Lecture 2. BASICS OF HEAT TRANSFER

2.1 SUMMARY OF LAST WEEK LECTURE

- There are three modes of heat transfer: conduction, convection and radiation.
- We can use the analogy between Electrical and Thermal Conduction processes to simplify the representation of heat flows and thermal resistances.

$$q = \frac{\Delta T}{\sum R}$$

- Fourier's law relates heat flow to local temperature gradient.

$$q_x = -A_x k \left(\frac{\partial T}{\partial x} \right)$$

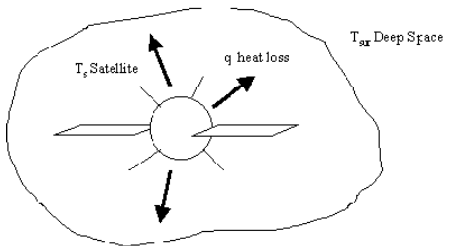
- Convection heat transfer arises when heat is lost/gained by a fluid in contact with a solid surface at a different temperature.

$$q = hA_s (T_W - T_s) \quad \text{[Watts]} \qquad \text{or} \qquad q = \frac{(T_W - T_s)}{1/hA_s} = \frac{(T_W - T_s)}{R_{conv}}$$

Where: $R_{conv} = \dfrac{1}{hA_s}$

- Radiation heat transfer is dependent on **absolute** temperature of surfaces, surface properties and geometry. For case of small object in a large enclosure.

$$q = \varepsilon_s \, A_s \, \sigma \left(T_s^4 - T_{surr}^4 \right)$$



2.2 CONTACT RESISTANCE

In practice materials in thermal contact may not be perfectly bonded and voids at their interface occur. Even a flat surfaces that appear smooth turn out to be rough when examined under microscope with numerous peaks and valleys.

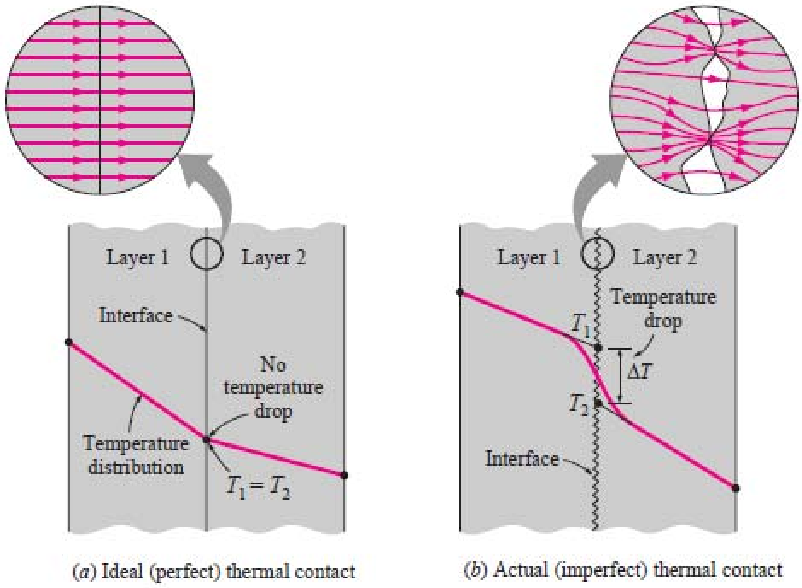


Figure 1. Comparison of temperature distribution and heat flow along two plates pressed against each other for the case of perfect and imperfect contact.

In imperfect contact, the "contact resistance", R_i causes an additional temperature drop at the interface

$$\Delta T_i = R_i \, q_x \tag{1}$$

R_i is very difficult to predict but one should be aware of its effect. Some order-of-magnitude values for metal-to-metal contact are as follows.

Material	Contact Resistance R_i [m^2 W/K]
Aluminum	5×10^{-5}
Copper	1×10^{-5}
Stainless steel	3×10^{-4}

We use grease or soft metal foil to improve contact resistance e.g. silicon grease between power transistor and mica sheet and heat sink.

2.3 THERMAL RESISTANCES IN PARALLEL

We use the electrical analogy to good effect where:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (2)$$

2.4 OVERALL HEAT TRANSFER COEFFICIEN, *U*

Up till now we have discussed the heat transfer coefficient (HTC) in relation to a fluid-surface pair. Often heat is transferred ultimately between two fluids. For example, heat must be exchanged between the air inside and outside an enclosure for telecommunications equipment.

