

3. Silicon Microdosimeter Requirements and Shape Analysis

This chapter analyses the requirements for the microdosimeter. Applications are presented followed by an analysis of the sources of fluctuations in microdosimetry. An extensive analysis of detector shape considerations is presented. Finally the microdosimeter requirements are summarised.

3.1 Intended Applications

Requirements are dictated by the intended applications. The use of a silicon based detector immediately places constraints on the range of applications for which such a device may be suitable. In particular, the interaction of radiation and the generation of secondary charged particles will differ in a given volume of silicon versus a tissue volume. In general, the silicon device will only be appropriate in radiation environments for which the dominant secondary charged particles produced in tissue have a range that is much longer than the mean chord length of the detector volume. That is, most events within the tissue volume must be crossers or stoppers as opposed to starters or insiders. The crossers or stoppers are generated by a tissue-equivalent converter on top of the detector. The definition of crossers, stoppers, starters and insiders is shown in Figure 3.1.

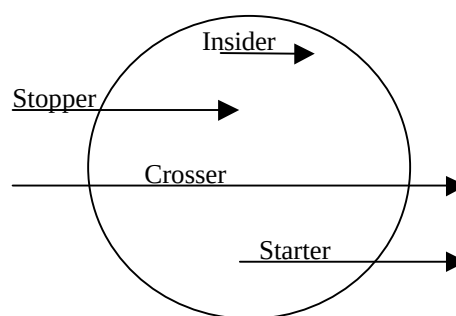


Figure 3.1. Definition of rays interacting with a sensitive volume

The extent to which the range of the secondary particles must exceed the dimensions of the detector volume is dependent on the application and the accuracy of the desired measurement. Typical ranges of recoil products in various radiation fields are shown in Table 3.1. In radiation protection, microdosimetric spectra may be used to determine the effective dose equivalent. Large uncertainties exist in the calculation method, which

relaxes the demands on high measurement accuracy. Measurement uncertainties of less than 30% are acceptable for radiation protection applications (p84, p251 [1]). Conversely, absorbed dose measurements in radiotherapy have a requirement for better than 5% accuracy due to the proximity of the curves relating tumor control and normal tissue complications to absorbed dose [98].

Table 3.1. Typical ranges of recoil products in various radiation fields

Environment	Typical Products	Range/ μm (Note2)
Fast Neutron (14 MeV), (Note1)	7 MeV protons	615
	3.5 MeV alpha	21.2
	0.6 MeV oxygen	1.6
	1.6 MeV carbon	3.3
BNCT	1.47 MeV alpha	7.7
	0.84 MeV lithium	4.1
	0.59 MeV nitrogen	10.5
	0.04 MeV carbon	0.2

Note 1: Calculated by integrating initial spectrum of charge particles from Caswell and Coyne.[125]

Note 2: Range data from SRIM [126] for ICRU striated muscle.

The main applications discussed in this work are several high LET radiation oncology modalities including proton therapy, fast neutron therapy and boron neutron capture therapy. For each of these applications, we should consider the errors introduced by using the silicon/converter structure as opposed to the traditional spherical proportional chamber. The errors due to geometrical considerations need to be compared to other sources of variation in the measurement to gauge their significance. Therefore, an analysis of the sources of fluctuations in regional microdosimetry is required.

3.2 Sources of Fluctuations in Regional Microdosimetry

The shape of the pulse height distribution obtained by a microdosimeter is influenced by several random processes. Analysis by Kellerer [127] and Rossi [1] uses the concept of relative variance to quantify the relative significance of various processes. The relative variance (V) of a probability distribution, $f(x)$ is a measure of the width of the distribution and is defined as:

$$V = \frac{s^2}{m_1^2} = \frac{m_2}{m_1^2} - 1 \quad (3.1)$$

where s^2 is the variance and m_1 and m_2 are the first and second moments of $f(x)$.

The total relative variance (V_T) of the measured pulse height distribution is the sum of the relative variance of several statistical factors each characterized by a probability distribution. For single event spectrum, the following main distributions are evident (with relative variance symbol in brackets):

- i) The distribution of *path lengths* traversed through the site. (V_t)
- ii) The distribution of *particle LET* .(V_L)
- iii) The distribution of the *number of collisions*.(V_c)
- iv) The distribution of *energy imparted in individual collisions*.(V_e)
- v) The distribution of the *fraction of energy retained in the site* (not escaping as delta radiation). (V_d)
- vi) The distribution of the *number of electron/hole pairs* (or ions) formed by particles of the same energy (the Fano factor). (V_f)
- vii) The distribution due to *measurement electronic noise and other measurement phenomena*. (V_m)

The most common geometry for microdosimetric measurements is a spherical volume using a proportional counter. This is the only geometry with an isotropic response. Furthermore, the relative variance due to the varying path lengths (V_t) is conjectured to be the minimum for any geometry (although this is unproven) [128]. Integrating the chord length distribution, given in Table 3.2, yields a value of 1/8 for V_t under the condition of large track lengths. As the particle range approaches the site diameter V_t drops and approaches zero for very short tracks. For other shapes with non-isotropic response, such as our proposed silicon detector with a rectangular parallelepiped (RPP) geometry, we need to consider the influence of shape on V_t . Rossi (p 80 [1]) notes that frequently $V_t \ll V_T$ so that non-spherical counters may be used without significantly affecting the measurement. A detailed discussion of detector geometry influences on spectrum shape, microdosimetric quantities and relative variance is provided in the next section.

Most microdosimetric applications involve multi-particle complex radiation fields with varying LET. The relative LET variance (V_L) varies considerably from about 0.3 for Co

γ radiation to greater than 0.8 for high energy ($>2\text{MeV}$) neutrons [1]. It is a significant contribution to the total variance in neutron measurements.

Factors (iii-v) are three aspects of range and energy straggling. They are best characterized by experimental measurements with monoenergetic sources since they are very geometry and radiation type dependent. Monte-Carlo simulations such as SRIM [126] are also useful for quantify straggling effects.

Factors (vi and vii) are features of the particular measurement system used. The number of electron-hole pairs formed in a semiconductor for a given energy deposited is subject to variation. Not all of the energy deposited in a semiconductor (or gas) is used to break covalent bonds. If the partitioning of energy between phonon production (lattice vibration) and bond breaking were completely uncorrelated the process would be described by Poisson statistics. However, some correlation exists and Fano [43] introduced an empirical correction (F) to the Poisson relation to give the experimentally observed variance.

$$V_f = \frac{F}{n} \quad (3.2)$$

where F = Fano factor and n is the average number of electron-hole pairs. F is usually around 0.1 for Silicon [23] and the minimum n is around 1000 (for silicon detectors at the electronic noise level). Therefore, V_f is negligible in comparison to other sources of variation in a microdosimeter.

The electronic noise for a semiconductor microdosimeter is dominated by the preamplifier electronics, which typically have noise rms levels (s_{pa}) of around 5 keV.

$$V_m = \frac{s_{pa}^2}{E_m^2} \quad (3.3)$$

Thus for even moderate mean energies (E_m), V_m is negligible in comparison to other sources. Electronic noise will be discussed in detail in section 4.5 since it is particularly important for determining the low energy threshold of the system.

The total relative variance of a microdosimetric spectrum is [1]:

$$\begin{aligned} V_T &= V_L + V_t + V_s + V_f + V_m \\ &\approx V_L + V_t + V_s \end{aligned} \quad (3.4)$$

where V_s is the total relative variance due to straggling and we assume that measurement variations (V_f and V_m) are negligible. Ideally in a microdosimeter, one would like to minimize the variation due to path length variations (V_t). This is particularly important for a non-isotropic radiation incident on a non-spherical detector. Therefore, a thorough analysis of the importance of detector shape and path length variations follows.

3.3 Detector Shape Considerations

Usually the intent of experimental microdosimetry is to model the energy deposited in volumes that are similar to critical tissue components such as cells or cell nuclei. Such components are frequently modeled as oblate spheroids or more simplistically as spheres. A spheroid is an ellipsoid with two axis of equal length and it is oblate if the third axis is smaller than the common axis length. The cells are usually randomly orientated unless they are in a flattened state characteristic of a tissue culture. Thus, even with a directional radiation source, the average response is that of a single cell in an isotropic field.

Traditionally, microdosimeters using proportional counters have employed a spherical or cylindrical counter. The spherical counter is preferred for two main reasons; the sphere is the only volume with an isotropic response and it has the lowest known relative variance. Cylindrical counters have evolved due to their simpler electrode design. Such counters do not have an isotropic response and simulation of an isotropic response, by the rotation of the counter under a constant fluence, is generally impractical. Kellerer provides an excellent paper [129] outlining the criteria for the equivalence of spherical and cylindrical proportional counters in microdosimetry. This work will be discussed in detail since it provides a basis for the comparison of other shapes with such counters.

The processing capability of integrated circuit technology is based on planar lithographic processes. Thus, structures with plane surfaces such as rectangular parallelepipeds (RPP) and, less readily, cylinders or hemispheres are manufacturable. In this section, chord and segment length distributions are analyzed for several shapes. Criteria for the equivalence of RPPs (in particular cubes) with spheres are discussed. Finally we compare lineal energy spectra produced by various shapes and include the effects of straggling on such spectra. The overall aim of the study is to obtain optimal

design criteria for the geometry of a silicon based microdosimeter. It is important to understand the conditions under which a planar geometry detector (particularly RPP) is sufficiently similar in performance to a spherical detector.

3.3.1 Comparison of Chord Length Distributions

In order to compare the microdosimetric spectrum produced by a spherical counter with other shapes such as RPP, infinite slab, hemisphere or cylinder we need to first calculate the chord-length distribution for each shape.

Chord-length distributions (CLD) result when convex bodies are intercepted by random straight lines. Several types of randomness result from the intersection of straight lines and convex bodies. The main types considered in the literature (originally defined by Coleman [130, 131] with revisions by Kellerer [132]) include *m-randomness* or *isotropic uniform randomness* in which the body is exposed to a uniform isotropic field of infinite straight lines; *n-randomness* or *weighted randomness* in which the ray passes in a uniformly random direction through an independently uniformly random point in the interior of the body (sometimes also called *I-randomness* or interior randomness); and *l-randomness* or *two point randomness* in which the ray passes through two points chosen independently and uniformly in the interior of the particle. Typically *m-randomness* conditions are used for comparing microdosimetric detectors since they most closely resemble microdosimetric experimental situations.

A review of the basic concepts of geometrical probability required in the calculation of chord length distributions may be found in Kendall and Moran [133]. There have been several attempts to derive analytical expressions for CLDs in various geometrical shapes. Simple analytical expressions of the CLD for the sphere and the infinite slab have been calculated [133, 134]. Kellerer [128] derived the expression for general cylinders and Mader [128, 135] presented numerical solutions for these expressions. Kellerer [132] also derived the formulae for prolate and oblate spheroids. The CLD for a hemisphere was derived by Langworthy [136]. Coleman [131] and Bradford [137] have derived solutions for RPPs based on rectangular CLDs calculated by Coleman [130, 138] and a formalism developed by Kellerer [128] to convert two-dimensional distributions into three-dimensional expressions. Note, that Bradford's work contains typographical errors with corrections presented by Ziegler [139]. Borak [140] recently presented a Monte-Carlo method for computing μ -randomness distributions. Appendix

B contains a Mathematica [141] based notebook which calculates all the CLDs and other calculations relevant to the shape analysis. Each of the distributions ($f(s)$) in the notebook was verified by calculating the mean chord length ($MCL=\bar{l}$) via integration techniques and comparing with Cauchy's equation as follows.

$$\bar{l} = \frac{4V}{S} = \int_0^{\infty} s f(s) ds \quad (3.5)$$

where V = volume and S = surface area

Table 3.2 summarizes some of the main CLDs. Equations provided in the notebook are correct (verified using the above method), however, almost all the papers cited contain typographical equation errors.

