
A five-round algebraic property of AES and its application to the ALPHA-MAC

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Abstract: We present a five-round algebraic property of the advanced encryption standard (AES), and we show that this algebraic property can be used to analyse the internal structure of ALPHA-MAC whose underlying block cipher is AES. In the proposed property, we modify 20 bytes from 5 intermediate values at some fixed locations in 5 consecutive rounds, and we show that after 5 rounds of operations, such modifications do not change the intermediate result and finally, still produce the same ciphertext. By employing the proposed five-round algebraic property of AES, we provide a method to find second preimages of the ALPHA-MAC based on the assumption that a key or an intermediate value is known. We also show that our idea can also be used to find internal collisions of the ALPHA-MAC under the same assumption.

Keywords: AES; advanced encryption standard; algebraic property; ALPHA-MAC; internal collisions; second preimages.

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1 Introduction

The block cipher Rijndael, invented by Daemen and Rijmen (2001), was selected as the advanced encryption standard (AES) by National Institute of Standards and Technology. Rijndael has a simple and elegant structure, and it was designed carefully to withstand two well-known cryptanalytic attacks: differential cryptanalysis, proposed by Biham and Shamir (1993) and linear cryptanalysis, described by Matsui (1994). Most operations of Rijndael are based on the algebraic Galois field $GF(2^8)$, which can be implemented efficiently in dedicated hardware and in software on a wide range of processors.

Since Rijndael was adopted as a standard by National Institute of Standards and Technology (2001), there have been many research efforts aiming to evaluate the security of this cipher. A block cipher, named big encryption system (BES), was defined by Murphy and Robshaw (2002), and Rijndael can be embedded into BES. The extended linearisation (XL) proposed by Courtois et al. (2000) and the extended sparse linearisation (XSL) provided by Courtois and Pieprzyk (2002) are new methods to solve non-linear algebraic equations. The concept of dual ciphers was introduced by Barkan and Biham (2002), and a collision attack on seven rounds of Rijndael was described

by Gilbert and Minier (2000). The most effective attacks on reduced round variants of the AES are square attack which was found by Daemen et al. (1997). The idea of the square attack was later employed by Ferguson et al. (2001) to improve the cryptanalysis of Rijndael, and by Lucks (2000) to attack seven rounds of Rijndael under 192- and 256-bit keys. A multiplicative masking method of AES was proposed by Akkar and Giraud (2001) and further discussed by Golic and Tymen (2002). The design of an AES-based stream cipher LEX was described by Biryukov (2007). A new message authentication code (MAC) construction ALRED and a special instance ALPHA-MAC was designed by Daemen and Rijmen (2005). So far, no short-cut attack against the full-round AES has been found.

In this paper, we present a five-round property of the AES. We modify 20 bytes from 5 intermediate values at some fixed locations in 5 consecutive rounds, and we demonstrate that after 5 rounds of operations, such modifications do not change the intermediate result and finally, still produce the same ciphertext. We introduce an algorithm named δ , and the δ algorithm takes a plaintext and a key as two inputs and outputs 20 bytes, which are used in the 5-round property. By employing the δ algorithm, we define a modified version of the AES algorithm, the δ AES. The δ AES calls the δ algorithm to generate 20 bytes, and uses these 20 bytes to modify the AES round keys. For a plaintext and a key, the AES and the δ AES produce the same ciphertext. By employing the proposed algebraic property of the AES, we analyse the internal structure of the ALPHA-MAC. Firstly, we present a method to find second preimages of the ALPHA-MAC by solving eight groups of linear functions based on the assumption that an authentication key or an intermediate value of this MAC is known. Each of these eight groups of linear functions contains two equations. Secondly, we show that the second-preimage finding method can also be used to generate internal collisions. The proposed collision search method can find two five-block messages such that they produce 128-bit collisions under a selected key (or a selected intermediate value).

This paper is organised as follows: Section 2 provides a brief description of the AES algorithm and Section 3 describes a five-round algebraic property of the AES. A modified version of the AES is defined in Section 4. Section 5 shows a description of the ALPHA-MAC construction and Section 6 demonstrates how the proposed five-round property of the AES is used to find second preimages and internal collisions of the ALPHA-MAC. Section 7 concludes this paper. Some examples of the AES and the AES with 20 extra exclusive-or operations are provided in the Appendix.

