = I\times I\times I + I\times W\times J + J\times I\times W + W\times J\times I + W\times W\times W$$

with $W$ as defined in (4).

Now $DJ = 0$ so

$$\begin{aligned} AA^T &= I\times D^2\times J^2 + J^2\times I\times D^2 + D^2\times J^2\times I + D^2\times D^2\times D^2 \\ &= I\times[(n-1)\,I-J]\times(n-1)\,J + (n-1)\,J\times I\times[(n-1)\,I-J] \\ &\quad +[(n-1)\,I-J]\times(n-1)\,J\times I + [(n-1)\,I-J]\times[(n-1)\,I-J] \\ &\quad \times[(n-1)\,I-J] \\ &= (n-1)^3\,I_{(n-1)^3} - J_{(n-1)^3}\,, \end{aligned}$$

and clearly $A^T = A$.

So

$$M = \begin{bmatrix} 0 & 1 \cdots 1 \\ 1 & \\ \vdots & A \\ 1 & \end{bmatrix} + I_{(n-1)^3+1}$$

is an $n$-type matrix of order $(n-1)^3 + 1$.

LEMMA 1. *If there is an $n$-type matrix of order $n$, then there is a symmetric Hadamard matrix of order $2n$.*

PROOF: Let $N = I + P$ be the $n$-type matrix of order $n$. Then $P^2 = (n-1)\,I_n$ and $P^T = P$, Now choose

$$X = \begin{bmatrix} -P-I & P-I \\ P-I & P+I \end{bmatrix};$$

then $X^T = X$ and $XX^T = 2nI_{2n}$. So $X$ is the required symmetric Hadamard matrix.

Since there are skew-Hadamard matrices of orders 16, 36, and 40 using class II we see there are $n$-type matrices of order 226, 1226, and 1522. Then using the above lemma there are symmetric Hadamard matrices of orders 452, 2452, and 3044. These orders are new.

LEMMA 2. *If there is an $n$-type matrix of order $n$ and an Hadamard matrix of order $h$, then there is an Hadamard matrix order $hn$.*

PROOF: Let $H$ be the Hadamard matrix and define $K = HS$ where

$$S = \begin{bmatrix} & & & & & & 1 \\ & & & & & -1 & \\ & & & & 1 & & \\ & & & -1 & & & \\ & & & \cdot & & & \\ & & \cdot & & & & \\ & \cdot & & & & & \\ & 1 & & & & & \\ -1 & & & & & & \end{bmatrix} \tag{5}$$

is of order $h$ then $SS^T = I_h$ and $S^T = -S$, so

$$HK^T = HS^TH^T = -HSH^T = -KH^T \quad \text{and}$$
$$hI_h = HH^T = HSS^TH^T = HS(HS)^T = KK^T.$$

Then if $N = I + R$ is an $n$-type matrix of order $n$, $R^2 = (n-1)I_n$ and $R^T = R$, so, if

$$L = I_n \times H + R \times K,$$

then

$$\begin{aligned} LL^T &= I_n \times HH^T + R \times KH^T + R^T \times HK^T + RR^T \times KK^T \\ &= I_n \times hI_h + (n-1)I_n \times hI_n \\ &= nhI_{nh}\,. \end{aligned}$$

So $L$ is the required Hadamard matrix of order $nh$.

THEOREM 3. *If $N = I + R$ is an $n$-type matrix of order $n$, then there is a symmetric Hadamard matrix of order $2n(n-1)$.*

PROOF: Let $D$ be as defined in (1) above, then $D^T = D$ and

$$D^2 = (n-1)I_{n-1} - J_{n-1}\,.$$

Further $R^T = R$, $R^2 = (n-1)I_n$ and $JD = 0$.

So, if $J$ is of order $n-1$, let

$$X = \begin{bmatrix} J & J \\ J & -J \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} D+I & D-I \\ D-I & -D-I \end{bmatrix};$$

then

$$X^T = X,\ Y^T = Y,\ XX^T = 2(n-1)\,J_{n-1} \times I_2,\ YY^T = (2nI_{n-1} - 2J_{n-1}) \times I_2,$$

and $XY^T + YX^T = 0$.