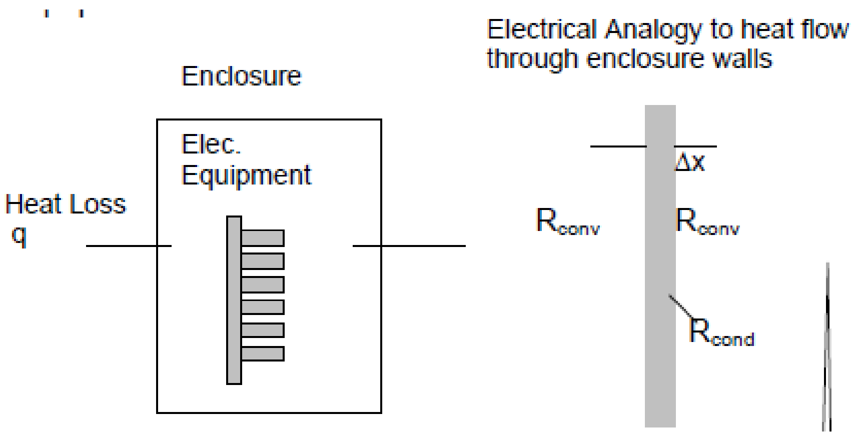


Figure 2. Heat transfer between air inside and outside an electrical enclosure.

The heat flow is given

$$q = \frac{T_2 - T_1}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}} \qquad (3)$$

For such situation it is often convenient to use the "overall heat transfer coefficient" defined as:

$$U = \left(\frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2} \right)^{-1} \tag{4}$$

And therefore the total heat flow through the wall from one fluid to the other is given by

$$q = UA(T_2 - T_1) \tag{5}$$

2.5 CONDUCTION WITH INTERNAL HEAT GENERATION

This situation is often encountered in engineering situations e.g. electrical heating, chemical reactions (endothermic or exothermic).

2.5.1 Heat Generation in a Slab

When there is heat generation in the body, the term $\dot{q}$ in the general equation is non-zero. For one dimensional problem such a slab, the conduction equation is

$$k \frac{d^2T}{dx^2} = -\dot{q} \tag{6}$$

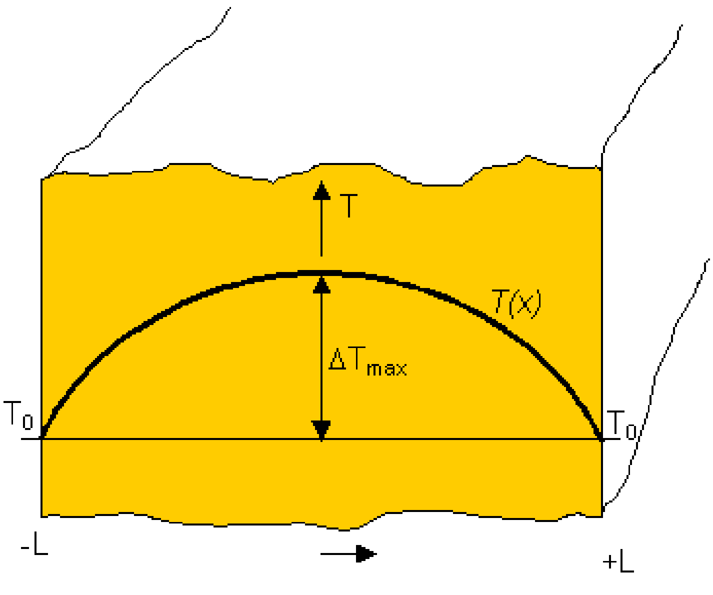


Figure 3. Temperature distribution in a slab with heat generation.

And integrating twice with respect to distance x and solving for the unknown constants using the boundary conditions $\left.\frac{dT}{dx}\right|_{x=0} = 0$ and T(L) = T_o gives:

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 = T_o + \frac{\dot{q}L^2}{2k}\left(1 - \left(\frac{x}{L}\right)^2\right) \qquad (7)$$

Which is a parabolic temperature distribution with the max temperature given by

$$\Delta T_{\max} = \frac{\dot{q}L^2}{2k} \qquad (8)$$

2.5.2 Heat Generation in a Solid Cylinder

The conduction equation for a solid cylinder assuming no axial heat conduction is reduced to

$$\frac{1}{r}\frac{d}{dr}\left(k\,r\frac{dT}{dr}\right) = -\dot{q} \qquad (9)$$

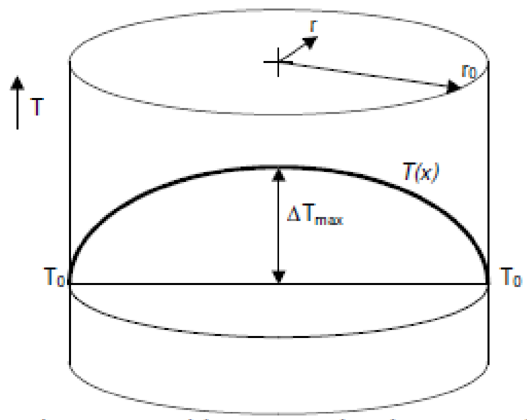


Figure 3. Temperature distribution in a solid cylinder with heat generation.