Table 3.2. Summary of chord length distributions for various shapes

Shape	Chord length distribution, $f(s)$
Sphere [132]	$\frac{2s}{d^2} \quad 0 < s < d, d = \text{diameter}$
Spheroid [132]	$\frac{2s}{c_1 d^2} \left[c_2 + \frac{\sqrt{1-e^2}}{4(e^2-1)} \left\{ \sqrt{\frac{d^2}{s^2}-1} \left(\frac{d^3}{s^3} + \frac{3d}{2s} \right) + \frac{3}{2} ci\left(\frac{d}{s}\right) \right\} \right]$ $ci(x) = \begin{cases} \cos^{-1}(x) & \text{for } 0 \leq x \leq 1 \\ \cosh^{-1}(x) & \text{for } x > 1 \end{cases}$ $c_1 = \frac{1}{2} + \frac{e^2}{2\sqrt{1-e^2}} ci\left(\frac{1}{e}\right), \quad c_2 = \frac{1}{4e^2} + \frac{3}{4} c_1$ $d = \text{diameter}, e = \text{elongation}$
Hemisphere [136]	$\frac{2s}{3p^2} H(r-s) + \frac{1}{pr} c_1 + \frac{2r}{3ps^2} \left(c_1 + 2\frac{r}{s} \left(\cos^{-1}\left(\frac{s}{2r}\right) - \frac{p}{2} H(r-s) \right) \right)$ $c_1 = \sqrt{1 - \frac{s^2}{4r^2}}$ $r = \text{radius}, H = \text{step function}$
Cylinder	Very lengthy (see Mader [135], Kellerer [132] and Appendix B)
Infinite Slab [132]	$\begin{cases} 0 & \text{for } s \leq h \\ \frac{2h^2}{s^3} & \text{for } s > h \end{cases}$ $h = \text{thickness}$
Cube [130]	$\begin{aligned} &8/(3ap) && x = 0 \\ &k(8x^3 - 3x^4) && 0 < x \leq 1 \\ &k(6p + 6x^4 - 1 - 8(2x^2 + 1)\sqrt{x^2 - 1}) && 1 < x \leq \sqrt{2} \\ &k(6p - 3x^4 - 5 + 8(x^2 + 1)\sqrt{x^2 - 2} - 24Tan^{-1}(\sqrt{x^2 - 2})) && \sqrt{2} < x \leq \sqrt{3} \\ &0 && x > \sqrt{3} \end{aligned}$ $\text{where } x = s/a, a = \text{side length and } k=1/(3apx^3)$
RPP	Very lengthy(see Bradford [137] corrections Ziegler [139] Kellerer [132], AppendixB)

Figure 3.2 to Figure 3.6 shows the CLD under μ -randomness for various shapes with an equal mean chord length ($2/3$) for all shapes. The mean chord length and relative variance for each of the shapes is given in Table 3.3. The relative variance is higher than the sphere relative variance for all shapes. Figure 3.2 illustrates that the prolate spheroid with a lower relative variance is a better approximation to the sphere than the oblate spheroid. Langworthy [136] identified some interesting similarities between the chord length distribution for an oblate spheroid, with $d=2r_{hemi}$ and elongation=0.5, and the hemisphere as shown in Figure 3.6 with the shapes illustrated in Figure 3.7. The distributions are remarkably close for $s > r_{hemi}$. For $s < r_{hemi}$ the functions are quite linear although the hemisphere exhibits a non-zero termination at $s=0$. Langworthy conjectures that all figures with dihedral corners such as cylinders, RPPs and hemispheres have a non-zero density at $s=0$. Furthermore, the oblate spheroid ($e=0.5$) and hemisphere have identical CLDs for unidirectional radiation parallel to the primary axis. This is evident on examination of Figure 3.7 by a simple geometrical argument. If we align at their midpoints chords in the hemisphere which are parallel to the x-axis then we obtain the spheroid shape. The importance of these results is that the hemisphere is a good approximation for modeling oblate spheroids ($e=0.5$) which in turn may be used to model a cell nucleus or cell. It is conceivable that a hemisphere could be constructed using integrated circuit (IC) processing methods. Alternatively, it may be possible to imitate the behavior of a hemisphere by using a number of different thickness RPP volumes connected electrically in parallel. This idea, however, will not be investigated in this work.

Figure 3.3 to Figure 3.5 indicate that the cube and cylinder with equal height and width have a CLD with a closer approximation to a sphere than shapes with non-unity elongation. In both the cylinder and RPP as we increase elongation the CLD broadens as the maximum chord length increases and it appears that the coarse approximation to a sphere significantly worsens.

The oblate cylinder or RPP is more amenable to IC processing and minimizes variance for directional radiation from a tissue equivalent converter located above the device. Figure 3.8 indicates the relative variance for various oblate shapes as a function of $1/\text{elongation}$. For $\text{elongation} > 0.2$ the spheroid offers the minimum variance. The RPP always has a higher relative variance than a cylinder of equivalent elongation, although

not significantly. Relative variance increases as the elongation differs from 1 due to a higher maximum chord length increasing the spread of possible chord lengths.

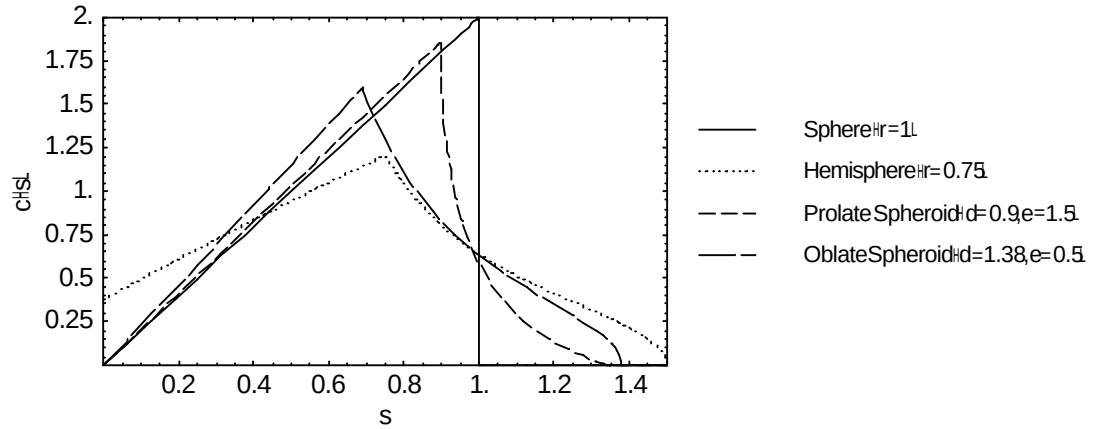


Figure 3.2. Comparison of CLD under μ -randomness for a sphere, hemisphere and spheroid. $\bar{l} = 2/3$ in all cases.

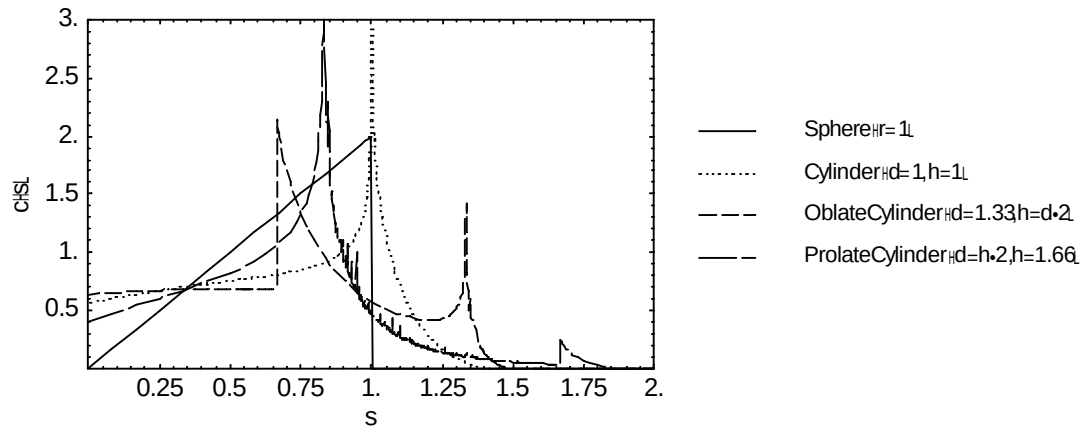


Figure 3.3. Comparison of CLD under μ -randomness for a sphere various cylinders. $\bar{l} = 2/3$ in all cases. Numerical integration errors create small spikes in the cylinder CLDs.

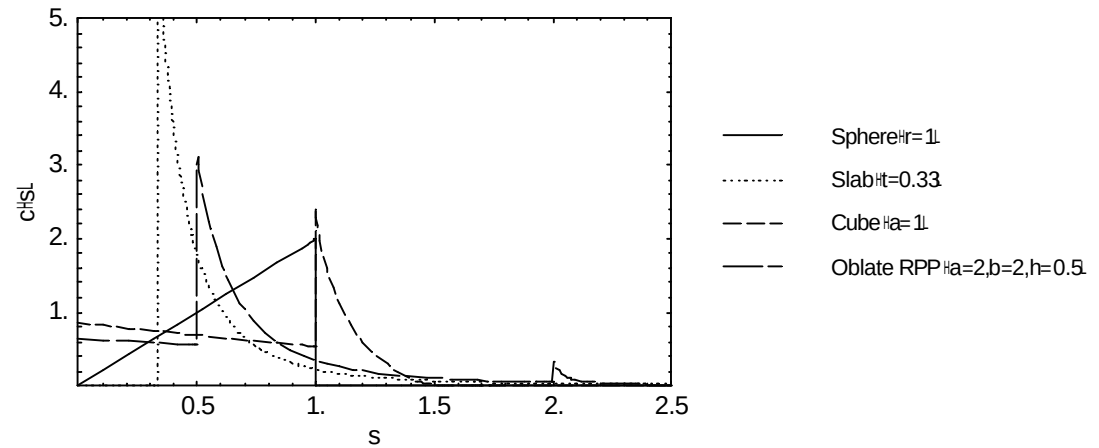


Figure 3.4. Comparison of CLD under μ -randomness for a sphere, infinite slab and RPP. $\bar{l} = 2/3$ in all cases.

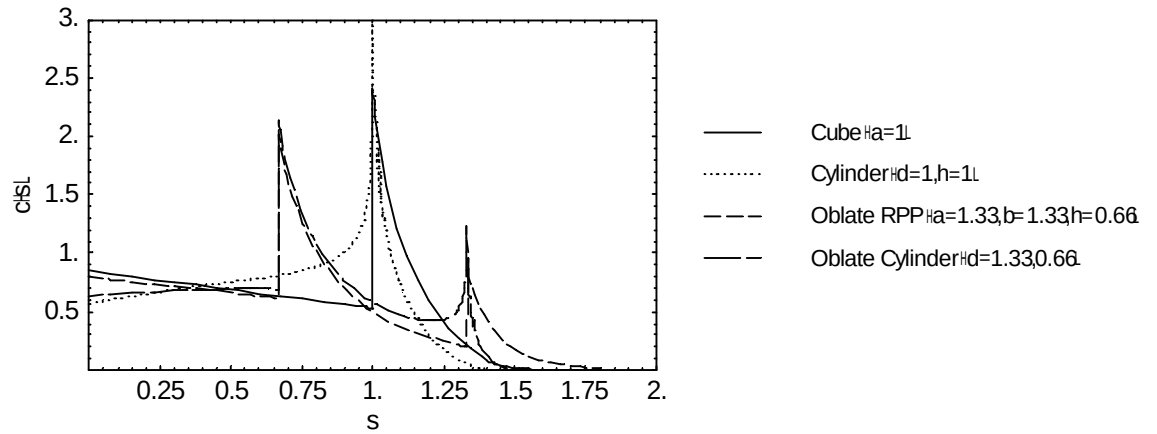


Figure 3.5. Comparison of CLD under μ -randomness for a cube, cylinder and oblate cylinder and oblate RPP. $\bar{l} = 2/3$ in all cases.

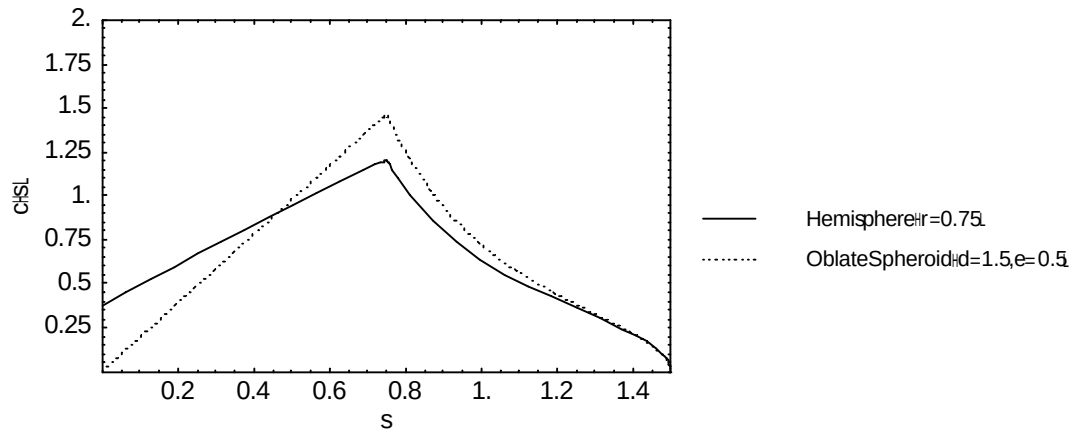


Figure 3.6. Comparison of CLD under μ -randomness for a hemisphere and oblate spheroid. Volume is the same in both cases (0.88). $\bar{l} = 0.66$ for hemisphere and 0.72 for spheroid.