2 Description of the AES

AES is a block cipher with a 128-bit block length and supports key lengths of 128, 192 or 256 bits. For encryption, the input is a plaintext block and a key, and the output is a ciphertext block. The plaintext is first copied to 4×4 array

of bytes, which is called the state. The bytes of a state is organised in the following format:

α_0	α_4	α_8	α_{12}
α_1	α_5	α_9	α_{13}
α_2	α_6	α_{10}	α_{14}
α_3	α_7	α_{11}	α_{15}

where α_i denote the i th byte of the block. After an initial round key addition, the state array is transformed by performing a round function 10, 12 or 14 times (for 128-, 192- or 256-bit keys, respectively), and the final state is the ciphertext. We denote the AES with 128-bit keys by AES-128, with 192-bit keys by AES-192 and with 256-bit keys by AES-256. Each round of AES consists of the following four transformations (the final round does not include AddRoundKey (ARK)):

- 1 *The SubBytes (SB) transformation*: it is a non-linear byte substitution that operates independently on each byte of the state using a substitution table.
- 2 *The ShiftRows (SR) transformation*: the bytes of the state are cyclically shifted over different numbers of bytes. Row 0 is unchanged and row i is shifted to the left i byte cyclicly, $i \in \{1, 2, 3\}$.
- 3 *The MixColumns (MC) transformation*: it operates on the state column-by-column, considering each column as a four-term polynomial. The columns are treated as polynomials over $\text{GF}(2^8)$ and multiplied modulo $x^4 + 1$ with a fixed polynomial, written as $\{03\}x^3 + \{01\}x^2 + \{01\}x + \{02\}$.
- 4 *The ARK transformation*: a round key is added to the state by a simple bitwise exclusive-or (XOR) operation.

The key expansion of the AES generates a total of $\text{Nb}(\text{Nr}+1)$ words: the algorithm needs an initial set of Nb words, and each of the Nr rounds requires Nb words of key data, where Nb is 4 and Nr is set to 10, 12 or 14 for 128-, 192- or 256-bit key sizes, respectively. For a 128-bit key K , we denote the round keys by

K_0^i	K_4^i	K_8^i	K_{12}^i
K_1^i	K_5^i	K_9^i	K_{13}^i
K_2^i	K_6^i	K_{10}^i	K_{14}^i
K_3^i	K_7^i	K_{11}^i	K_{15}^i

where i is the round number, $i \in \{1, 2, \dots, 10\}$. We note that the round key used in the initial round is the secret key K itself, and the secret key is represented without the superscript i . The combinations of the key length, block size and number of rounds are listed below:

Key length	Block size	Number of round
128 bits	4	10
192 bits	4	12
256 bits	4	14

3 A five-round property of AES

We describe a five-round property of the AES in this section. In the proposed property, we modify 20 bytes from 5 intermediate values at some fixed locations in 5 consecutive rounds, and we show that after 5 rounds of operations, such modifications do not change the intermediate result and finally, still produce the same ciphertext. The modifications are carried out by performing four extra XOR operations at the end of each round (i.e. after the ARK transformation), and in total, we perform 20 extra XOR operations in 5 rounds. We require that each of these 5 rounds must contain SB, SR, MC and ARK transformations.

We use Figures 1–3 to describe this property. The layout of the 20 bytes in the 5 intermediate values is shown in Figure 1, and the 20 bytes are $G'_0, G'_2, G'_8, G'_{10}, M'_0, M'_2, M'_8, M'_{10}, R'_0, R'_2, R'_8, R'_{10}, V'_0, V'_2, V'_8, V'_{10}, Z'_0, Z'_2, Z'_8, Z'_{10}$.

Figure 1 20 bytes

G'_0	0	G'_8	0
0	0	0	0
G'_2	0	G'_{10}	0
0	0	0	0
M'_0	0	M'_8	0
0	0	0	0
M'_2	0	M'_{10}	0
0	0	0	0
R'_0	0	R'_8	0
0	0	0	0
R'_2	0	R'_{10}	0
0	0	0	0
V'_0	0	V'_8	0
0	0	0	0
V'_2	0	V'_{10}	0
0	0	0	0
Z'_0	0	Z'_8	0
0	0	0	0
Z'_2	0	Z'_{10}	0
0	0	0	0