Again we integrate and use the boundary conditions to find that

$$T(r) = T_o + \frac{\dot{q}r_o^2}{4k}\left(1 - \left(\frac{r}{r_o}\right)^2\right) \quad (10)$$

With max temperature

$$\Delta T_{\max} = \frac{\dot{q}r_o^2}{4k} \quad (11)$$

A typical example of heat generation in solid cylinder is: **Heat generation due to electrical resistance in wires.** Heat generation in the wire

$$\dot{q} = \frac{I^2 R_{elec}}{volume} \quad (12)$$

Electrical resistance is given by

$$R_{elec} = \frac{\gamma L}{A} \quad (13)$$

Where
γ is electrical resistivity (Ω-m)
L is length of wire (m)
A is wire cross-section (m^2)

Thus,

$$\dot{q} = \frac{I^2 \gamma}{A^2} \quad (14)$$

2.6 USE OF HEAT SINKS FOR ELECTRICAL COOLING

The term "heat sink" can be used in the general sense of a cool object that absorbs or dissipates heat without a significant rise in temperature.

In the case of cooling of electronic equipment a "heat sink" is usually taken to mean a metal plate onto which electronic components are mounted and which is "finned" to increase the surface area. Commercial heat sinks are rated in terms of their thermal resistance [^{o}C/W]. This resistance includes BOTH the conduction resistance through the metal (usually aluminum) and the convection resistance from the metal surfaces to the air.

There are as many different types of heat sink available as there are situations where electronics require cooling!!

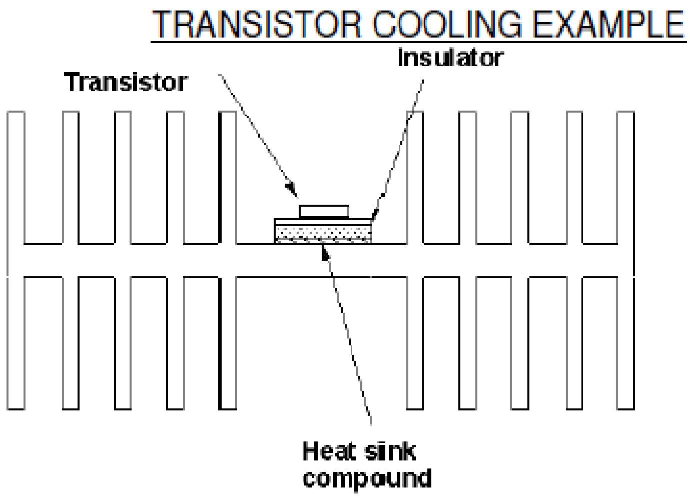


Figure 4. Example of transistor cooling.

2.7 HEAT TRANSFER ENHANCEMENT USING FINS

We use a fin on a solid object to increase convective heat transfer by increasing surface area. The fin must be made of a good thermal conductor. Examples of this type of heat transfer enhancement include:

- Heat sinks on electrical equipment
- Satellite cooling panels
- Radiator panels and oil coolers on power transformers
- Fins on the outside of motors

We are seeking to decrease the total resistance to heat flow when surface convection/radiation presents the dominant resistance i.e. by INCREASING THE SURFACE AREA.

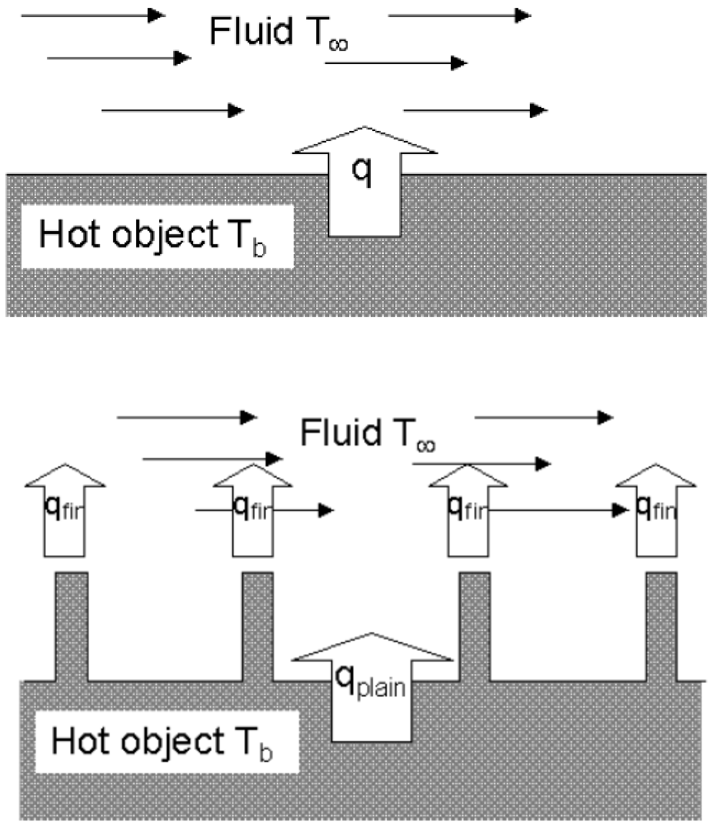


Figure 5. Increasing the surface area by adding fins in low heat transfer coefficient situation.

Some innovative fin designs are shown in Figure 6.