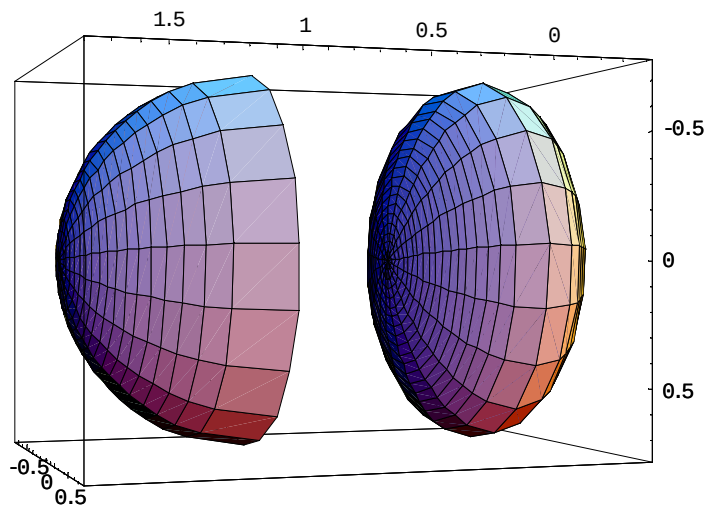
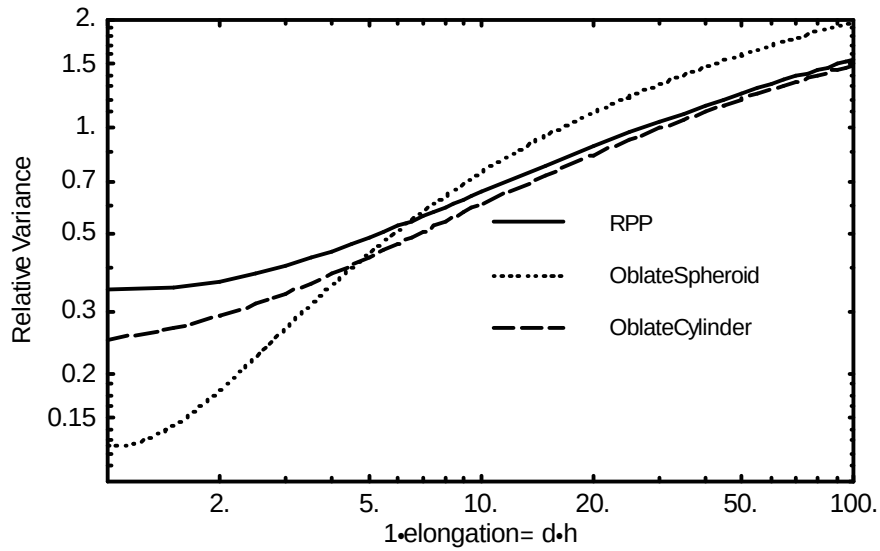


Figure 3.7. Hemisphere ($r=0.75$) and oblate spheroid ($d=1.5, e=0.5$) with CLDs shown in previous figure. These shapes provide identical CLDs for unidirectional radiation along the x-axis

Table 3.3. Summary of mean chord length and relative variance for various shapes

Shape	Mean Chord Length, $\bar{l}$	Relative Variance, V_t
Sphere	$\frac{2d}{3}$, d = diameter	0.125
Spheroid	$\frac{8de\sqrt{1-e^2}}{6\sqrt{1-e^2} + 3e^2 \text{Log}\left(\frac{1+\sqrt{1-e^2}}{1-\sqrt{1-e^2}}\right)}$ e = elongation, d = main diameter	See Figure 3.8
Hemisphere	$\frac{8r}{9}$, r = radius	0.266
Cylinder	$\frac{2dh}{d+2h}$, d = diameter, h = height	See Figure 3.8
Infinite Slab	$2h$, h = height	Does not exist [135]
Cube	$\frac{2a}{3}$, a = side length	0.345
RPP	$\frac{2abh}{bh+ab+ah}$, h = height, a, b = side lengths	See Figure 3.8

**Figure 3.8. Relative variance V_t of the chord length distribution as a function of elongation for oblate shapes**

3.3.2 Comparison of Segment Length Distributions and finite range effects

3.3.2.1 Segment length distributions for RPP, cube and sphere under μ -randomness

The previous section analyzed chord length distributions assuming μ -randomness with infinite range rays. Segment length distributions (SLD) are generated under similar conditions of randomness but with finite range rays. We can thus see the effect of finite range particles on the microdosimetric spectrum by considering SLDs. This is particularly important when our detector construction relies on a converter lying above a silicon volume.

Kellerer [128] provides an extremely useful expression for calculating SLDs given the CLD. The essential equations are repeated here due to their importance. Sum distributions are given in upper case and probability density distributions are in lower case. Assume that a convex body K has the sum distribution $F(s)$ of chord length under μ -randomness and that it is exposed to an isotropic field of straight random tracks whose length u has the sum distribution $R(u)$. Note that the sum distributions are equal to the probability of exceeding a given value s . The resulting sum distribution of segment length in K is:

$$P(s) = k \left(F(s) \int_s^\infty R(x) dx + R(s) \int_s^\infty F(x) dx \right) \quad (3.6)$$

where $k = 1/(\bar{u} + \bar{l})$ and $\bar{u}$ = mean range, $\bar{l}$ = MCL

In the special case where all tracks are of fixed length, u the formula reduces to

$$P(s) = \frac{1}{\bar{u} + \bar{l}} \left((u - s)F(s) + \int_s^\infty F(x) dx \right) \quad s < u \quad (3.7)$$

The mean segment length is given by Kellerer [128] as

$$\bar{s} = 1/(\bar{u}^{-1} + \bar{l}^{-1}) \quad (3.8)$$

Differentiating equation (3.6) we obtain the segment length probability density, $p(s)$,

$$p(s) = \frac{1}{\bar{u} + \bar{l}} \left(f(s) \int_s^\infty R(x) dx + r(s) \int_s^\infty F(x) dx + 2F(s)R(s) \right) \quad (3.9)$$

and in the special case of a fixed range u ,

$$p(s) = \frac{1}{\bar{u} + \bar{l}} \left((u-s)f(s) + d(s-u) \int_u^\infty F(x)dx + 2F(s) \right) \quad s < u \quad (3.10)$$

Kellerer categorizes the three terms in equation (3.10) as distributions associated with crossers, insiders and starters+stoppers. If we split the sum segment distribution $P(s)$ given by equation (3.7) into these categories, we obtain

$$P(s) = \frac{1}{\bar{u} + \bar{l}} \left(\begin{aligned} &F(s) \int_s^\infty R(x)dx - \int_s^\infty F(x)R(x)dx && \text{crossers} \\ &+ R(s) \int_s^\infty F(x)dx - \int_s^\infty F(x)R(x)dx && \text{insiders} \\ &+ 2 \int_s^\infty F(x)R(x)dx && \text{starters + stoppers} \end{aligned} \right) \quad (3.11)$$

The SLD, $p(s)$, in the case of a fixed range u , has been calculated for the sphere, cube, and RPP as given in Appendix B. The sphere is a tractable integration whilst the RPP requires numerical integration methods. The effect of varying the fixed range on the distribution $p(s)$ is shown for the sphere and RPP in Figure 3.9 and Figure 3.10, respectively. Clearly, the distribution shape changes significantly as u approaches the dimensions of K . The relative variance of the distributions is a maximum when the range u corresponds closely to the main dimension of the volume as shown in Figure 3.11. As will be seen in the following analysis, this is near the peak in the fraction of stoppers and starters.

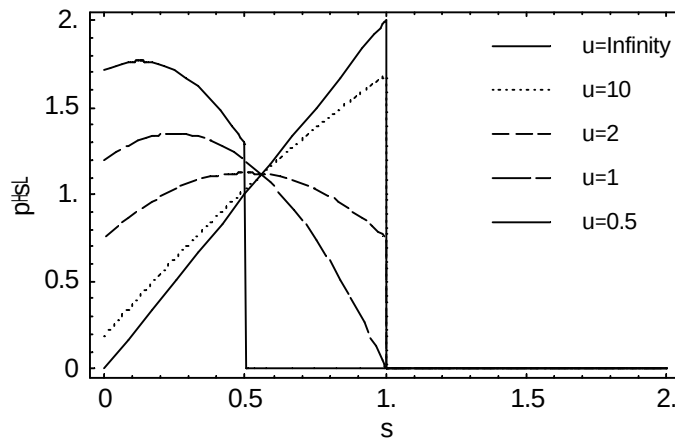


Figure 3.9. Effect of varying range (u) on segment length distribution for a sphere ($r=1$)

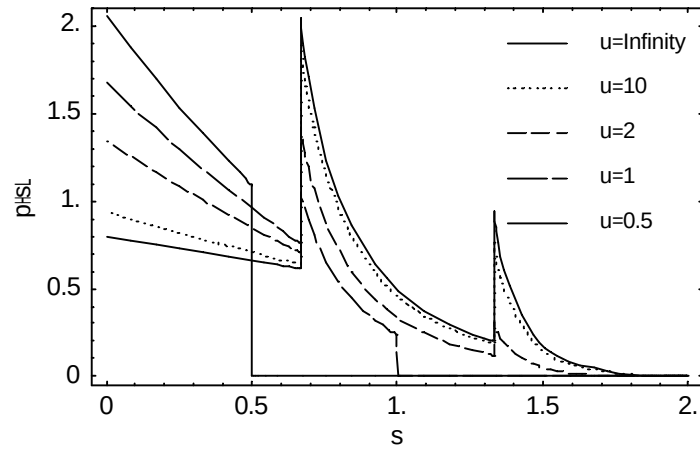


Figure 3.10. Effect of varying range (u) on segment length distribution for a RPP ($a=b=1.33, h=0.66$)

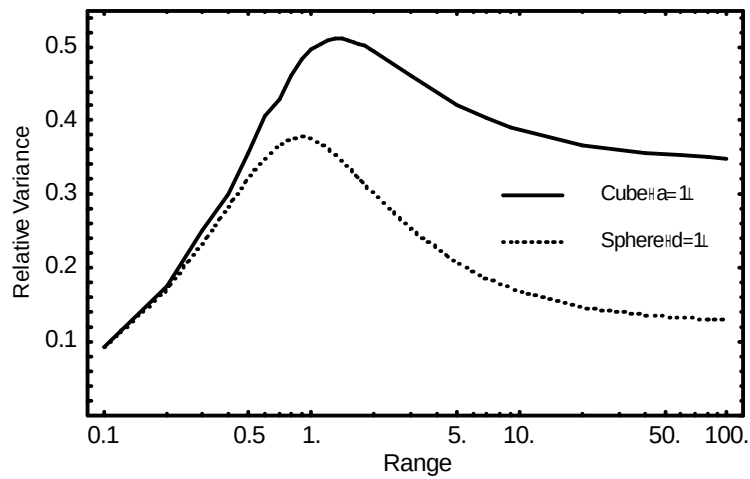


Figure 3.11. Relative variance of the segment length distribution as a function of particle fixed range for a sphere and cube

As noted previously, the use of a silicon structure requires that a high proportion of the chords are crossers or stoppers since we do not want to consider chords generated within the non-tissue equivalent silicon structure. It is useful to determine an expression for the fraction of chords that are generated from outside of K (i.e. stoppers and crossers). The equations for the fraction of crossers (b_{cross}) and stoppers (b_{stop}) are given by the first and third term of equation (3.11) at $s=0$.

$$b_{cross} = \frac{1}{\bar{u} + \bar{l}} \left(\bar{u} - \int_0^\infty F(x)R(x)dx \right) \quad (3.12)$$

$$b_{stop} = \frac{1}{\bar{u} + \bar{l}} \left(\int_0^\infty F(x)R(x)dx \right) \quad (3.13)$$

Note that we make use of the fact that $\bar{l} = \int_0^\infty F(s)ds$ and $\bar{u} = \int_0^\infty R(x)dx$.

If u is always greater than the maximum chord length ($u_{\min} \geq s_{\max}$), in which case there are no insiders and the sum range distribution $R(x)=1$ $x \leq s_{\max}$, then we may simplify equations (3.12) and (3.13) to

$$b_{\text{cross}} = \frac{\bar{u} - \bar{l}}{\bar{u} + \bar{l}} \quad u_{\min} \geq s_{\max} \quad (3.14)$$

$$b_{\text{stop}} = \frac{\bar{l}}{\bar{u} + \bar{l}} \quad u_{\min} \geq s_{\max} \quad (3.15)$$

The summation of the fraction of stoppers and crossers to give the fraction of segments from outside K ($b_{\text{start_outside}}$) requires no range assumptions to obtain a simple form. From equations (3.8), (3.12) and (3.13) we obtain:

$$\begin{aligned} b_{\text{start_outside}} &= b_{\text{cross}} + b_{\text{stop}} \\ &= \frac{\bar{u}}{\bar{u} + \bar{l}} = \frac{\bar{s}}{\bar{l}} \end{aligned} \quad (3.16)$$

Solving equation (3.16) for $a = \frac{\bar{u}}{\bar{l}}$, the ratio of mean range to mean chord length, gives

$$a = \frac{b_{\text{start_outside}}}{1 - b_{\text{start_outside}}} \quad (3.17)$$

Note that these previously unreported equations are independent of shape for objects of the same MCL. Examples of equation (3.12) and (3.13) applied to a cube, RPP and sphere of equal MCL are shown in Figure 3.12 and Figure 3.13. As u decreases the proportion of crossers continuously drops whilst the proportion of insiders steadily rises. The proportion of stoppers and starters peaks at around the mean chord length. At range values greater than the maximum chord length, we observe identical crosser and stopper fractions for each shape reflecting the expected shape independence. The fraction which start outside (crossers + stoppers) is shown in Figure 3.14 with complete shape independence for all values of u .

Figure 3.15 shows the ratio of $a = \text{Range/MCL}(\bar{l})$ required for a given fraction of start-outsiders. If we require a minimum of 80% of segments from outside of the detection volume K then the minimum allowable mean particle range is $4\bar{l}$. This provides a useful guideline for using a RPP volume with a converter structure for which we expect segments exclusively from outside of the RPP.