Figure 2 The intermediate values of AES-128

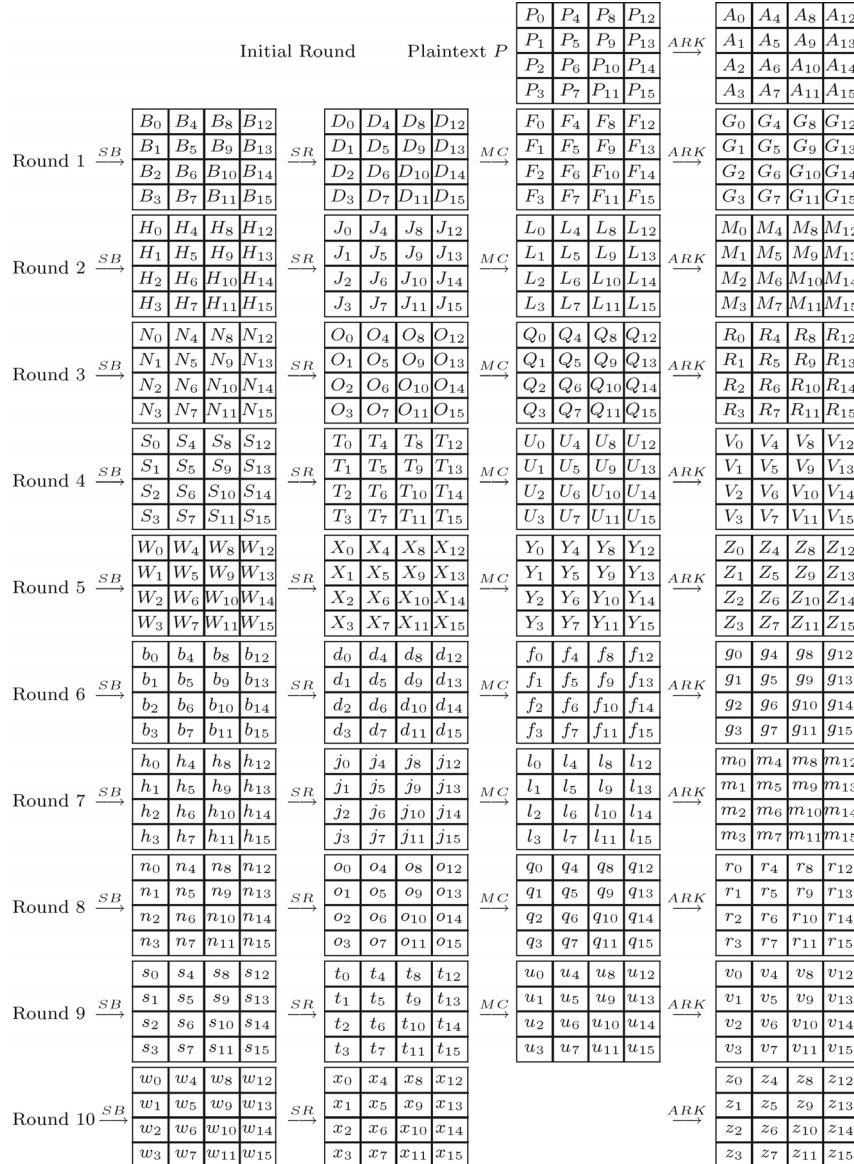
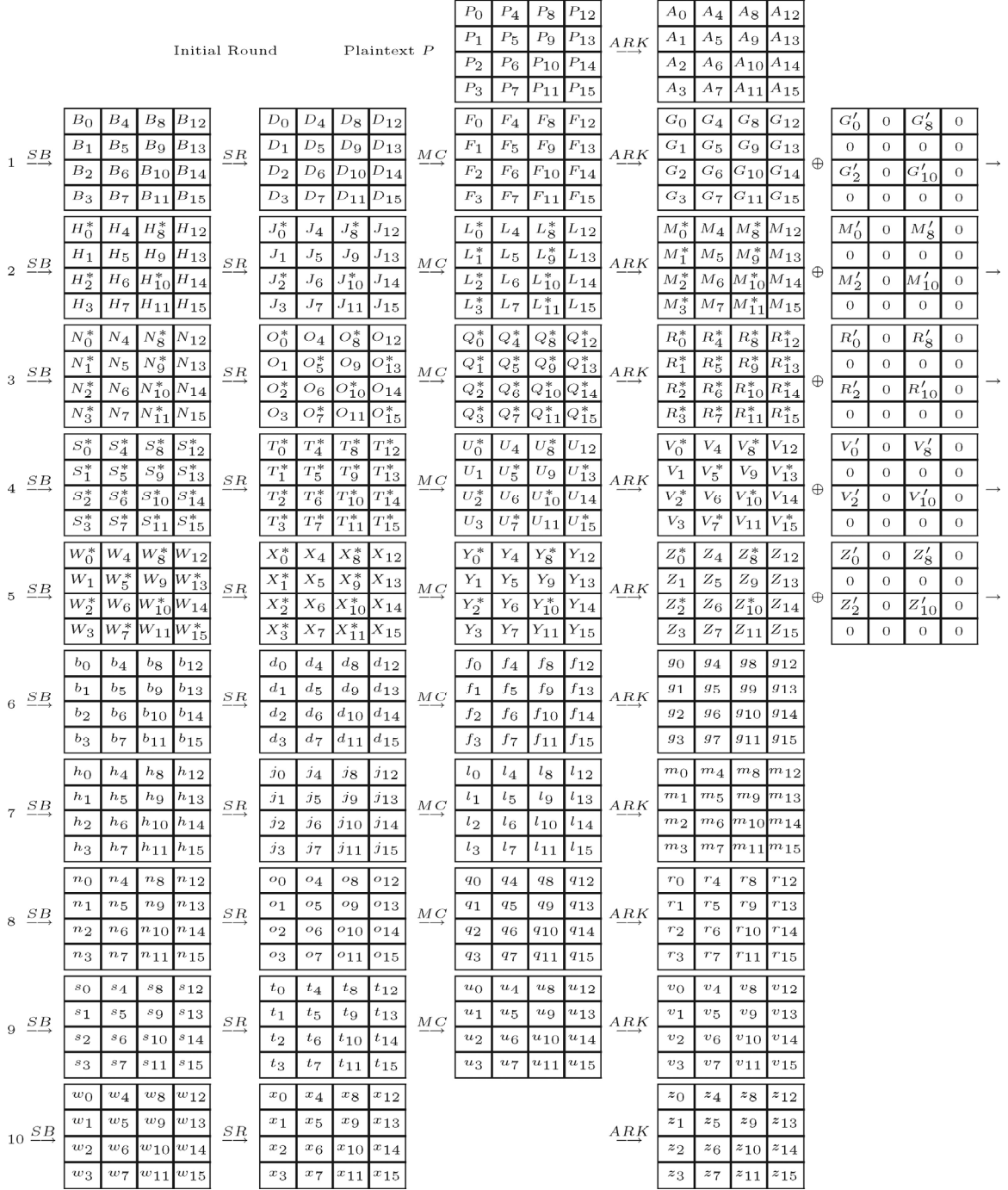