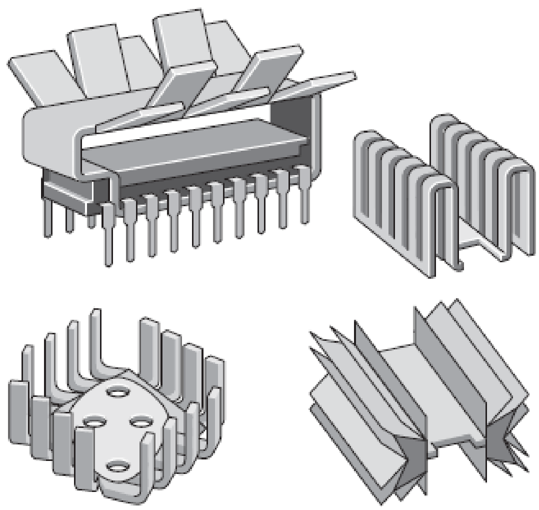


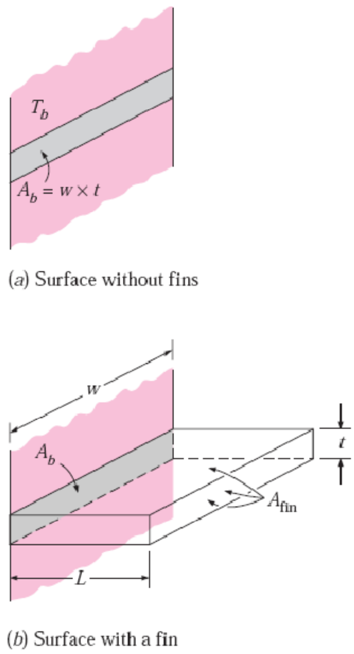
Figure 6. Some innovative fin designs.

Heat transfer from a surface is increased by adding fins. If fins have an extremely high thermal conductivity ($k \rightarrow \infty$) then their surface temperature will be equal to that of the body, T_b, and the heat loss will be given by:

$$q = h(A_{plane} + A_{fin})(T_b - T_\infty) \quad \text{only true if } (k \rightarrow \infty) \qquad (15)$$

But real fins have a finite thermal conductivity so the temperature must change from the base to the end of the fin. We must first determine what the temperature distribution on the fin will be before finding q_{fin}. The local rate of heat loss per unit surface area, q''_x, from the fin is dependent on the local fin temperature, T(x).

$$q''_x = h(T(x) - T_\infty) \qquad (16)$$



$$A_{fin} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$

Figure. Fins enhance heat transfer from a surface by enhancing surface area.

2.7.1 Temperature Distribution in fins

We use a one-dimensional approximation and assume that fin cross-section is constant and perform an *energy balance* on a small element of the fin.

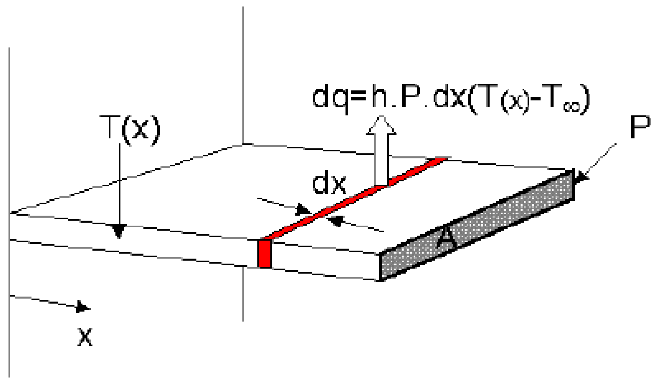


Figure 7. Fin element for energy balance analysis.

Energy balance on element is therefore:

HEAT CONDUCTED IN - HEAT CONDUCTED OUT = HEAT CONVECTED OUT

From the Fourier's Law the heat flow in x-direction

$$q_x = -A_x k\left(\frac{\partial T}{\partial x}\right) \tag{17}$$

which when applied in the energy balance equation gives

$$dq_x = d\left(-A_x k\frac{\partial T}{\partial x}\right) = -h(P\,dx)(T - T_\infty) \tag{18}$$

Let $\theta = T - T_\infty$

$$\frac{d}{dx}\left(kA_x\frac{d\theta}{dx}\right) - h\,P\theta = 0 \tag{19}$$

Or when the fin is of constant cross-section and constant k,

$$\frac{d^2\theta}{dx^2} - \left(\frac{hP}{kA}\right)\theta = 0 \tag{20}$$

We can define a characteristic length, λ with which to non-dimensionalise our equation

$$\lambda = \left(\frac{kA}{hP}\right)^{1/2} \tag{21}$$

And the general solution to the 2nd order DE becomes

$$\theta = C_1 e^{-x/\lambda} + C_2 e^{-x/\lambda} \tag{22}$$

To solve this equation the fin boundary conditions must be specified. The temperature of the plate to which the fins are attached is normally known in advance. Therefore, at the fin base we have a specified temperature boundary condition. At fin root

$$\theta = \theta_b = T_b - T_\infty \quad \text{at } x = 0 \tag{23}$$

At the fin tip there are several possibilities, including specified temperature, negligible heat loss (idealized as an insulated tip), convection, and combined convection and radiation, namely:

Case 1: fin is very long, temperature at the end of the fin = T_∞

Case 2: fin is of finite length with end of fin insulated.