Then, if

$$H = I_n \times X + R \times Y,$$

$H$ is symmetric and, since

$$\begin{aligned} HH^T &= I_n \times XX^T + R \times (YX^T + XY^T) + RR^T \times YY^T \\ &= I_n \times 2(n-1)\,J_n \times I_2 + (n-1)\,I_n \times (2nI_{n-1} - 2J_{n-1}) \times I_2 \\ &= 2n(n-1)\,I_{2n(n-1)}, \end{aligned}$$

$H$ is Hadamard.

COROLLARY 4. *If $p^r \equiv 1 \pmod 4$ is a prime power, then there is a symmetric Hadamard matrix of order $2p^r(p^r + 1)$.*

PROOF: This follows by putting $n$ equal to orders of class I.

This corollary is important as it means class VII of Marshall Hall [2, p. 207] may be partially rewritten. The construction gives an Hadamard matrix of order 612 which was previously not known.

THEOREM 5. *If $N = I + R$ is an $n$-type matrix of order $m$, then, if there are symmetric $(1, -1)$ matrices $D + I$ and $M$ satisfying $D^2 = nI_n - J_n$, $M^2 = (n + 1 - m)\,I_n + (m-1)\,J_n$, and $MD = DM$, where $D$ has zero diagonal, there is an Hadamard matrix of order $4mn$.*

PROOF: Choose

$$X = \begin{bmatrix} M & M & M & M \\ M & -M & -M & M \\ M & -M & M & -M \\ M & M & -M & -M \end{bmatrix}$$

and

$$Y = \begin{bmatrix} D+I & -D-I & D-I & -D+I \\ -D-I & -D-I & D-I & D-I \\ -D+I & -D+I & -D-I & -D-I \\ +D-I & -D+I & -D-I & D+I \end{bmatrix};$$

then $XY^T + YX^T = 0$, $XX^T = [4(n + 1 - m) I_n + 4(m - 1) J_n] \times I_4$ and $YY^T = [4(n + 1) I_n - 4J_n] \times I_4$, so, if

$$H = I_m \times X + R \times Y$$

then

$$\begin{aligned} HH^T &= I_m \times XX^T + R \times (XY^T + YX^T) + R^2 \times YY^T \\ &= I_m \times [4(n+1-m) I_n + 4(m-1) J_n] \times I_4 \\ &\quad + (m-1) I_m \times [4(n-1) I_n - 4J_n] \times I_4 \\ &= 4nmI_{4nm} . \end{aligned}$$

So $H$ is an Hadamard matrix of order $4nm$.

COROLLARY 6. *If $m$ is the order of an $n$-type matrix, then, if there is a back-circulant $(1, -1)$ matrix $M$ of order $p \equiv 1 \pmod 4$, a prime, satisfying $M^2 = (p + 1 - m) I + (m - 1) J$,* then *there is an Hadamard matrix of order $4pm$.*

PROOF: This follows with $n$ of the theorem of class I and $D$ defined as in (2). By Theorem 1 of [3], a circulant $D$ and a back-circulant $M$ satisfy $MD^T = DM^T$ and since $M$ and $D$ are both symmetric the conditions of the theorem are satisfied.

COROLLARY 7. *If there is a $n$-type matrix of order*:

(i) $p^r - 3$;

(ii) $p - 4q^{n-1} + 1$, *where* $p = q^n + q^{n-1} + \cdots + 1$, *$n$ integer*;

(iii) $q^2 - 3q + 2$, *where* $p = q^2 + q + 1$;

(iv) $x^2 + 1$, *where* $p = 4x^2 + 1$;

(v) $x^2 + 1$, *where* $p = 4x^2 + 9$;

(vi) $36b^2 + 6$, *where* $p = 8a^2 + 1 = 64b^2 + 9$, *$b$ odd*;

(vii) $36b^2 + 250$, *where* $p = 8a^2 + 49 = 64b^2 + 441$, *$b$ even*;

*with $p$* (prime) $\equiv 1 \pmod 4$, *$q$ prime power, and $x$ and $a$ odd, then there are Hadamard matrices of orders*:

(i) $4p^r(p^r - 3)$;

(ii) $4(q^n + q^{n-1} + \cdots + 1)(q^n - 3q^{n-1} + q^{n-2} + \cdots + q^2 + q + 2)$;

(iii) $4(q - 1)(q - 2)(q^2 + q + 1)$;

(iv) $4(x^2 + 1)(4x^2 + 1)$;

(v) $4(x^2 + 1)(4x^2 + 9)$;

(vi) $24(6b^2 + 1)(64b^2 + 9)$;

(vii) $8(18b^2 + 125)(64b^2 + 441)$;

*respectively.*

PROOF: In each case but (i) $M$ of the corollary is a back-circulant matrix generated by a difference set: the notation we use for these difference sets is that of Marshall Hall [2, p. 141]. The proof follows with $M$ given by (i) $J$-$2I$; (ii) $S$; (iii) $S$ with $n = 3$; (iv) $B$; (v) $B_0$; (vi) $O$; (vii) $O_0$; respectively.

THEOREM 8. *If $h > 1$ is the order of an Hadamard matrix, $n$ is the order of an $n$-type matrix and $v \equiv 1 \pmod 4$ the order of three $(1, -1)$ matrices $A$, $B$, and $D + I$ which satisfy $DD^T = vI - J$, $AA^T = aI + (v - a)J$, $BB^T = [2(v - n + 1) - a]I + [-v + 2(n - 1) + a]J$, and $AB^T$, $BD^T$, and $AD^T$ all symmetric, where $D$ has zero diagonal, then there is an Hadamard matrix of order $2nvh$.*

PROOF: Let $I_n + R$ be the $n$-type matrix, $H$ the Hadamard matrix, and $K = HS$ be as defined in the proof of Lemma 2. Now choose

$$X = \begin{bmatrix} A & B \\ B & -A \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} D + I & D - I \\ -D + I & D + I \end{bmatrix};$$

then $XY^T = YX^T$. So

$$W = I_n \times H \times X + R \times K \times Y$$

is the required matrix.

In the constructions of Williamson shown in Marshall Hall [2, pp. 214–216], $m$ and $n$ are defined as being of the form $p^r + 1$, where $p^r \equiv 1 \pmod 4$ is a prime power; if we read instead that $m$ and $n$ are the orders of $n$-type matrices, then the same proofs give us the theorem

THEOREM 9. *If Hadamard matrices of orders $h_1$ and $h_2$ exist, $h_1 > 1$, $h_2 > 1$ and* (i) *$n$;* (ii) *$n$ and $n + 4$, are the orders of $n$-type matrices; there exist Hadamard matrices of orders*

(i) $h_1h_2n(n - 1)$;

(ii) $h_1h_2n(n + 3)$;

*respectively.*

THEOREM 10. *If there is an $n$-type matrix of order $n$ and $k$ is the order of three $(1, -1)$ matrices $A$, $C$, and $D + I$ satisfying*

$$AA^T = aI + (k - a)J,\ CC^T = (2k - 2n + 2 - a)I + (-k + 2n - 2 + a)J,$$

*$DD^T = kI - J$, and $AC^T$, $CD^T$, and $AD^T$ all symmetric where $D$ has zero diagonal, then there is an Hadamard matrix of order $4kn$.*

PROOF: Define

$$M = \begin{bmatrix} A & A & C & C \\ A & -A & C & -C \\ C & C & -A & -A \\ C & -C & -A & A \end{bmatrix}$$

and

$$P = \begin{bmatrix} -D-I & D+I & D-I & -D+I \\ D+I & D+I & -D+I & -D+I \\ -D+I & D-I & -D-I & D+I \\ D-I & D-I & D+I & D+I \end{bmatrix};$$

then, since $AC^T$, $CD^T$, and $AD^T$ are all symmetric, $MP^T + PM^T = 0$. Also

$$MM^T = \{4(k - n + 1) I_k + 4(n - 1) J_k\} \times I_4 ,$$

$$PP^T = \{4(k + 1) I_k - 4J_k\} \times I_4 .$$

So, if $N = I + R$ is the $n$-type matrix, $R^T = R$ and $R^2 = (n - 1) I_n$. Now consider

$$H = I_n \times M + R \times P,$$

then

$$\begin{aligned} HH^T &= I_n \times MM^T + R \times \{MP^T + PM^T\} + R^2 \times PP^T \\ &= 4knI_{4kn} . \end{aligned}$$

So $H$ is the required Hadamard matrix.