Figure 3 The intermediate values of AES-128 with extra 20 XOR operations

In Figure 1, a zero occupied byte means that there is no change in that byte, and a variable occupied byte indicates that there is a modification in that byte. In Figure 2, all intermediate values are listed when using the AES algorithm to encrypt a plaintext P under a 128-bit key K , and all bytes of the intermediate values are denoted by plain variables. Correspondingly, Figure 3 enumerates all intermediate values of the AES with 20 extra XOR operations. The 20-byte modifications take place in Rounds 1–5, and after ARK transformation in each of these 5 rounds, we perform XOR operations on Bytes 0, 2, 8 and 10. We show that the

20-byte modifications do not change the input to Round 6, that is, both the AES and the AES with 20 extra XOR operations generate the same input to Round 6. In Figure 3, a variable marked by an asterisk indicates that the value at that location has been affected by the 20-byte modifications, and a plain variable shows that the value at that location is not affected by the 20-byte modifications. For example, after ARK in Round 1 in Figure 3, Byte G_i is XORed with Byte G'_i , and after SB, we have four modified bytes H_i^* , $i \in \{0, 2, 8, 10\}$ and 12 unchanged bytes: $H_1, H_3, H_4, H_5, H_6, H_7, H_9, H_{11}, H_{12}, H_{13}, H_{14}$ and H_{15} .

3.1 The δ algorithm

To decide the values of the 20 bytes: G'_i, M'_i, R'_i, V'_i and Z'_i , $i \in \{0, 2, 8, 10\}$, we introduce an algorithm named δ . For any plaintext P and any key K used in the AES algorithm, the δ algorithm accepts P and K as two inputs, and generates an output which contains 20 bytes $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}$, where G'_i, M'_i, R'_i, V'_i and Z'_i are bytes, $i \in \{0, 2, 8, 10\}$.