Case 3: fin is of finite elength with heat convected from the end.

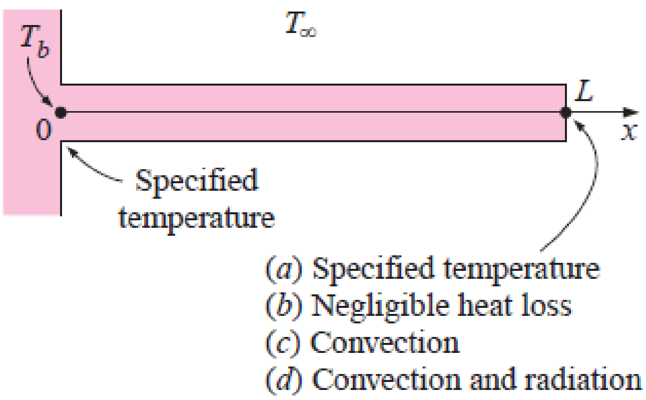


Figure 8. Boundary conditions at the fin base and the fin tip.

Case 1- fin is very long, temperature at the end of the fin = T_∞

In this case, $\theta = \theta_b$ at x = 0 and $\theta = 0$ at x = L, thus the temperature distribution is an exponential decay towards the ambient fluid temperature.

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x/\lambda} \tag{24}$$

It can be seen that the temperature along the fin in this case decreases exponentially from T_b to T_∞.

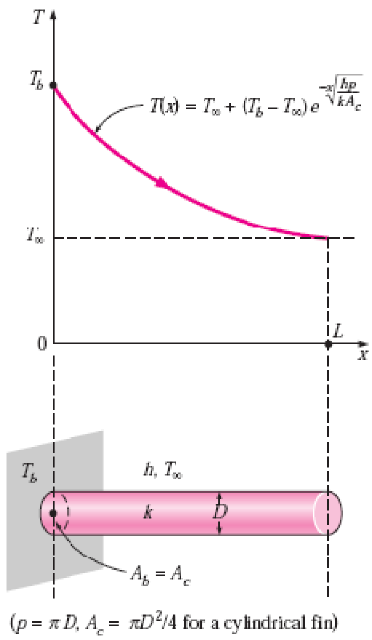


Figure 9. Variation of temperature along very long fin.

Case 2- fin is of finite length with end of fin insulated

Generally fins are not very long that their temperature approaches the surrounding temperature at the tip. It is sometimes more accurate to consider the heat transfer from the tip to be negligible since it is proportional to its surface area. Since the surface area of the fin tip is usually very small fraction of the total fin area the tip can be assumed to be insulated. In this case the boundary condition at the tip is $\frac{d\theta}{dx} = 0$ at x = L, and the condition at the base remains the same as in case 1. The application of these two conditions on the general solution Eq. (22) yields, after some manipulations, the relation for temperature distribution

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh \lambda(L - x)}{\cosh \lambda L} \qquad (25)$$

Case 3- fin is of finite elength with heat convected from the end.
In this case, the boundary condition at the tip is

$$\frac{d\theta}{dx} = -\frac{h}{k}(T_{tip} - T_\infty) \qquad (26)$$

The solution of the general equation gives the temperature distribution

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[(L - x)/\lambda] + (h\lambda/k)\sinh[(L - x]/\lambda)}{\cosh(\lambda/L) + (h\lambda/k)\sinh(L/\lambda)} \qquad (27)$$

Limitations of the analysis above include:

- Assumes one-dimensional conduction i.e. temperature only varies along the fin major axis
- Assumes constant surface heat transfer coefficient, h

2.7.2 Heat Transfer from Fins

To determine the total heat loss from fin, we use the Fourier's Law at the base of the fin

$$q_{fin} = -Ak\left(\frac{\partial T(x)}{\partial x}\right)\bigg|_{x=0} \qquad (28)$$

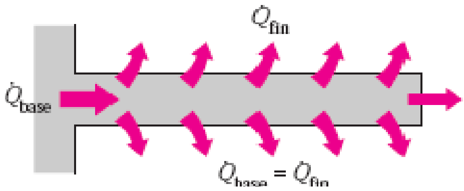


Figure 10. Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

Case 1- fin is very long, temperature at the end of the fin = T_∞

The steady rate of heat transfer from the entire fin can be determined from Fourier's Law of heat conduction.

$$q_{fin} = -Ak\left.\frac{\partial T}{\partial x}\right|_{x=0} = -Ak\left.\left(\frac{\partial}{\partial x}(T_b - T_\infty)e^{-x/\lambda}\right)\right|_{x=0} = \left(AkPh\right)^{1/2}(T_b - T_\infty) \qquad (29)$$

Or

$$q_{fin} = (T_b - T_\infty)/R_{fin} \qquad (30)$$

With

$$R_{fin} = \frac{1}{(AkPh)^{1/2}} \qquad (31)$$

Case 2- fin is of finite length with end of fin insulated

Similarly for adiabatic (insulated) tip fin, the heat transfer from the fin can be determined

$$q_{fin} = -Ak\left.\frac{dT}{dx}\right|_{x=0} = \left(AkPh\right)^{1/2}(T_b - T_\infty)\tanh(\lambda L) \qquad (32)$$

Thus,

$$R_{fin} = \frac{1}{(AkPh)^{1/2}\tanh(\lambda L)} \qquad (33)$$

Case 3- fin is of finite elength with heat convected from the end.