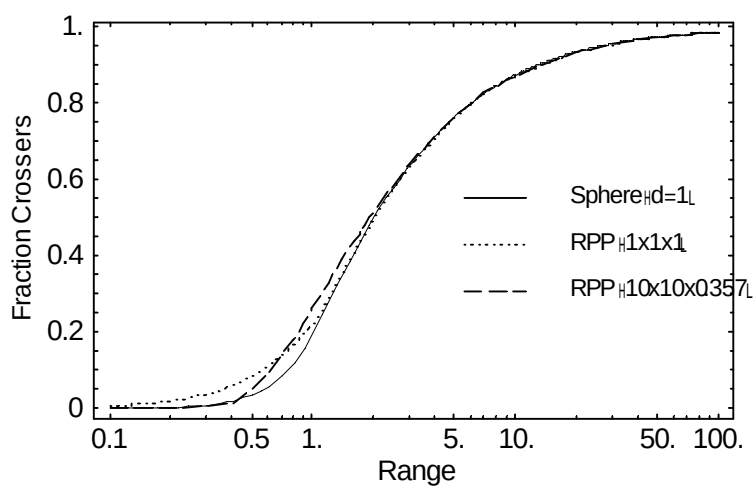


Figure 3.12. Fraction of crossers as a function of range for three different shapes with the same MCL

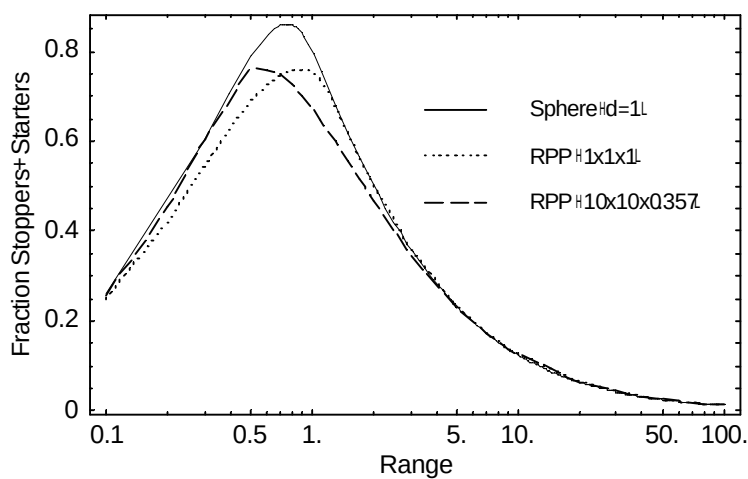


Figure 3.13. Fraction of stoppers+starters as a function of range for three different shapes with the same MCL

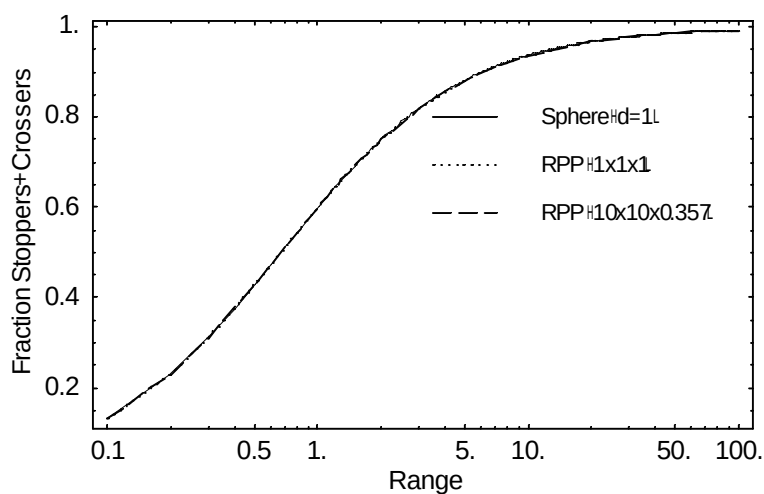


Figure 3.14. Fraction of stoppers+crossers as a function of range for three different shapes with the same MCL

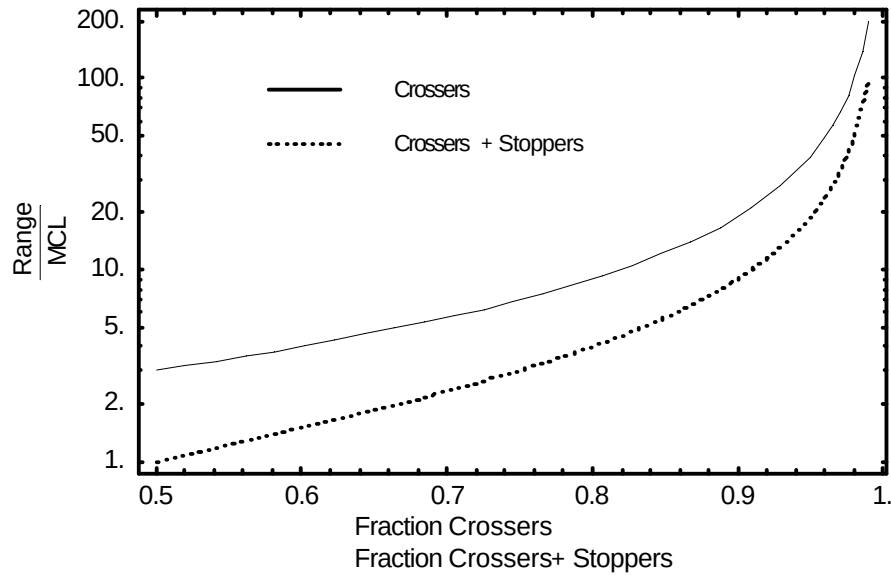


Figure 3.15. Ratio of Range/MCL required for the given fraction of crossers or crossers and stoppers. This function applies for any convex shape.

3.3.2.2 Segment length distributions for infinite slab with converter overlayer

The previously described segment length distributions considered the case of μ -randomness with finite range tracks. Practical implementations of silicon based detectors will initially incorporate a converter which is only above the detector. We define a new type of randomness which will be termed *c-randomness* (for converter) to describe the geometry shown in Figure 3.16 with finite range rays originating from a point uniformly distributed within the converter volume and emanating isotropically. The thickness of the converter and overlayer exceeds the particle range conforming to the typical dosimetry requirement of "dose equilibrium".

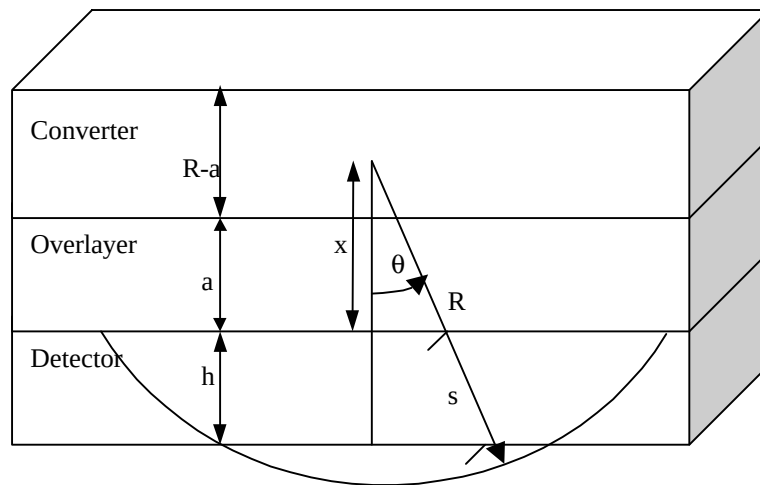


Figure 3.16. Geometry of RPP detector with overlayer (thickness = a) and converter (thickness = h). Infinite slab case has the same cross section with the RPP having infinite length and width.

It is relatively simple to derive an analytic solution for the case of an infinite slab and converter. The RPP case with c-randomness involves complex integrals so it is most simply handled using Monte-Carlo methods. The infinite slab calculation proceeds by considering a point x above the sensitive volume of thickness h with a ray of length (R) emanating isotropically from point x at an angle q with respect to the slab surface normal. The probability $p(q)dq$ of emitting at an angle between q and $q+dq$ is proportional to the partial area of the hemisphere $dA=2\pi r^2 \sin(q)dq$ since the emission is isotropic. Normalising to the area of the hemisphere gives

$$p(q) = \sin(q) \quad (3.18)$$

Simple geometry gives the relationship between the segment length s and q .

For $x < R-h$, we have

$$s = \begin{cases} \frac{h}{\cos(q)}, & 0 \leq q \leq \cos^{-1}\left(\frac{x+h}{R}\right) \\ R - \frac{x}{\cos(q)}, & \cos^{-1}\left(\frac{x+h}{R}\right) \leq q \leq \cos^{-1}\left(\frac{x}{R}\right) \\ 0, & \cos^{-1}\left(\frac{x}{R}\right) \leq q \leq \frac{\pi}{2} \end{cases} \quad (3.19)$$

and for $x \geq R-h$ there are no crossers and we have

$$s = \begin{cases} R - \frac{x}{\cos(q)}, & 0 \leq q \leq \cos^{-1}\left(\frac{x}{R}\right) \\ 0, & \cos^{-1}\left(\frac{x}{R}\right) \leq q \leq \frac{\pi}{2} \end{cases} \quad (3.20)$$

These relationships are shown in Figure 3.17 for the two cases. We can now apply change of variable methods to determine the distribution of segment length.

$$p(s) = p(q) \left| \frac{dq}{ds} \right| \quad (3.21)$$

Care must be taken in applying the above equation since the inverse of $s(q)$ is not unique for $s > h$ as shown in Figure 3.17. We define two regions: A ($h < s \leq hR/(h+R)$) and B ($0 \leq s \leq h$). For region A, we need to add the contribution due to $p(s)$ for both solutions to $q(s)$. Applying equation (3.21) to (3.19) and (3.20) we obtain:

$$\begin{aligned}
 p(s, x) &= \frac{x}{R} & s = 0 \\
 &= \frac{x}{(R-s)^2} & \text{Region B: } 0 < s \leq h \\
 &= \frac{h}{s^2} + \frac{x}{(R-s)^2} & \text{Region A: } h < s \leq \frac{hR}{x+h}
 \end{aligned} \tag{3.22}$$

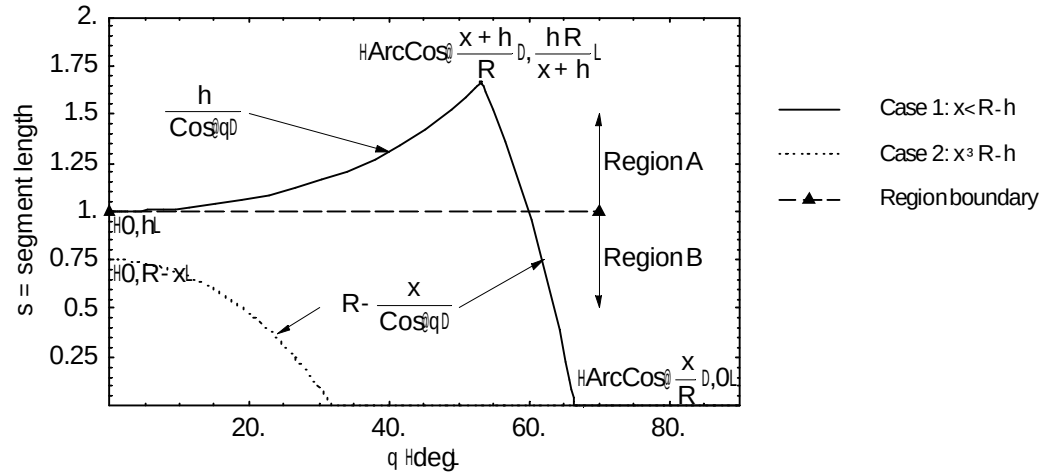


Figure 3.17. Segment length as a function of angle q for the infinite slab and overlayer/converter geometry. Examples given use $R=5$, $h=1$, and $x=2$ (Case 1) and $x=4.25$ (Case 2). In Region A, $s > h$ and two values of q correspond to each s value whilst for region B, $s \leq h$ and the inverse is unique.

Now we sum the probability over all x to remove the x variable and normalize appropriately. Care should be taken in selecting the integral bounds. The range of x is from the start of the converter at $x = a$ to the maximum value of x for each region given the finite range R .

$$\begin{aligned}
 p(s) &= k \int_a^{R-s} p(s, x) dx & \text{Region B: } 0 < s \leq h \\
 &= k \int_a^{\frac{hR}{s}-h} p(s, x) dx & \text{Region A: } h < s \leq \frac{hR}{x+h}
 \end{aligned} \tag{3.23}$$

Evaluating we obtain

$$\begin{aligned}
 p(s) &= k \left(\frac{1}{2} - \frac{a^2}{2(R-s)^2} \right) & \text{Region B: } 0 < s \leq h \\
 &= k \left(\frac{h^2 R}{s^3} - \frac{a^2}{2(R-s)^2} - \frac{h(2a+h)}{2s^2} \right) & \text{Region A: } h < s \leq \frac{hR}{x+h}
 \end{aligned} \tag{3.24}$$

$$k = \frac{2R}{(a-R)^2} \text{ and } 0 \leq a < R$$

An example of this function is shown in Figure 3.18. It is evident that the proportion of large segments for a given range is reduced by increasing the overlayer thickness. This function will be used later in section 3.3.3.

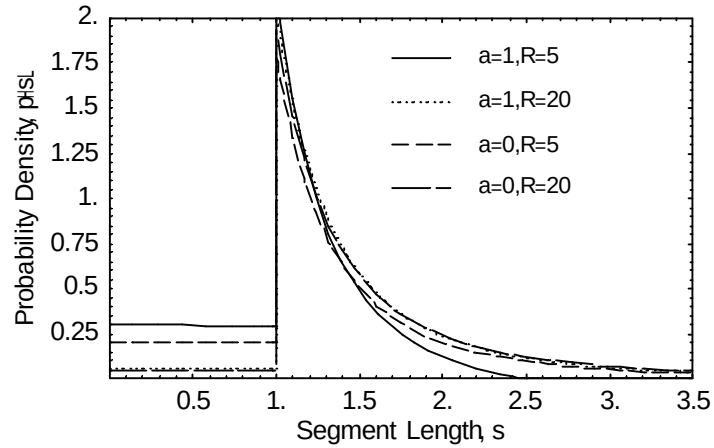


Figure 3.18. Segment length distributions for an infinite slab under c-randomness. Various overlayer thickness (a) and particle range (R) configurations are compared.