The δ algorithm includes a number of steps:

- 1 Process the first five rounds of the AES algorithm by taking the plaintext P and the key K as the inputs, that is, start with the initial round, and process Rounds 1–5 of the AES. Therefore, we know all intermediate values in Figure 2, from initial round to Round 5.
- 2 Initialise G'_i, M'_i, R'_i, V'_i and Z'_i to zero, $i \in \{0, 2, 8, 10\}$.
- 3 Choose G'_0, G'_2, G'_8 and G'_{10} freely. The only requirement is that at least one of these four bytes is not equal to zero, namely, G'_0, G'_2, G'_8 and G'_{10} cannot be all zeros. If G'_0, G'_2, G'_8 and G'_{10} are all zeros, the δ algorithm outputs 20 zero bytes. Once G'_0, G'_2, G'_8 and G'_{10} are decided, the remaining 16 bytes will be computed by the procedures described in Sections 3.1.1–3.1.4.
- 4 Decide M'_0, M'_2, M'_8 and M'_{10} .
- 5 Decide R'_0, R'_2, R'_8 and R'_{10} .
- 6 Decide V'_0, V'_2, V'_8 and V'_{10} .
- 7 Decide Z'_0, Z'_2, Z'_8 and Z'_{10} .

Remark 1: There are $2^{32}-1$ combinations of $\{G'_0, G'_2, G'_8, G'_{10}\}$ because each byte can have 2^8 possible values.

3.1.1 Deciding M'_0, M'_2, M'_8 and M'_{10}

After we have decided the values of G'_0, G'_2, G'_8 and G'_{10} , we carry out a four-round computation (of the AES with extra 12 XOR operations), called Routine Computation One, which starts with the initial round and ends with MC in Round 4 (see Figure 3).

Routine Computation One

Initial round : ARK

Round 1: SB SR MC ARK $\oplus$

Round 2: SB SR MC ARK $\oplus$

Round 3: SB SR MC ARK $\oplus$

Round 4: SB SR MC.

All intermediate values from the computation of this time are stored in array called Buffer One (note that Routine Computation One produces 19 intermediate values). We denote the input and output of MC in Round 4 by

$$\begin{bmatrix} T_0^* & T_4^* & T_8^* & T_{12}^* \\ T_1^* & T_5^* & T_9^* & T_{13}^* \\ T_2^* & T_6^* & T_{10}^* & T_{14}^* \\ T_3^* & T_7^* & T_{11}^* & T_{15}^* \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} U_0^* & U_4^* & U_8^* & U_{12}^* \\ U_1^* & U_5^* & U_9^* & U_{13}^* \\ U_2^* & U_6^* & U_{10}^* & U_{14}^* \\ U_3^* & U_7^* & U_{11}^* & U_{15}^* \end{bmatrix}$$

Next, we will show that there is an algebraic relation between Bytes $\{M'_0, M'_2, M'_8, M'_{10}\}$ and Bytes $\{U_4^*, U_6^*, U_{12}^*, U_{14}^*\}$. Based on this relationship, we can change the values of $\{U_4^*, U_6^*, U_{12}^*, U_{14}^*\}$ to the values of $\{U_4, U_6, U_{12}, U_{14}\}$ by setting the values of $\{M'_0, M'_2, M'_8, M'_{10}\}$. After we have decided the values of $\{M'_0, M'_2, M'_8, M'_{10}\}$, we aim to have an intermediate value after MC in Round 4 in the format of

$$\begin{bmatrix} U_0^* & U_4^* & U_8^* & U_{12}^* \\ U_1^* & U_5^* & U_9^* & U_{13}^* \\ U_2^* & U_6^* & U_{10}^* & U_{14}^* \\ U_3^* & U_7^* & U_{11}^* & U_{15}^* \end{bmatrix}$$

The steps of deciding $\{M'_0, M'_2, M'_8, M'_{10}\}$ are listed as follows:

$$\begin{aligned} \{M'_0, M'_2, M'_8, M'_{10}\} &\leftarrow \{N_0^*, N_2^*, N_8^*, N_{10}^*\} \leftarrow \{O_0^*, O_2^*, O_8^*, O_{10}^*\} \\ &\leftarrow \{Q_1^*, Q_3^*, Q_9^*, Q_{11}^*\} \leftarrow \{R_1^*, R_3^*, R_9^*, R_{11}^*\} \leftarrow \{S_1^*, S_3^*, S_9^*, S_{11}^*\} \\ &\leftarrow \{T_5^*, T_7^*, T_{13}^*, T_{15}^*\} \leftarrow \{U_4^*, U_6^*, U_{12}^*, U_{14}^*\} \end{aligned}$$