Finally for convecting tip fin, the heat transfer is

$$q_{fin} = (AkPh)^{1/2}\left(\frac{\sinh(L/\lambda)+(h\lambda/k)\cosh(L/\lambda)}{\cosh(L/\lambda)+(h\lambda/k)\sinh(L/\lambda)}\right)(T_b - T_\infty) \qquad (34)$$

And

$$R_{fin} = \left\lfloor (AkPh)^{1/2}\left(\frac{\sinh(L/\lambda)+(h\lambda/k)\cosh(L/\lambda)}{\cosh(L/\lambda)+(h\lambda/k)\sinh(L/\lambda)}\right)\right\rfloor^{-1} \qquad (35)$$

Note that A above is the cross sectional area of the fin.

2.7.3 Fin Efficiency

The idea of fin is to increase the surface area, however from the base to the tip the fin surface temperature decreases. In the limiting case of zero thermal resistance or infinite thermal conductivity (k → ∞), the temperature of the fin will be uniform at the base value T_b. The heat transfer from the fin will be maximum in this case and can be expressed as

$$q_{fin,\max} = h\,A_{fin}\,(T_b - T_\infty) \tag{36}$$

In reality, as the temperature drops the fin heat transfer will be less than this. To account for the effect of this decrease in temperature on heat transfer, we define **fin efficiency**.

$$\eta = \frac{\textit{actual heat transfer}}{\textit{heat transfer if entire fin at base temp.}} = \frac{q_{actual}}{q_{\max}} \tag{37}$$

This relation can help us to determine the efficiency of very long fins and fins with insulated tips.

$$\eta_{long\ \ fin} = \frac{q_{long\ \ fin}}{q_{\max}} = \frac{(Ak\,P\,h)^{1/2}\,(T_b - T_\infty)}{h\,A_{fin}(T_b - T_\infty)} = \frac{1}{L}(Ak\,P\,h)^{1/2} = \frac{1}{\lambda\,L} \tag{38}$$

And

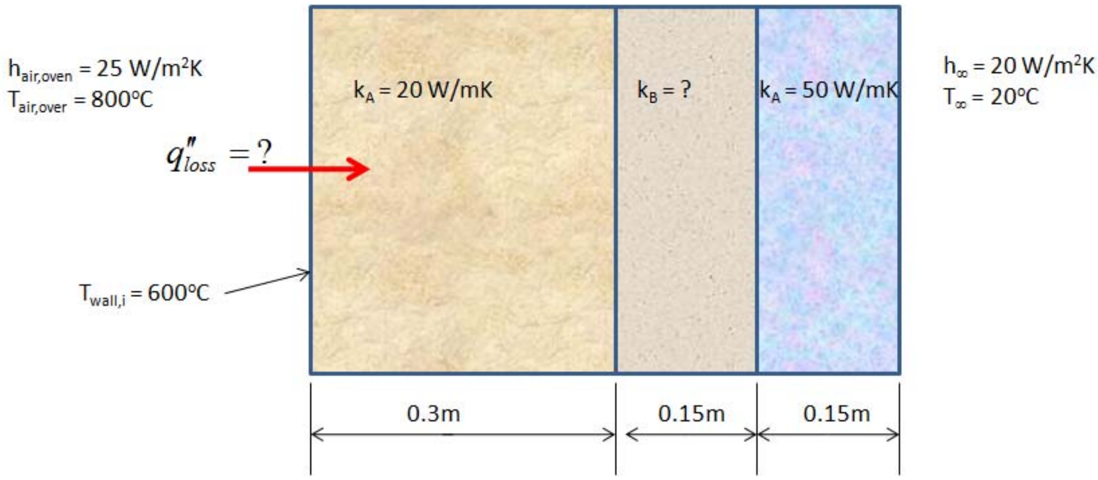
$$\eta_{insulated\ tip\ \ fin} = \frac{q_{fin}}{q_{\max}} = \frac{(Ak\,P\,h)^{1/2}\,(T_b - T_\infty)\tanh(\lambda L)}{h\,A_{fin}\,(T_b - T_\infty)} = \frac{\tanh(\lambda L)}{\lambda\,L} \tag{39}$$

2.8 WORKED EXAMPLES

We will solve these examples in the lecture together.

Example 1

The composite wall of an oven consists of three materials as shown below, what is the heat flux, q'' through the wall? And the thermal conductivity of the middle layer, k_b?