3.3.2.3 Segment length distributions for RPP with converter and overlayer

The structure of a RPP separated from a converter volume by an overlayer volume is the simplest geometry for a silicon based detector constructed using integrated circuit technology. The geometry is shown in Figure 3.16 with the tissue equivalent converter volume defining the region in which ionizing particles are generated. The randomness defined by such a construction was termed c-randomness in the previous section. An analytical solution to the segment length distribution for the RPP under c-randomness may be obtained following similar methods to the infinite slab case. However, the calculations are tedious so the Monte-Carlo method is employed. The process begins by randomly selecting points in the converter volume from which rays of a user-defined range are isotropically emitted. The segment lengths for rays intercepting the user defined sensitive volume are calculated and stored in order to form the segment length distribution. A more detailed description of the technique is provided in section 5.4.2. An example comparing the SLD for a unit cube under μ -randomness with the SLD under c-randomness using a ray with a range of 10 is shown in Figure 3.19. The calculations indicate that c-randomness is well approximated by μ -randomness particularly for larger ranges and minimum overlayer thickness. Increasing the overlayer thickness has the effect under c-randomness of reducing the probability of large segment lengths, which occur due to high angle intercepts (with respect to the

overlayer normal). A similar effect occurs for decreasing ranges under μ -randomness. Thus, an approximation to a c-random configuration with an overlayer of thickness a and particle range R is a cube under u-randomness with range $R-a$. An example of such an approximation is shown in Figure 3.19.

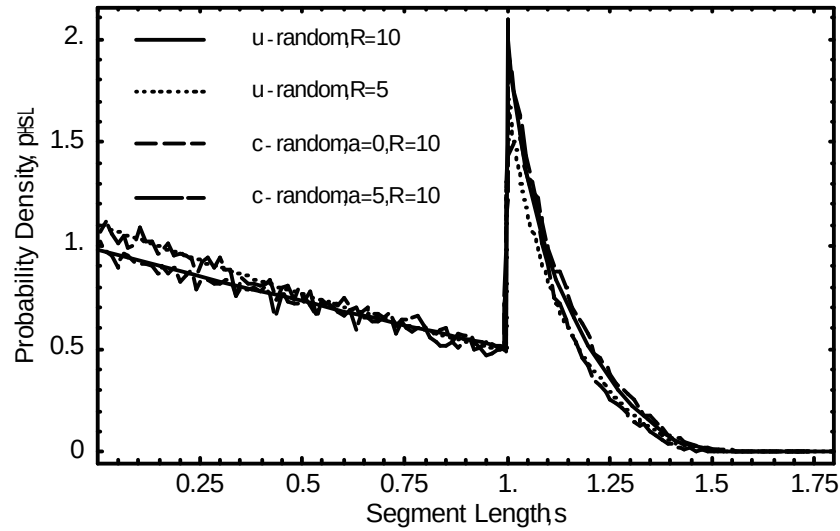


Figure 3.19. Segment length distributions for a unit side length cube under c-randomness for two different overlayer thickness, a . The range R is 10. SLDs calculated assuming μ -randomness are shown for comparison

3.3.3 Equivalence of Shapes using Mean Energy per Event (ϵ_d)

The previous sections detailed calculations of CLD and SLD are useful for comparing microdosimetric $f(y)$ spectra between shapes and predicting the expected number of crossers, stoppers, starters and insiders. In this section, we investigate parametric criteria for the equivalence of shapes.

Kellerer [129] proposed that a particularly suitable parameter for comparing the performance of microdosimetric shapes is the mean energy imparted per event ($\bar{e}_d$). This quantity is closely related to the dose average of lineal energy since

$$y_d = \frac{\bar{e}_d}{\bar{l}} \quad (3.25)$$

Both of these quantities are frequently used in microdosimetric applications since they often relate to the radiobiological effectiveness of the radiation. Moreover, they vary substantially with size and shape of the sensitive volume and radiation quality. From a

microdosimetric viewpoint, if the quantity $\bar{e}_d$ does not vary substantially between two detector shapes then the detectors may be considered largely equivalent.

There are two methods to calculate $\bar{e}_d$; one may integrate the previously calculated chord length distributions or use the proximity function method proposed by Kellerer [129, 142, 143]. Proximity functions applied to a volume are a measure of the probability distributions of the distances between points in a volume and when applied to a track relate to the distribution between energy transfers. The integral proximity function, $S(x)$, of a region S is equal to the expected volume of the region that is contained in a sphere of radius x centered at a random point of S . The differential proximity function is the expected volume of S contained in a spherical shell of radius x and thickness dx centered at a random point of S . Kellerer [142] has shown that $\bar{e}_d$ may be calculated for a uniform isotropic radiation field given the proximity function $t(x)$ of the radiation and the proximity function $s(x)$ of a region.

$$\bar{e}_d = \int_0^{x_{\max}} \frac{s(x)t(x)}{4\pi x^2} dx = \int_0^{x_{\max}} U(x)t(x) dx \quad (3.26)$$

where $U(x)$ is termed the geometric reduction factor. If a spherical shell of radius x is centered at a random point of S , then $U(x)$ is equal to the average fraction of the shell that lies within S . The relation shown above applies rigorously for all radiations and arbitrary volumes and indicates that reference volumes with similar $U(x)$ functions will be largely equivalent for all types of radiations. The advantage of using proximity functions over integrating the chord length distribution is that they are often simpler than the chord length distribution and contain fewer singularities.

3.3.3.1 Equivalence under μ -randomness

The geometric reduction factors for the sphere, cylinder and RPP under μ -randomness have been calculated by Kellerer [143] and are reproduced in Table 3.4 and calculated in Appendix B.

Kellerer assumes that radiation may be approximated by simple infinite straight-line tracks of constant energy transfer rate L . The proximity function is then equal to $t(x)=2L$. An improvement on Kellerer's work is provided by assuming finite range tracks which have a proximity function [142] given by:

Table 3.4. Summary of geometric reduction factor for various shapes

Shape	Geometric Reduction Factor, U(x)
Sphere	$U(x) = 1 - \frac{3x}{2d} + \frac{x^3}{2d^3}, \quad x \leq d, \quad d = \text{diameter}$
Cylinder	$U(x) = \frac{1}{2px} \int_{z_1}^{z_2} \left(1 - \frac{z}{h}\right) \frac{s_c(\sqrt{x^2 - z^2})}{\sqrt{x^2 - z^2}} dz$ $z_1 = \left(\text{Max}(0, x^2 - d^2)\right)^{1/2}; \quad z_2 = \text{Min}(x, h); \quad 0 < x \leq \sqrt{h^2 + d^2}$ For cylinder, $s_c(x) = 4x \left(\text{Cos}^{-1}\left(\frac{x}{d}\right) - \frac{x}{d^2} \sqrt{d^2 - x^2} \right), \quad x \leq d$ $h = \text{height}, d = \text{diameter}$
RPP	As above but with $\text{For RPP, } s_c(x) = 2x \begin{cases} \frac{x^2}{d^2} - \frac{4x}{d} + p & x \leq d \\ p - 2 - 4\text{Cos}^{-1}\left(\frac{d}{x}\right) + 4\sqrt{\frac{x^2}{d^2} - 1} - \frac{x^2}{d^2} & d \leq x \leq \sqrt{2}d \end{cases}$ and $h = \text{height}, d = \text{maximum diameter} = \sqrt{2}d, d = \text{side length}$

$$t(x) = 2L \left(1 - \frac{x}{R}\right), \quad x \leq R \quad (3.27)$$

Using Kellerer's assumption that R is infinite, the mean energy imparted is proportional to the integral over $U(x)$ which is shown to be simply related to be mean chord length $\bar{l}_t$ for I-randomness (random chords through a random point in a convex body).

$$\begin{aligned} \bar{e}_d &= 2L \int_0^{x_{\max}} U(x) dx \\ &= L \bar{l}_t \end{aligned} \quad (3.28)$$

For the case of finite range tracks, $\bar{e}_d$ is obtained using equation (3.27) with the appropriate $U(x)$ applied to equation (3.26). A unit sphere may be considered a reference for the mean energy deposited. For a given range R , it is useful to determine the dimensions of a RPP or cylinder for which the calculated $\bar{e}_d$ is the same as for the reference unit sphere. Kellerer [129] considered first the infinite range case and compared a unit sphere with a cylinder and RPP both of unit elongation. The sphere has $\bar{e}_d = 0.75 L$ whilst for a cube of unit side length, $\bar{e}_d = 0.897 L$. Therefore, the required dimensions for a cube to obtain the same $\bar{e}_d$ as for the sphere is $0.75/0.897 = 0.837$. The results are summarized in Table 3.5.

Table 3.5. Dimensions of a cylinder and cube with the same $\bar{e}_d$ as a unit diameter sphere.
(assuming μ -randomness with infinite range tracks and constant LET)

Shape	$\bar{e}_d$	Required Dimensions
Sphere ($d=1$)	$0.75 L$	$d = 1$
Cylinder ($d=1, h=1$)	$0.838 L$	$d = h = 0.895$
RPP (side length $d=1, h=1$)	$0.897 L$	$d = h = 0.837$

This process may be applied for various elongations' of a cylinder and RPP. Figure 3.20 gives those values of height and side length for a RPP which are associated with the same value of $\bar{e}_d$ as a unit diameter sphere. Table 3.6 lists corresponding numerical values. Figure 3.21 shows the value of $U(x)$ for the sphere, cube, cylinder ($e=1$) and RPP ($e=0.1$). The best approximation to the sphere $U(x)$ is obtained with an elongation of 1 for both the cylinder and RPP. The poor correspondence for the RPP with $e=0.1$ indicates that, although $\bar{e}_d$ is equivalent, the spectral shape and performance with varying range particles will be poor for elongation values significantly different from unity. Conversely, excellent correspondence of the cube with the sphere $U(x)$ is found over all x with the conclusion that a cube of side length $d=0.837$ is very nearly equivalent to a sphere of diameter d . Using a numerical analysis, Kellerer [129] showed that the difference between a cube and sphere $\bar{e}_d$ must always be less than 3.5% provided the function $t(x)$ is monotonically decreasing. The proximity function $t(x)$ is always a decreasing function of x except for small distances x below the microdosimeters resolution. Kellerer also claims that the difference between a cylinder and sphere must always be less than 1.7% under similar conditions for $t(x)$.

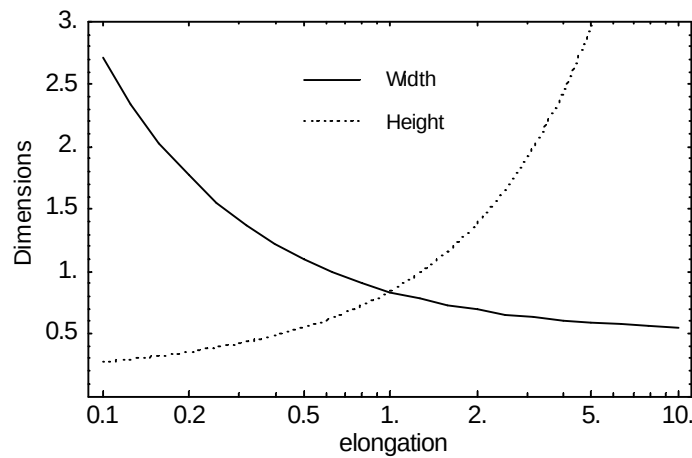


Figure 3.20. Height h and width d of the RPPs that have the same mean energy deposited as the unit diameter sphere (see also Table 3.6). For the specified elongations these RPPs are most closely equivalent to the unit diameter sphere

**Table 3.6. Dimensions of a RPP with the same $\bar{e}_d$ as a unit diameter sphere.
(assuming μ -randomness with infinite range tracks and constant LET)**

Elongation (e)	Height (h)	Diameter (d)	$\frac{V}{V_{sphere}}$	$\frac{S}{S_{sphere}}$	$\frac{\bar{l}_m}{\bar{l}_{m,sphere}}$
1/32	0.194	6.192	14.17	25.94	0.546
1/16	0.234	3.747	6.279	10.06	0.624
1/10	0.272	2.721	3.849	5.658	0.680
1/8	0.294	2.354	3.117	4.413	0.706
1/4	0.389	1.556	1.800	2.313	0.778
1/2	0.549	1.098	1.262	1.534	0.823
1	0.837	0.836	1.118	1.336	0.836
2	1.380	0.690	1.255	1.516	0.828
4	2.434	0.609	1.722	2.122	0.811
8	4.504	0.563	2.727	3.431	0.795
10	5.531	0.553	3.232	4.090	0.790
16	8.599	0.537	4.743	6.067	0.782
32	16.73	0.523	8.740	11.32	0.772

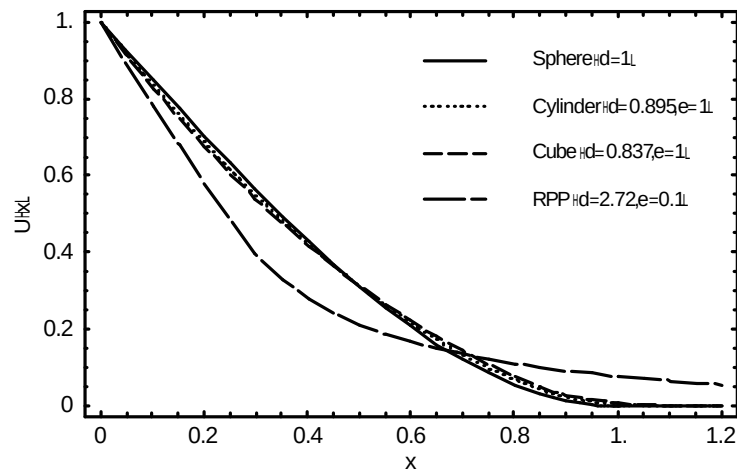


Figure 3.21. The geometric reduction factor $U(x)$ for the unit diameter sphere and for equivalent cylinder, cube and RPP. The largest difference occurs for the elongation that is not unity

The previous discussion determined the optimal geometry for various shapes under the criteria that the mean energy deposited is identical to a unit diameter sphere assuming infinite range particles. If the particle range is finite, the mean energy deposited drops as the proportion of crossers drops. Figure 3.22 indicates that the rate of decrease is almost identical for the sphere and cube with deviations less than 2.1% for all ranges. For the RPP with small elongation ($e=0.1$) the equivalence with a sphere is quite poor (up to 15% difference) for ranges less than 10 as expected given the disparity in the $U(x)$ functions.