After we change the values of $\{U_4^*, U_6^*, U_{12}^*, U_{14}^*\}$ to the values of $\{U_4, U_6, U_{12}, U_{14}\}$, the input and output of MC in Round 4 become

$$\begin{bmatrix} T_0^* & T_4^* & T_8^* & T_{12}^* \\ T_1^* & T_5^* & T_9^* & T_{13}^* \\ T_2^* & T_6^* & T_{10}^* & T_{14}^* \\ T_3^* & T_7^* & T_{11}^* & T_{15}^* \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} U_0^* & U_4^* & U_8^* & U_{12}^* \\ U_1^* & U_5^* & U_9^* & U_{13}^* \\ U_2^* & U_6^* & U_{10}^* & U_{14}^* \\ U_3^* & U_7^* & U_{11}^* & U_{15}^* \end{bmatrix}$$

Our next target is to modify the values $\{T_5^*, T_7^*, T_{13}^*, T_{15}^*\}$ of according to the values of $\{U_4, U_6, U_{12}, U_{14}\}$. From the MC transformation, we have the following formula:

$$\begin{aligned} &\begin{bmatrix} U_0^* & U_4^* & U_8^* & U_{12}^* \\ U_1^* & U_5^* & U_9^* & U_{13}^* \\ U_2^* & U_6^* & U_{10}^* & U_{14}^* \\ U_3^* & U_7^* & U_{11}^* & U_{15}^* \end{bmatrix} \\ &= \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} T_0^* & T_4^* & T_8^* & T_{12}^* \\ T_1^* & T_5^* & T_9^* & T_{13}^* \\ T_2^* & T_6^* & T_{10}^* & T_{14}^* \\ T_3^* & T_7^* & T_{11}^* & T_{15}^* \end{bmatrix} \end{aligned}$$

To find out the values of $\{T_5^*, T_7^*, T_{13}^*, T_{15}^*\}$, we need to solve the following two groups of linear functions, which are marked by (1) and (2).

$$\left\{ \begin{array}{l} [02 \ 03 \ 01 \ 01] \begin{bmatrix} T_4^* \\ T_5^* \\ T_6^* \\ T_7^* \end{bmatrix} = U_4 \\ [01 \ 01 \ 02 \ 03] \begin{bmatrix} T_4^* \\ T_5^* \\ T_6^* \\ T_7^* \end{bmatrix} = U_6 \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} [02 \ 03 \ 01 \ 01] \begin{bmatrix} T_{12}^* \\ T_{13}^* \\ T_{14}^* \\ T_{15}^* \end{bmatrix} = U_{12} \\ [01 \ 01 \ 02 \ 03] \begin{bmatrix} T_{12}^* \\ T_{13}^* \\ T_{14}^* \\ T_{15}^* \end{bmatrix} = U_{14} \end{array} \right. \quad (2)$$

In Equation (1), there are two linear equations with two undecided variables T_5^* and T_7^* and thus we can solve (1) to obtain the values of T_5^* and T_7^* . Similarly, there are two linear equations in (2) with two undecided variables T_{13}^* and T_{15}^* and therefore we can solve (2) to get the values of T_{13}^* and T_{15}^* . After having T_5^*, T_7^*, T_{13}^* and T_{15}^* , perform SR^{-1} (inverse SR) and SB^{-1} (inverse SB), and we have the values of R_1^*, R_3^*, R_9^* and R_{11}^* after ARK in Round 3. Apply the ARK transformation to R_1^*, R_3^*, R_9^* and R_{11}^* , we have the values of Q_1^*, Q_3^*, Q_9^* and Q_{11}^* . Our next task is to modify the values of O_0^*, O_2^*, O_8^* and O_{10}^* . In Round 3, the input and output of MC are as follows:

$$\begin{bmatrix} Q_0^* & Q_4^* & Q_8^* & Q_{12}^* \\ Q_1^* & Q_5^* & Q_9^* & Q_{13}^* \\ Q_2^* & Q_6^* & Q_{10}^* & Q_{14}^* \\ Q_3^* & Q_7^* & Q_{11}^* & Q_{15}^* \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} O_0^* & O_4^* & O_8^* & O_{12}^* \\ O_1^* & O_5^* & O_9^* & O_{13}^* \\ O_2^* & O_6^* & O_{10}^* & O_{14}^* \\ O_3^* & O_7^* & O_{11}^* & O_{15}^* \end{bmatrix}$$

We can form two groups of linear equations, which are named (3) and (4), and solve them to decide

O_0^*, O_2^*, O_8^* and O_{10}^* . There are two linear equations in (3) with two undetermined variables O_0^* and O_2^* , and we can solve them to determine the values of O_0^* and O_2^* . Also, there are two linear equations in (4) with two undecided variables O_8^* and O_{10}^* , and we can get O_8^* and O_{10}^* and by solving (4).