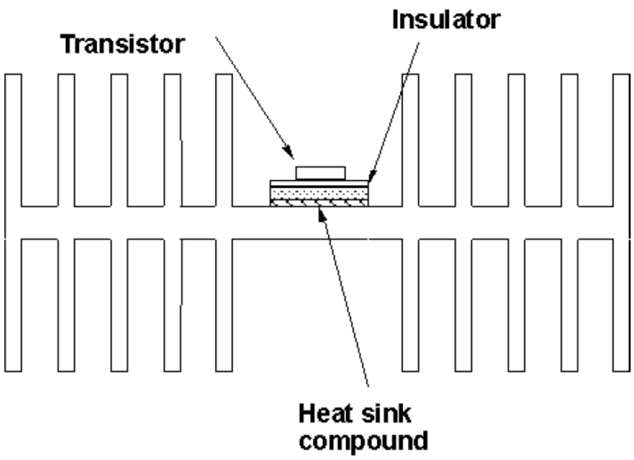
Example 2

A current of 200A is passed through a stainless steel wire (k = 19 W/mK) 3mm in diameter and 1.0m long. The resistivity of the wire is 70μΩ.cm. The wire is submerged in a liquid at 110°C and experiences a heat transfer coefficient of 4.0kW/m^2K. Calculate the wire centre temperature.

Example 3

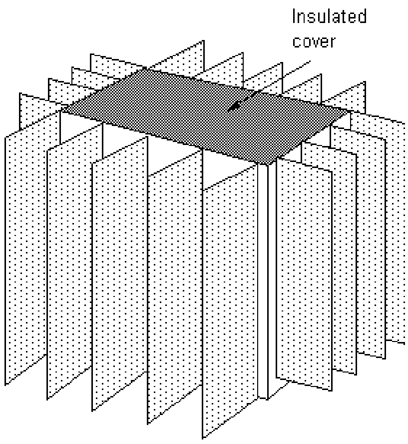
Calculate the transistor case temperature and junction temperature if the transistor dissipates 20W assuming that heat loss by convection from the top of the transistor case is negligible.

Let the cross-sectional areas of the insulator and heat sink compound be 130mm^2 and the thickness be 1.6mm and 0.025mm, respectively. Thermal conductivities are: insulator k = 15.0 W/mK, heat sink compound k = 0.39 W/mK. The overall thermal resistance of the heat sink is 0.23^{o}C/W and junction to case thermal resistance of the transistor is 2.2^{o}C/W. First draw the electrical analogy of this thermal situation.



Example 4:

An oil-filled, high voltage power transformer is to be cooled by natural convection. The transformer containment comprises a steel tank measuring 600mm long, 500mm wide and 800mm high with fins on the vertical surfaces as shown below.



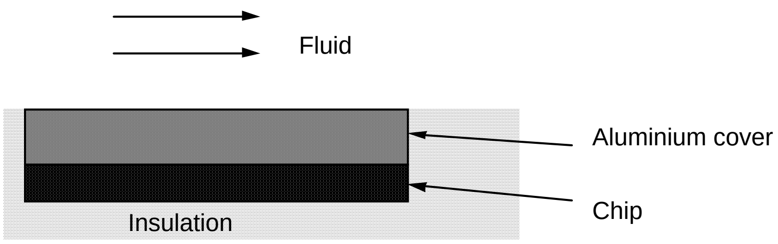
Oil-filled Transformer Containment

The walls of the tank may be taken as thick and highly conductive (and thus of uniform temperature). The 18 fins attached to the walls are 2mm thick, 150mm long and 800mm high. The steel has a thermal conductivity, k=55W/mK. The heat transfer coefficient for heat flow from the oil to the inside walls of the tank is 120W/m^{2}K and the heat transfer coefficient between all outside surfaces and the air is 18W/m^{2}K. (Assume heat loss from the top of the tank is negligible).

a) Draw an electrical analogy for the flow of heat from the hot oil inside the tank to the ambient air.
b) If the walls of the tank are at 40°C and the air temperature T = 20°C, calculate the rate of heat loss from the fins.
c) What is the efficiency and thermal resistance of a single fin?
d) Determine the heat loss from the plain (unfinned) area of the tank walls.

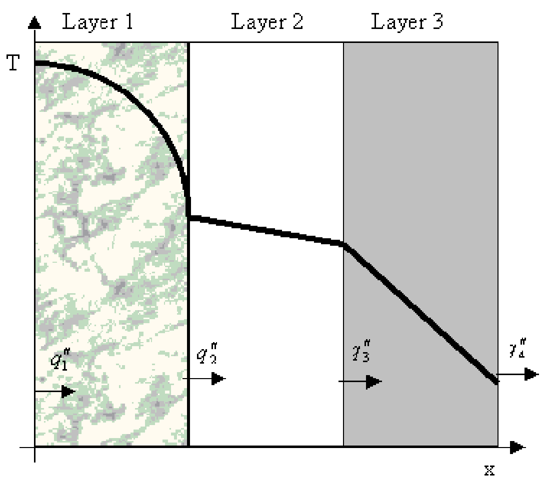
2.8 TUTORIAL QUESTIONS

1. A silicon chip operating under steady state conditions is encapsulated so that all of the power it dissipates is transferred by convection to a fluid stream where h =1000 W/m^2K and T_∞ = 25°C. The chip is assumed to at a uniform temperature internally. It is separated from the fluid by a 2mm thick aluminium cover plate and the contact resistance of the chip-aluminium interface is 5 x 10^{-5} m^2K/W. a) Draw an electrical analogy to this thermal situation. b) If the chip surface area is $100mm^2$ and its maximum allowable temperature is 85°C, what is the maximum allowable power dissipation in the chip? (A 5.65W)