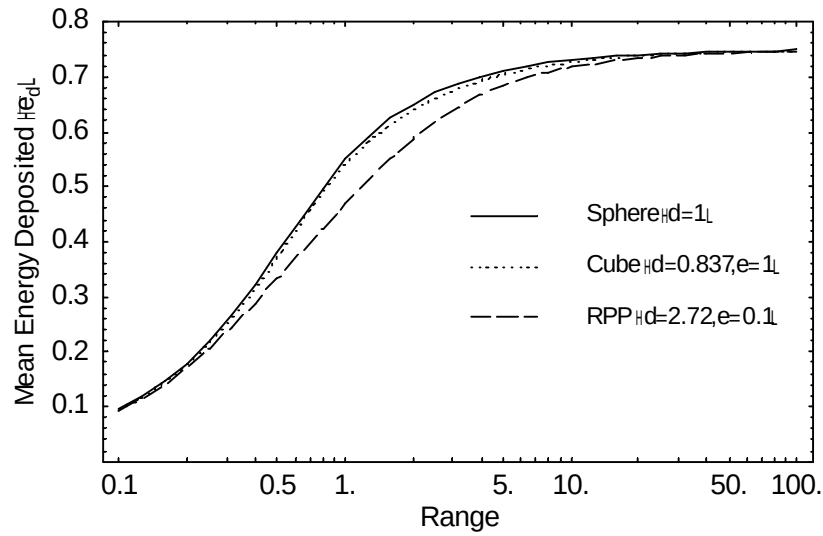


Figure 3.22. Variation of mean energy deposited per event with particle range. The cube closely approximates a sphere for all ranges. The dimensions of each object were selected to obtain equivalent $\bar{e}_d$ assuming an infinite range. The LET L is assumed equal to 1.

3.3.3.2 Equivalence under nonisotropic conditions and c-randomness

It is important to check the equivalence under non-isotropic conditions. Kellerer [143] performs this analysis for a cylinder by considering the extreme case of a unidirectional field perpendicular to the cylinder axis. This orientation is common in experimental applications with the particles then having a preferred direction in passing through the detector. A value of $\bar{e}_d = 0.849 L$ was derived for the unit elongation cylinder under a unidirectional field, which is close to the isotropic case of $\bar{e}_d = 0.838 L$. The conclusion is that the cylinder remains closely equivalent to a sphere even under non-isotropic conditions.

Similarly, the cube or RPP is commonly orientated with the normal to a face plane parallel to the radiation beam. We consider the limiting case of particle tracks that are all normal to the selected face plane. In this case, the proximity function is that of a straight line of length equal to the thickness of the RPP (d) in the beam direction.

$$U(x) = 2 \left(1 - \frac{x}{d} \right) \quad (3.29)$$

For an infinite range the mean energy deposited is simply $\bar{e}_d = d L$ since all chords intersecting the RPP have length d . Under these unidirectional conditions, the side length of a cube that is most nearly equivalent to a unit diameter sphere is $d = 0.75$ compared with a value of $d = 0.837$ for the isotropic case. Thus, a cubic detector

optimized for isotropic conditions will record in a unidirectional beam an $\bar{e}_d$ 11.6% higher than desired for equivalence to a sphere. However, if a cube with $d = 0.75$ is employed in unidirectional beams then the close equivalence with a sphere is preserved for all particle ranges as shown in Figure 3.23.

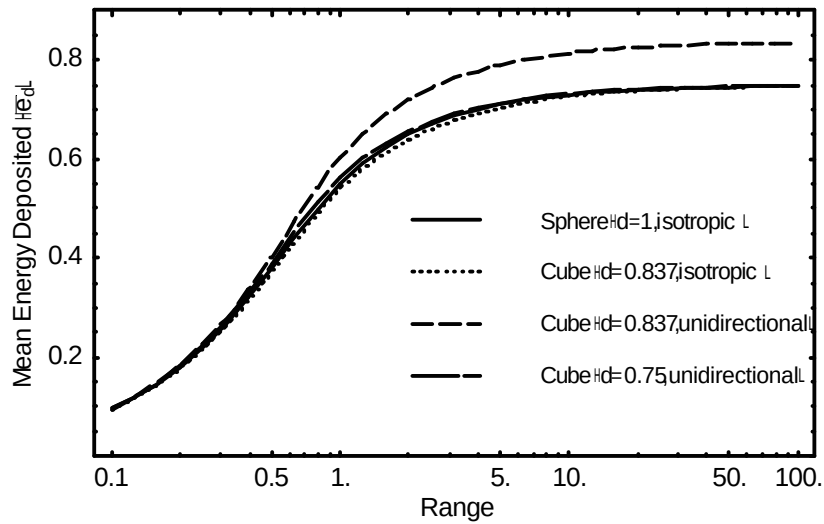


Figure 3.23. Variation of mean energy deposited per event with particle range for isotropic and unidirectional beams. The unidirectional beam is normal to a cube face. The LET L is assumed equal to 1. (See also Figure 3.22)

The use of a converter on top of a RPP under c-randomness is another form of non-isotropic radiation. For the infinite slab case discussed previously in section 3.3.2.2, the mean energy deposited $\bar{e}_d$ may be calculated by integrating equation (3.24) for the SLD $p(s)$ as follows:

$$\bar{e}_d = L \frac{\int_0^\infty s^2 p(s) ds}{\int_0^\infty s p(s) ds} = L \frac{\int_0^\infty s^2 p(s) ds}{\bar{l}} \quad (3.30)$$

For a given range, the slab thickness required to obtain the same $\bar{e}_d$ as a unit diameter sphere may be estimated. The thickness required varies considerably with range and overlayer thickness such that the equivalence between the infinite slab structure and a sphere is quite poor. An example of the large variation in $\bar{e}_d$ with range for an infinite slab structure is shown in Figure 3.24. The slab of height 0.176 and overlayer thickness 0.5 is selected to give a unit sphere equivalent $\bar{e}_d$ at a range of 100. The mean energy increases continuously with range due to the large segment lengths that arise as the

range increases. The large segments are at high angles with respect to the top face normal. RPPs of elongation closer to unity limit this rise and large variation of $\bar{e}_d$ with particle range since the maximum chord length is restricted.

Consider now the case of a cube under c-randomness. The SLD was calculated using Monte-Carlo methods in section 3.3.2.3. Again, the mean energy deposited may be estimated by numerically integrating the SLD as per equation (3.30) assuming for comparative purposes a fixed LET of 1. The number of segments in the calculated distribution was 10000, which leads to an error in $\bar{e}_d$ of around 0.7% (based on comparative studies with known spectra under μ -randomness). The cube under c-randomness with no overlayer closely approximates the sphere for ranges greater than the cube dimensions. When the range becomes close to the sum of the overlayer and detector thickness $\bar{e}_d$ drops faster than for a sphere since the proportion of crossers drops dramatically. The overlayer increases the range above which spherical equivalence is attained. In summary, the $\bar{e}_d$ equivalence of a RPP/overlayer/converter structure to a sphere is best attained by using a minimum sized cube (elongation=1) with no overlayer.

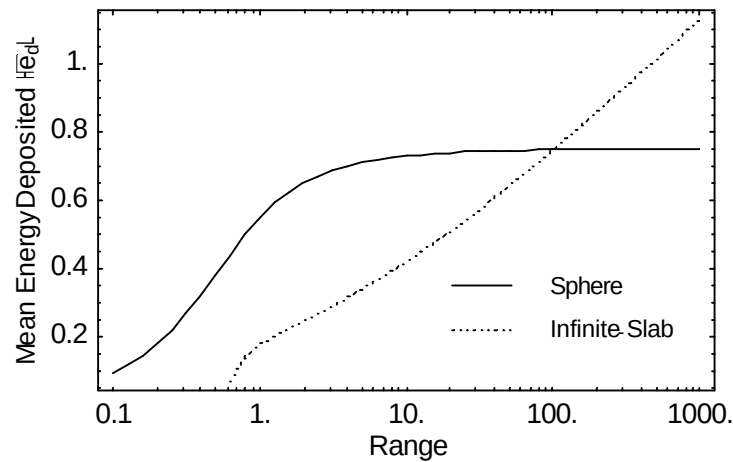


Figure 3.24. Comparison of mean energy deposited per event with particle range for a unit diameter sphere and infinite slab. The infinite slab has a height of 0.176 (calculated to give equivalence at range = 100) and is exposed to a radiation field isotropically emitted from a converter 0.5 units above the slab. The LET L is assumed equal to 1.

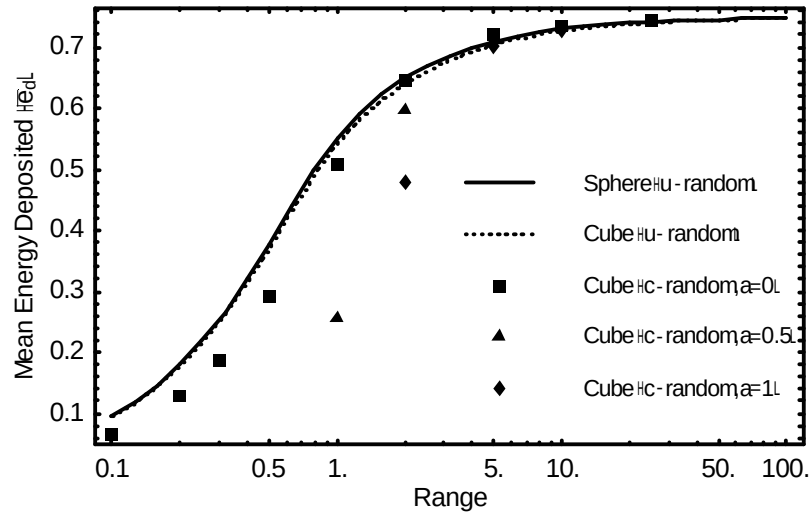


Figure 3.25. Comparison of mean energy deposited per event with particle range for a unit diameter sphere and cube with various overlayers/converters. The sphere and cube lines are calculated under the condition of μ -randomness whilst the cube points for various overlayers are calculated using Monte-Carlo methods assuming c-randomness. 10000 segments were used in the Monte-Carlo calculation. The LET L is assumed equal to 1.

3.3.4 Equivalence of Shapes using Frequency Mean Energy per Event (ϵ_f)

The previous sections have demonstrated a good equivalence between the sphere and cube with respect to the parameter $\bar{e}_d$. This equivalence is maintained even when using a converter/overlayer type geometry although the equivalence is reduced somewhat under the condition of unidirectional radiation. In most microdosimetric applications $\bar{e}_d$ and the closely related parameter $\bar{y}_d$ are of greatest interest. However, equivalence using the $\bar{e}_d$ parameter does not necessarily imply that other parameters and the microdosimetric spectra are in perfect agreement. Estimates of the remaining differences may be gauged by considering the frequency mean energy imparted per event $\bar{e}_f$ and the related event frequency Φ representing the average number of events per unit dose. Kellerer [143] performs the calculations for a sphere, cylinder and cube and the results are reproduced here supplemented by the non-isotropic case for the cube and consideration of the use of a converter with under c-randomness.

The relevant equations are

$$\bar{e}_f \Phi = m = rV \quad (3.31)$$

where m , V and r are the mass, volume and density of the volume. Kellerer considers two cases; short range and long range particles. For particles of short range $\bar{e}_f$ is equal

to the mean energy of the particles and is independent of the radiation directionality and detector shape. Therefore,

$$\frac{\bar{e}_F}{\bar{e}_{Fsphere}} = 1 \quad \text{and} \quad \frac{\Phi}{\Phi_{sphere}} = \frac{V}{V_{sphere}} = \frac{6a^3}{pd^3} \quad \text{for short range} \quad (3.32)$$

where a = side length of the cube and d = sphere diameter. For long range particles using the approximation of constant LET, L , $\bar{e}_F = L \bar{l}_m$ and applying equations (3.31) and (3.5) gives

$$\frac{\bar{e}_F}{\bar{e}_{Fsphere}} = \frac{\bar{l}_m}{\bar{l}_{msphere}} \quad \text{and} \quad \frac{\Phi}{\Phi_{sphere}} = \frac{S}{S_{sphere}} \quad \text{for long range} \quad (3.33)$$

which for isotropic conditions becomes

$$\frac{\bar{e}_F}{\bar{e}_{Fsphere}} = \frac{a}{d} \quad \text{and} \quad \frac{\Phi}{\Phi_{sphere}} = \frac{6a^2}{pd^2} \quad \text{for long range, isotropic} \quad (3.34)$$

and for unidirectional radiation

$$\frac{\bar{e}_F}{\bar{e}_{Fsphere}} = \frac{3a}{2d} \quad \text{and} \quad \frac{\Phi}{\Phi_{sphere}} = \frac{4a^2}{pd^2} \quad \text{for long range, unidirectional} \quad (3.35)$$

For long ranges, these equations are good approximations to the case of c-randomness with a converter and overlayer since the segment length distribution for c-randomness approximates μ -randomness. For short ranges, no rays intersect the sensitive volume under c-randomness.