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} O_0^* \\ O_1^* \\ O_2^* \\ O_3^* \end{bmatrix} = Q_4^* \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} O_0^* \\ O_1^* \\ O_2^* \\ O_3^* \end{bmatrix} = Q_3^* \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} O_8^* \\ O_9^* \\ O_{10}^* \\ O_{11}^* \end{bmatrix} = Q_9^* \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} O_8^* \\ O_9^* \\ O_{10}^* \\ O_{11}^* \end{bmatrix} = Q_{11}^* \end{array} \right. \quad (4)$$

Once knowing the values of O_0^*, O_2^*, O_8^* and O_{10}^* , we perform SR^{-1} and thus we get Bytes N_0^*, N_2^*, N_8^* and N_{10}^* after SB in Round 3. Finally, Bytes M_0', M_2', M_8' and M_{10}' are decided by the following computations (note that M_0^*, M_2^*, M_8^* and M_{10}^* are obtained from Buffer One):

$$M_0' = M_0^* \oplus SB^{-1}(N_0^*), M_2' = M_2^* \oplus SB^{-1}(N_2^*)$$

$$M_8' = M_8^* \oplus SB^{-1}(N_8^*), M_{10}' = M_{10}^* \oplus SB^{-1}(N_{10}^*)$$

At this stage, we have decided the values of $\{G_i', M_i'\}$ and $\{R_i', V_i', Z_i'\}$ are not yet decided (*note*: they are still initialised to zero), $i \in \{0, 2, 8, 10\}$.

3.1.2 Deciding R_0', R_2', R_8' and R_{10}'

Perform Routine Computation One second time, and all intermediate values from the computation of this time are stored in an array called Buffer Two. The intermediate value after MC in Round 4 is

$$\begin{bmatrix} U_0^* & U_4^* & U_8^* & U_{12}^* \\ U_1^* & U_5^* & U_9^* & U_{13}^* \\ U_2^* & U_6^* & U_{10}^* & U_{14}^* \\ U_3^* & U_7^* & U_{11}^* & U_{15}^* \end{bmatrix}$$

We will demonstrate that there is an algebraic relation between Bytes $\{R'_0, R'_2, R'_8, R'_{10}\}$ and Bytes $\{U_1^*, U_3^*, U_9^*, U_{11}^*\}$. By employing this relationship, we are able to change the values of $\{U_1^*, U_3^*, U_9^*, U_{11}^*\}$ to the values of $\{U_1, U_3, U_9, U_{11}\}$ by choosing the values of $\{R'_0, R'_2, R'_8, R'_{10}\}$. After we have determined the values of $\{R'_0, R'_2, R'_8, R'_{10}\}$ and perform Routine Computation One second time, our target is that the intermediate value after MC in Round 4 is

$$\begin{bmatrix} U_0^* & U_4 & U_8^* & U_{12} \\ U_1 & U_5^* & U_9 & U_{13} \\ U_2^* & U_6 & U_{10}^* & U_{14} \\ U_3 & U_7^* & U_{11} & U_{15}^* \end{bmatrix}$$

The moves of determining the values of $\{R'_0, R'_2, R'_8, R'_{10}\}$ are shown below:

$$\begin{aligned} \{R'_0, R'_2, R'_8, R'_{10}\} &\leftarrow \{S_0^*, S_2^*, S_8^*, S_{10}^*\} \leftarrow \{T_0^*, T_2^*, T_8^*, T_{10}^*\} \\ &\leftarrow \{U_1, U_3, U_9, U_{11}\} \end{aligned}$$

After we replace the values of $\{U_1^*, U_3^*, U_9^*, U_{11}^*\}$ with the values of $\{U_1, U_3, U_9, U_{11}\}$ the input and the output of MC in Round 4 are

$$\begin{bmatrix} T_0^* & T_4^* & T_8^* & T_{12}^* \\ T_1^* & T_5^* & T_9^* & T_{13}^* \\ T_2^* & T_6^* & T_{10}^* & T_{14}^* \\ T_3^* & T_7^* & T_{11}^* & T_{15}^* \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} U_0^* & U_4 & U_8^* & U_{12} \\ U_1 & U_5^* & U_9 & U_{13} \\ U_2^* & U_6 & U_{10}^* & U_{14} \\ U_3 & U_7^* & U_{11} & U_{15}^* \end{bmatrix}$$