COROLLARY 11. *If there is an $n$-type matrix of order*

(i) $q^n - q^{n-1} + q^{n-2} + \cdots + q^2 + q + 2$, *where* $p = q^n + q^{n-1} + \cdots + q + 1$, $n$ *integer*;

(ii) $q^n - q^{n-1} + q^{n-2} + \cdots + q^2 + q$, $p$ *as in* (i);

(iii) $(5x^2 - 1)/2$, *where* $p = 4x^2 + 1$;

(iv) $(5x^2 + 7)/2$, *where* $p = 4x^2 + 9$;

(v) $50b^2 + 8$, *where* $p = 8a^2 + 1 = 64b^2 + 9$, $b$ *odd*;

(vi) $50b^2 + 346$, *where* $p^r = 8a^2 + 49 = 64b^2 + 441$, $b$ *even*;

*with $p$* (prime) $\equiv 1 \pmod 4$, *$q$ prime power, and $x$ and $a$ odd, then there are Hadamard matrices of orders*

(i) $4(q^n + q^{n-1} + \cdots + 1)(q^n - q^{n-1} + q^{n-2} + \cdots + q^2 + q + 2)$;

(ii) $4(q^n + q^{n-1} + \cdots + 1)(q^n - q^{n-1} + q^{n-2} + \cdots + q^2 + q)$;

(iii) $2(5x^2 - 1)(4x^2 + 1)$;

(iv) $2(5x^2 + 7)(4x^2 + 9)$;

(v) $8(25b^2 + 4)(64b^2 + 9)$;

(vi) $8(25b^2 + 173)(64b^2 + 441)$.

PROOF. In each case $C$ of the theorem is a back-circulant matrix generated by a difference set; we again use the notation of Marshall Hall [2, p. 141]. The proof follows with

(i) $A = J$, $C = S$, $q$ powers of 2;

(ii) $A = J - 2I$, $C = S$, $q \equiv 3 \pmod 4$;

(iii) $A = J - 2I$, $C = B$;

(iv) $A = J - 2I$, $C = B_0$;

(v) $A = J$, $C = O$;

(vi) $A = J$, $C = O_0$.

*Note Added in Proof.* Dr. J. M. Goethals has pointed out to me that some of the results of this paper overlap those of [9].

## REFERENCES

1. K. GOLDBERG, Hadamard Matrices of Order Cube Plus One, *Proc. Amer. Math. Soc.* **17** (1966), 744–746.
2. M. HALL, JR., *Combinatorial Theory*, Blaisdell, Waltham, Mass., 1967.
3. J. WALLIS, A Class of Hadamard Matrices, *J. Combinatorial Theory* **6** (1969), 40–44.
4. J. WALLIS, $(v,k,\lambda)$ Configurations and Hadamard Matrices, to appear in *J. Australian Math. Soc.*
5. J. WILLIAMSON, Hadamard's Determinant Theorem and the Sum of Four Squares, *Duke Math. J.* **11** (1944), 65–81.
6. D. BLATT AND G. SZEKERES, A Skew Hadamard Matrix of Order 52, *Canad. J. Math.* **22** (1970), 1319–1322.
7. G. SZEKERES, Tournaments and Hadamard Matrices, *Enseignment Math.* **XV** (1969), 269–278.
8. J. M. GOETHALS AND J. J. SEIDEL, A Skew Hadamard Matrix of Order 36, to appear in *J. Australian Math. Soc.*
9. J. M. GOETHALS AND J. J. SEIDEL, Orthogonal Matrices with Zero Diagonal, *Canad. J. Math.* **19** (1967), 1001–1010.

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