The ratios of these microdosimetric quantities for a cube and sphere have been calculated in Table 3.7 with relevant ratios calculated for various RPP elongations in Table 3.6. These results indicate that the best overall equivalence to a sphere occurs for elongations close to 1. The event frequency and frequency average vary significantly with radiation direction for a cube due to the large difference in mean chord length with orientation. Given a cube optimized for a sphere equivalent $\bar{e}_d$, the ratio $\bar{e}_F / \bar{e}_{Fsphere}$ varies from 0.84 to 1.26 as we go from isotropic to unidirectional radiation. However, in practice $\bar{e}_F / \bar{e}_{Fsphere}$ will be somewhere between these values since the radiation pattern will be neither ideal isotropic nor unidirectional and a value closer to 1 may be expected. The condition of equal volumes for a sphere and cube provides a

reasonable overall equivalence for each of the parameters particularly if a case between isotropic and unidirectional is considered. However, it is important to note that equivalence based on event parameters is not as important as equivalence based on mean energy deposited, a point which will be further emphasized in the following section.

Table 3.7. Ratio of microdosimetric quantities in a cube of side length a and a sphere of diameter d

	Equal $\bar{e}_d$ $a = 0.837d$		Equal Volume $a = 0.806d$	
	Isotropic	Unidirectional	isotropic	Unidirectional
$\bar{e}_d / \bar{e}_{d,sphere}$	1 - 1	1 - 1.12	1 - 0.96	1 - 1.07
$\bar{e}_F / \bar{e}_{F,sphere}$	1 - 0.84	1 - 1.26	1 - 0.81	1 - 1.21
Φ / Φ_{sphere}	1.12 - 1.34	1.12 - 0.89	1 - 1.24	1 - 0.82

Note: The values to the left of the hyphen apply to short tracks whilst the values to the right apply to long tracks

3.3.5 Equivalent Lineal Energy Spectra and Straggling

Previously, the equivalence of various shapes to a sphere was studied using parameters such as $\bar{e}_d$. Attention must also be given to obtaining equivalence for the lineal energy spectrum and the important parameter of dose average lineal energy ($\bar{y}_d$). In this section, example lineal energy spectra will be presented, along with a study of the effect of straggling on the final spectrum. First, a method for constructing lineal energy spectra that are equivalent to a sphere must be derived.

The mean energy deposited in a volume K is proportional to the mean chord length assuming a constant LET and particles of range much larger than the dimensions of K . Therefore,

$$\frac{\bar{e}_{d,K}}{\bar{e}_{d,sphere}} = \frac{\bar{l}_K}{h \bar{l}_{sphere}} \quad (3.36)$$

$$\text{where } h = \frac{\bar{l}_{K_{eq}}}{\bar{l}_{sphere_{eq}}} = \begin{cases} 0.837 & \text{if shape = cube} \\ 0.895 & \text{if shape = cylinder}(e = 1) \end{cases}$$

and $\bar{l}_{K_{eq}}$ = mean chord length (μ -random) of K with dimensions such that mean energy deposited in K , $\bar{e}_{d,K_{eq}}$ is equivalent to the reference sphere energy $\bar{e}_{d,sphere}$ and $\bar{l}_{sphere_{eq}}$ = mean chord length for the reference sphere = $2d/3$. So h is the ratio of the mean chord lengths for the volume K and for a sphere when the dimensions are selected to give

equal dose mean energy deposited per event. Using equation (3.25) and rearranging equation (3.36) provides an expression for the dose average lineal energy in a reference sphere, $\bar{y}_{d,sphere}$.

$$\begin{aligned}\bar{y}_{d,sphere} &= \frac{\bar{e}_{d,sphere}}{\bar{l}_{sphere}} \\ &= \frac{\bar{e}_{d,K}}{\bar{l}_K} h = \frac{\bar{e}_{d,K}}{\bar{l}_{Keff}}\end{aligned}\quad (3.37)$$

where the effective mean chord length $\bar{l}_{Keff}$ is given by:

$$\bar{l}_{Keff} = \frac{\bar{l}_K}{h} \quad (3.38)$$

Thus to obtain the same $\bar{y}_d$ as a reference sphere, the lineal energy for a given shape (composed of the same material as the reference sphere) must be calculated using an effective mean chord length $\bar{l}_{Keff}$ equal to $\bar{l}_K / h$. Ideally, one should select the dimensions of K such that $\bar{l}_K = \bar{l}_{K_{eq}}$ since the scaling will then apply for finite range particles. The equation above is generally valid for ranges greater than the maximum of $(\bar{l}_K, \bar{l}_{K_{eq}})$ such that a high proportion of crossers exists.

As an example, a cube of side length a may be used to approximate a 1 μm sphere provided the particle ranges are much greater than a . (e.g. $a = 2\mu\text{m}$, $h = 0.837$, $\bar{l}_K = 1.33 \mu\text{m}$ and $\bar{l}_{Keff} = 1.59 \mu\text{m}$). Nevertheless, the preferred option is to use a cube of side length 0.837 μm since the proportion of each type of segment (e.g. crosser) is better matched over a wide sweep of ranges as discussed in section 3.3.2.

Consider a cube, cylinder and unit diameter sphere with dimensions selected such that $\bar{e}_d$ is equivalent for each volume. A comparison of the chord length distributions is shown in Figure 3.26 with an improved qualitative agreement compared with distributions provided in Figure 3.3 to Figure 3.5 (where the MCL is constant).

Straggling varies considerably with ion energy and type. A typical case is provided here to illustrate the influence of straggling on the expected microdosimetric spectrum. Consider first some data obtained in chapter 5 for a 5.3 MeV alpha incident on a 2 μm thick detector. The energy deposited, calculated using SRIM [126], is gaussian

distributed with a mean of 262 keV and a standard deviation, induced by energy and range straggling, of about 1/10 the mean (28.8 keV). For simplicity, consider a 1 μm diameter sphere and equivalent cube and cylinder in a μ -random particle environment with the particle LET = 100 keV/ μm . The lineal energy spectra are gaussian filtered with a standard deviation of 10 keV/ μm to approximate straggling effects. The resulting lineal energy spectra in y vs. $f(y)$ format is shown in Figure 3.27. Straggling improves the match between the cube spectrum and the sphere, particularly at higher lineal energies. The cylinder is a slightly better approximation to the sphere than a cube; a result consistent with the previous section's analysis.

The poor approximation at lower lineal energies becomes negligible when the data is displayed in the most common format for microdosimetric data, the $\log(y)$ vs. $yd(y)$ format as shown in Figure 3.28. If one were to introduce other sources of variation (such as LET and range as described in section 3.2 or increased straggling), it is clear that the cube and cylinder would provide lineal energy spectra $yd(y)$ and $\bar{y}_d$ values which are indistinguishable from the reference sphere. Note, that if the appropriate calculation for lineal energy, as provided by equation (3.37), is not employed (such as setting $h=1$) then the spectra shift significantly and an extremely poor correspondence with a sphere is observed.

An example of the effect of finite range on equivalence and lineal energy spectra is shown in Figure 3.29 for a range of 5 μm . The lineal energy spectrum differs for spheres of diameter 1 μm and 2 μm due to the differing contribution of segment types (e.g. crossers). Similarly, a cube of side length 1.674 μm is a poor match to a 1 μm sphere, although its equivalence to a 2 μm sphere is maintained even for low ranges. This illustrates the importance of selecting dimensions for equivalence such that $\bar{l}_K = \bar{l}_{K_eq}$ and therefore $\bar{e}_{d,sphere} = \bar{e}_{d,K}$, particularly when relatively small ranges are expected.

The final plot in Figure 3.30 shows the close spherical equivalence of a cube exposed to a c-random field. The range of the particles is 10 μm . This plot, along with previous discussions on the equivalence of $\bar{e}_d$, demonstrates the feasibility of utilizing a silicon detector with a converter structure to record a lineal energy spectra equivalent to sphere. Increasing the overlayer thickness to 5 μm did not significantly alter the $yd(y)$ spectrum

for the given range of 10 μm indicating that the equivalence of such a spectrum is robust to moderate overlayer thickness.

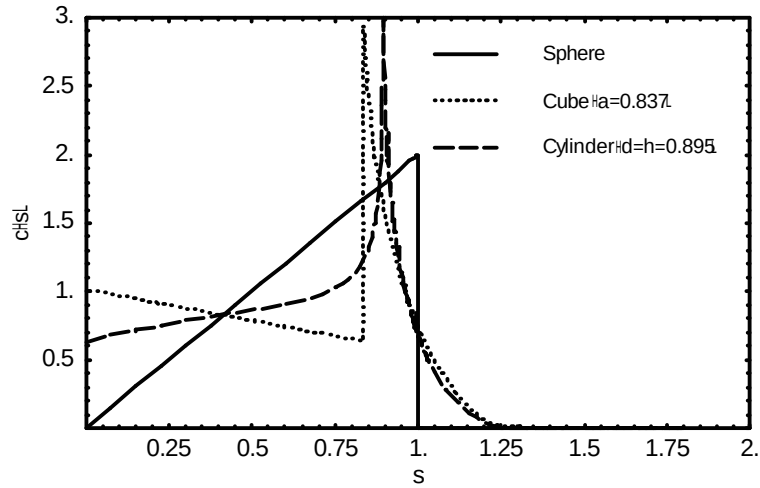


Figure 3.26. Comparison of chord length distribution under μ -randomness for a sphere, cube and cylinder with dimensions selected so that the $\bar{e}_d = 0.75$ assuming an LET of 1.

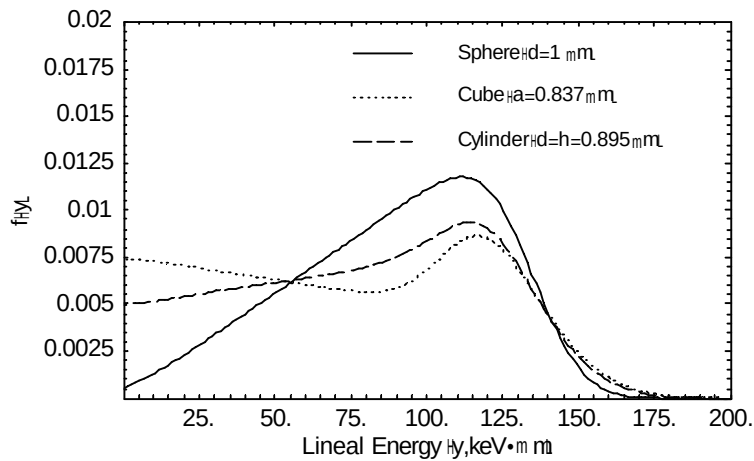


Figure 3.27. Lineal energy spectra $f(y)$ under μ -randomness for equivalent sphere, cube and cylinder with straggling. LET assumed to be 100 keV/ μm . Straggling standard deviation of 10 keV/ μm .

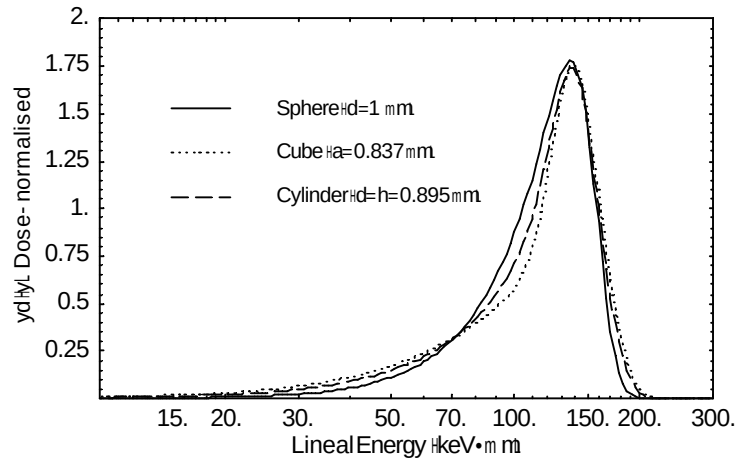


Figure 3.28. Lineal energy spectra $y_d(y)$ under μ -randomness for equivalent sphere, cube and cylinder with straggling (10keV/ μ m). LET assumed to be 100 keV/ μ m. See also Figure 3.27.

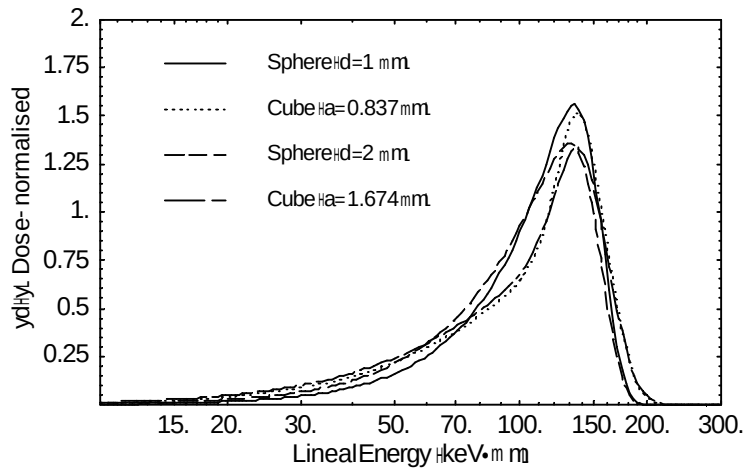


Figure 3.29. Lineal energy spectra $y_d(y)$ under μ -randomness with a finite range = 5 μ m. For range values approaching the shape dimensions the equivalence criteria is stricter. LET assumed to be 100 keV/ μ m. Straggling standard deviation of 10keV/ μ m. See also Figure 3.28.