2. Calculate the surface temperature and the maximum internal temperature of a 10mm diameter steel conductor carrying 5000A which is forced convection cooled to a fluid at 15°C with a convection coefficient of 5.55 kW/m^2K. For the conductor, take the electrical resistivity as 8 x 10^{-8} Ω.m, and the thermal conductivity as 120 W/mK. (ans. 161.3°C and 178.0°C).

3. A semiconductor device is made up of a number of distinct layers of differing thermal properties and power dissipation. The temperature distribution through the device is shown in the diagram below. a) Rank the heat fluxes, q", in descending order of magnitude; b) rank the thermal conductivities of the three layers; c) sketch the heat flux through the device as a function of distance, x.

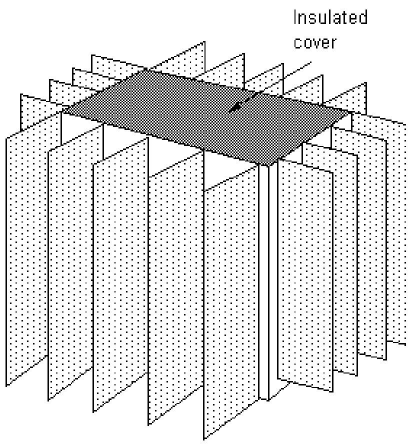


(based on a tutorial problem in Incropera and De Witt)

4. (worked example in the lecture) A plane slab of electrical conductor carries a very heavy current for electrical heating purposes. The slab is 50mm thick, is perfectly insulated on the left side (i.e. q”=0) and subjected to convection on the right side. For k = 12 W/mK, ambient temperature = 20°C and heat transfer coefficient h = 18 W/m^{2}K. The temperature of the insulated face is 400°C. Find the uniform heat generation rate per unit volume and the right-side temperature under these conditions. [Hint: the insulated face is effectively a plane of symmetry - i.e. you may consider the slab to one half of a slab of twice the thickness with convection either side. Try sketching the situation/temperature distribution first.]

Ans. $\dot{q}$ = 1.32 x 10^5 W/m^3 and T = 386°C.

5. An oil-filled, high voltage power transformer is to be cooled by natural convection. The transformer containment comprises a steel tank measuring 600mm long, 500mm wide and 800mm high with fins on the vertical surfaces as shown below.



Oil-filled Transformer Containment

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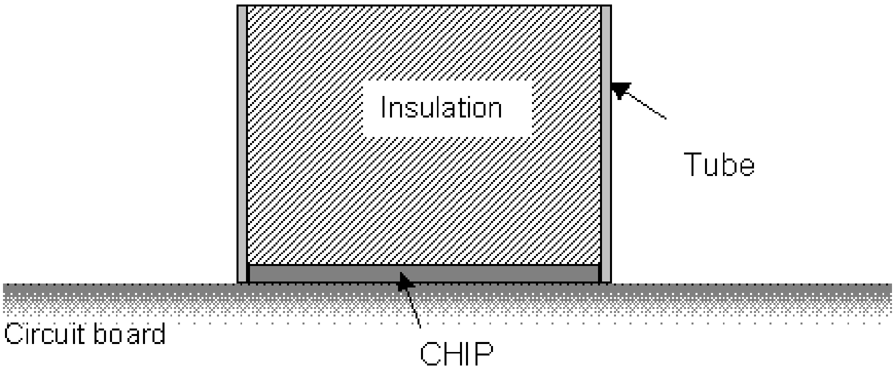
e) Draw an electrical analogy for the flow of heat from the hot oil inside the tank to the ambient air.
f) If the walls of the tank are at 40°C and the air temperature T_∞ = 20°C, calculate the rate of heat loss from the fins.
g) What is the efficiency and thermal resistance of a single fin?
h) Determine the heat loss from the plain (unfinned) area of the tank walls.

6. A chip, circular in plan view is held in a special heat sink arrangement as shown below and dissipates 0.3W. A thin metal tube of height 16mm, inside diameter 12mm, thickness 0.3mm and k=400W/mK is connected to the chip with negligible contact

resistance. The inside of the tube is filled with foam insulation of negligible thermal conductivity. The heat transfer coefficient on all external surfaces is $55 W/m^2K$ and the surrounding air temperature is 22°C. Assuming heat transfer to the circuit board is negligible, a) what is the chip temperature in this cooling arrangement? b) what would the chip temperature be if it was not connected to the tube/insulation?



(based on a tutorial problem in Incropera and De Witt)