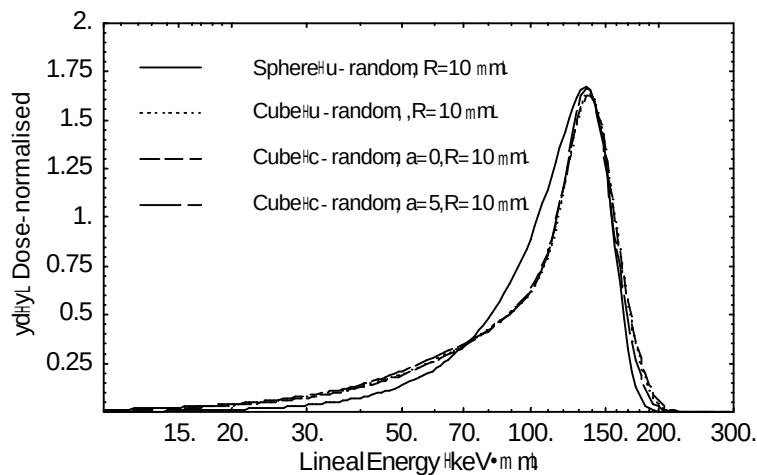


Figure 3.30. Comparison of lineal energy spectra $y_d(y)$ for a cube (side length=0.837 μ m) and sphere (d=1 μ m) under μ -randomness and c-randomness with a particle range of 10 μ m. Spectra calculated for two different overlayer thickness (a) are shown indicating that for the particle range selected the dose lineal energy is not significantly affected. LET assumed to be 100 keV/ μ m.

3.3.6 Summary: Criteria for the Equivalence of Various Shapes in Microdosimetry

The following summarizes the shape analysis study.

- Should select detector volumes to minimize the relative variance due to chord length variations. A sphere has the minimum known relative variance. For a RPP and cylinder the minimum relative variance occurs for unit elongation. Large elongations increase the maximum chord length possible and thus the relative variance.
- The primary criterion for equivalence of microdosimetry volumes is equality of the dose mean energy imparted per event $\bar{e}_d$
- To obtain the same $\bar{y}_d$ as a reference sphere, the lineal energy for a given shape K must be calculated using a mean chord length equal to $\bar{l}_K / h$ where h is the ratio of the mean chord lengths for the volume K and for a sphere when the dimensions are selected to give equal $\bar{e}_d$. ($h = 0.837$ for a cube)
- A cube of side length $0.837d$ is closely equivalent to a sphere of diameter d for all radiation types and ranges. ($\bar{e}_d$ varies less than 2% over all ranges for constant LET, Kellerer [129] estimates maximum deviations of 4% under all isotropic conditions)
- A RPP detector optimized for isotropic conditions will record in a unidirectional beam an $\bar{e}_d$ 11.6% higher than desired for equivalence to a sphere.
- A RPP with an overlayer/converter structure (c-randomness) is closely equivalent to a sphere and RPP under μ -randomness. The equivalence holds within 10% for particle ranges $R > 2(a + \bar{l})$ and within 2% for $R > 4(a + \bar{l})$ where a = overlayer thickness.
- The overlayer thickness should be a minimum; ideally zero.
- The thickness of the detector should be minimized commensurate with the radiobiological requirements of the measurement.

- A hemisphere is an excellent approximation to an oblate spheroid with elongation = 0.5. This may prove useful in microdosimetric applications in which cells are modeled as oblate spheroids.
- Given a cube optimized for a sphere equivalent $\bar{e}_d$, the event frequency ratio $\bar{e}_F / \bar{e}_{F_{\text{sphere}}}$ varies from 0.84 to 1.26 as we go from isotropic to unidirectional radiation.

3.4 Combining Criteria for Shape Equivalence and Tissue Equivalence

The shape analysis thus far has assumed that both the reference sphere and an alternative shape K are composed of the same material. In chapter 5, a correction factor that enables the conversion of energy spectra measured in silicon shapes to tissue shapes is derived. Thus, to provide equivalent dose mean energy deposited and $\bar{y}_d$ as a reference sphere, two corrections to the tissue sphere diameter are required to determine the dimensions of the silicon detector.

1. *Shape Corrections:* To ensure the same dose mean energy deposited and $\bar{y}_d$ as a reference sphere, the lineal energy for a given shape K must be calculated using a mean chord length equal to $\bar{l}_K / h$ as determined in section 3.3.5.
2. *Tissue Equivalence(TE) Corrections:* A silicon volume, K_{si} , of a given shape, will record the same energy deposition spectrum as a tissue volume, K_t , of the same shape, if the linear dimensions of K_{si} , are scaled by a TE scaling factor $z = 0.63$ times the dimensions of K_t . This factor arises due to the differing range-energy relationships (and LET) for particles in silicon versus tissue and is discussed in detail in chapter 5. The TE lineal energy is calculated using the tissue mean chord length $\bar{l}_{Kt}$ equal to $\bar{l}_{Ksi} / h$.

Therefore, given a measurement of energy deposited e_{Ksi} in a silicon detector the TE lineal energy, y_{Kt} , is calculated using:

$$\begin{aligned}
 y_{Kt} &= \frac{e_{Ksi}}{\bar{l}_{Ksi}} h z \\
 &= \frac{e_{Ksi}}{\bar{l}_{Keff}}
 \end{aligned}
 \tag{3.39}$$

where the effective MCL $\bar{l}_{Keff}$ is now given by:

$$\bar{l}_{Keff} = \frac{\bar{l}_{Ksi}}{hz} \quad (3.40)$$

This equation may be used for designing the silicon volume dimensions to meet the demands of $\bar{y}_d$ equivalence to a reference tissue sphere. The effective MCL is actually the reference tissue sphere MCL $\bar{l}_{Kt}$, if we desire equal dose mean energy deposited in both shapes (as is generally the case, see also section 3.3.5). Therefore, given a reference tissue sphere with a MCL = $\bar{l}_{Kt}$, the required MCL of a silicon volume $\bar{l}_{Ksi}$ is

$$\bar{l}_{Ksi} = \bar{l}_{Kt} hz$$

where $hz = \begin{cases} 0.5 & \text{for a cube} \\ 0.54 & \text{for a cylinder (elongation} = 1) \end{cases} \quad (3.41)$

Therefore, a silicon cube with side length 0.5 μm will have a similar lineal energy deposition spectrum and $\bar{y}_d$ as a 1 μm tissue sphere. Note, that the MCL used for calculating the TE lineal energy spectrum is not $\bar{l}_{Ksi}$, as would normally be the case, but is instead the tissue MCL, $\bar{l}_{Kt}$ (given by equation (3.40) with $\bar{l}_{Keff} = \bar{l}_{Kt}$). This is evident by inspection of the required lineal energy expression, equation (3.39).

3.5 Summary of Requirements

3.5.1 Geometrical Requirements and Considerations

The previous section discussed in detail the shape of the detector sensitive volume. Given the shape of the volume, dimensions of the detector should be selected based on two considerations. Firstly, the detector size should closely match the size of the biological entity that the detector is attempting to model in order to maximize the potential radiobiological information of the microdosimetric spectrum. Secondly, the size should be minimized in order to maintain as many crossers as possible. The microdosimetric spectrum will be more representative of particle LET, and possibly radiobiological effectiveness, if crossers dominate. Moreover, the use of a converter above a silicon detector is applicable under such circumstances.

Typical equivalent diameters for spherical and cylindrical proportional counters vary from around 1 μm to 10 μm . However, nanodosimetry miniature counters produced by

Kliauga [144-146] are capable of measurements down to a simulated site size of 10 nm. Some cell dimensions found in the literature are given in Table 3.8. Hartman [147] states that most types of tumor cells have cell diameters in the range 10-20 μm with temporal variations for each type depending on phase in the cell cycle. Smaller (8-10 μm for lymphoid tumors) and larger ($\sim 50 \mu\text{m}$ for epitheloid sarcomas) tumor cells have also been reported [148]. From the data presented in Table 3.8 the typical cell diameter is around 13 μm with a nucleus diameter of 9 μm . The calculations assume a spherical model for the cell although an ellipsoid is often more accurate. The cell diameter provides an upper limit on a microdosimeter's effective diameter. Interesting structures from the radiobiological viewpoint are present at the sub-cell nucleus levels (e.g. Chromosomes and DNA strands). Such structures are at the nanodosimetric scale and vary in size from 1.4 μm for a metaphase chromosome to 2 nm for the DNA double helix [1]. Therefore, the theoretical lower limit on the desired microdosimeter diameter is of the order of a few nanometers, well below sizes achievable with current experimental techniques. In practice, a range of sizes is desirable in order to study the effects of varying spatial distribution of energy.

Table 3.8. Cell size data from the literature

Cell Type	Cell Volume (μm^3)	Cell Diameter (μm)	Nucleus Volume (μm^3)	Nucleus Diameter (μm)	Comments
HeLa	1025	12	-	-	Ref [149]
V79 Chinese Hamster	1150	13	230	7.6	Ref [150]
9L Gliosarcoma	2140	16	690	11	Ref [151]
Lymphoid tumors	380	8-10	-	-	Ref [147, 148]
epitheloid sarcomas	65450	50	-	-	Ref [147, 148]
Melanoma (MeWo)	1550	14.4	640	10.7	Ref [152]

In the TDRA, briefly described in section 2.4.2, it was stated that the domain in which specific energy should be considered depends upon the average distance over which sublesions combine. Microdosimetric measurements of the neutron beams employed in biological experiments indicate that the sites typically have diameters of 0.2-2 μm [12, 18]. For this reason, along with practical design limitations and the desire to conform to Bragg-Gray cavity theory (see chapter 5), TEPC measurements are frequently performed in 1 and 2 μm site sizes. In summary, the ideal silicon microdosimeter should

have dimensions that approximate a tissue sphere of diameter ranging from around 13 μm to the minimum achievable given the device construction. Radiobiological studies indicate site sizes in the range less than 2 μm are of importance.

The silicon microdosimeter should be constructed with an array of sensitive volumes to improve collection statistics. The effective cross-sectional area required is dependent on the flux of the radiation environment. The lowest flux appears in radiation protection applications whilst the highest flux is observed in certain synchrotron based fast proton therapy facilities.

Proportional counters used in radiation protection have diameters of the order of 10-20 cm with gas pressure selected to simulate 1 μm diameter sites in tissue. Alternatively arrays of proportional counters may be employed [72]. For the same application and similar count rates of 500/hr at 10 $\mu\text{s/h}$, the required silicon detector area is also around 10 cm. Conversely, it will be shown in chapter 8, that the required area to avoid pile-up in fast proton therapy facilities is around 1 mm^2 .

In summary the geometrical requirements are

- Given a reference tissue sphere with a $\text{MCL} = \bar{l}_{kt}$, the required MCL of a silicon volume K is $\bar{l}_{ksi} = \bar{l}_{kt} h z$ where $h z = 0.5$ for a cube. Therefore, a silicon cube with side length 0.5 μm will have a similar TE lineal energy spectrum and $\bar{y}_d$ as a 1 μm tissue sphere. In addition, $\bar{l}_{kt}$ (0.66 μm in this case) and not $\bar{l}_{ksi}$ is used for calculating the TE lineal energy in the silicon volume.
- The ideal silicon microdosimeter should have dimensions that approximate a tissue sphere of diameter ranging from around 13 μm (the typical cell diameter) to the minimum achievable given the device construction.
- The detector should use an array of sensitive volumes to improve collection statistics. The total sensitive area of the volumes should be approximately 100 cm^2 for radiation protection applications and as small as 1 mm^2 for fast proton therapy measurements.
- A RPP with an overlayer/converter structure is appropriate. Such a structure is easily manufacturable. Furthermore, a RPP under c-randomness is closely equivalent to a sphere and RPP under μ -randomness. The equivalence holds within

10% for particle ranges $R > 2(a + \bar{l})$ and within 2% for $R > 4(a + \bar{l})$ where a = overlayer thickness.

- The elongation of the RPP should be as close to unity as practicable in order to minimize shape relative variance.
- A tissue equivalent converter should be used with a thickness greater than the maximum particle range expected to ensure charged particle equilibrium.
- The overlayer thickness separating the converter and detector should be minimized.

3.5.2 Operational Requirements and Considerations

In addition to the geometrical requirements, one must examine operational requirements such as electronic noise and charge collection behavior.

The application requires:

- An accurately defined sensitive volume. Minimization of charge collection complexity in particular diffusion and funneling effects. These effects are explained fully in chapter 6.
- Capable of measuring a lineal energy down to a minimum of ~ 1 keV/ μm in tissue equivalent terms. For a silicon microdosimeter with a noise level e_{rms} (in rms electrons) and silicon MCL $\bar{l}_{Ksi}$, the minimum lineal energy in tissue $y_{Kt,min}$ (in eV) is approximately given by inserting equation (2.26) into (3.39):

$$y_{Kt,min} \approx \frac{5e_{rms}W}{\bar{l}_{Ksi}} \text{ hz} \approx \frac{9e_{rms}}{\bar{l}_{Ksi}} \quad (3.42)$$

The radiobiological effectiveness begins to rise above unity as the lineal energy increase above ~ 1 keV/ μm . From a microdosimetric standpoint, events below this lineal energy are not interesting since their RBE is close to unity [53]. Ideally one would like to measure the lineal energy > 0.1 keV/ μm to obtain the complete spectrum [88] in all radiation environments. However, in many practical examples of interest such as microdosimetric measurements in fast neutron therapy [153, 154] and proton therapy near the distal edge of the beam [155] the dose contribution below 1 keV/ μm is small. Note that the energy of a proton with an LET of 1

keV/ μm is 70 MeV. Therefore provided the radiation environment does not contain protons with energy greater than 70 MeV and is not dominated by gamma radiation then a minimum lineal energy of 1 keV/ μm should suffice. Clearly, this specification is very application dependent and there are some cases in which a lower minimum lineal energy is desirable and a specification closer to 0.5 keV/ μm may be more appropriate (e.g. see section 8.3.2). Low noise design and optimization is necessary to met such requirements and will be discussed in detail in chapter 4.

- The charge collection behavior should be resistant to radiation damage at typical application fluences. The radiation hardness of silicon detectors is discussed in chapter 6.