We need to modify the values of $\{T_0^*, T_2^*, T_8^*, T_{10}^*\}$ according to the values of $\{U_1, U_3, U_9, U_{11}\}$. We can form two groups of linear equations, which are named (5) and (6). There are two undecided variables T_0^* and T_2^* in Equation (5), and we can solve (5) to get the values of T_0^* and T_2^* . In Equation (6), there are two undetermined variables T_8^* and T_{10}^* , and we can find out the values of T_8^* and T_{10}^* by solving (6).

$$\left\{ \begin{aligned} [01 \ 02 \ 03 \ 01] \begin{bmatrix} T_0^* \\ T_1^* \\ T_2^* \\ T_3^* \end{bmatrix} &= U_1 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} T_0^* \\ T_1^* \\ T_2^* \\ T_3^* \end{bmatrix} &= U_3 \end{aligned} \right. \quad (5)$$

$$\left\{ \begin{aligned} [01 \ 02 \ 03 \ 01] \begin{bmatrix} T_8^* \\ T_9^* \\ T_{10}^* \\ T_{11}^* \end{bmatrix} &= U_9 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} T_8^* \\ T_9^* \\ T_{10}^* \\ T_{11}^* \end{bmatrix} &= U_{11} \end{aligned} \right. \quad (6)$$

After knowing the values of $\{T_0^*, T_2^*, T_8^*, T_{10}^*\}$, we perform SR^{-1} and have four corresponding values $\{S_0^*, S_2^*, S_8^*, S_{10}^*\}$ after SB in Round 4. Bytes $\{R'_0, R'_2, R'_8, R'_{10}\}$ are computed as follows: (note that R_0^*, R_2^*, R_8^* and R_{10}^* are obtained from Buffer Two):

$$\begin{aligned} R'_0 &= R_0^* \oplus \text{SB}^{-1}(S_0^*), R'_2 = R_2^* \oplus \text{SB}^{-1}(S_2^*) \\ R'_8 &= R_8^* \oplus \text{SB}^{-1}(S_8^*), R'_{10} = R_{10}^* \oplus \text{SB}^{-1}(S_{10}^*) \end{aligned}$$

At this moment, we have decided the values of $\{G'_i, M'_i, R'_i\}$ and $\{V'_i, Z'_i\}$ are not determined and they are still equal to their initial values, $i \in \{0, 2, 8, 10\}$.

3.1.3 Deciding V'_0, V'_2, V'_8 and V'_{10}

After having the values of R'_0, R'_2, R'_8 and R'_{10} , we carry out a five-round computation of the AES with 16 extra XOR operations, called Routine Computation Two, which begins with the initial round and ends with MC in Round 5 (See Figure 3). All intermediate values from the computation of this time are stored in an array named Buffer Three (note that Routine Computation Two generates 24 intermediate values).

Routine Computation Two

Initial round : ARK

Round 1: SB SR MC ARK $\oplus$

Round 2: SB SR MC ARK $\oplus$

Round 3: SB SR MC ARK $\oplus$

Round 4: SB SR MC

After MC in Round 5, we will have an intermediate value in the following format:

$$\begin{bmatrix} Y_0^* & Y_4 & Y_8^* & Y_{12} \\ Y_1^* & Y_5 & Y_9^* & Y_{13} \\ Y_2^* & Y_6 & Y_{10}^* & Y_{14} \\ Y_3^* & Y_7 & Y_{11}^* & Y_{15} \end{bmatrix}$$

There is an algebraic relation between Bytes $\{V'_0, V'_2, V'_8, V'_{10}\}$ and Bytes $\{Y_1^*, Y_3^*, Y_9^*, Y_{11}^*\}$, and we can change the values of $\{Y_1^*, Y_3^*, Y_9^*, Y_{11}^*\}$ to the values of $\{Y_1, Y_3, Y_9, Y_{11}\}$ by setting the values of $\{V'_0, V'_2, V'_8, V'_{10}\}$. The steps of determining the values of $\{V'_0, V'_2, V'_8, V'_{10}\}$ are shown below:

$$\begin{aligned} \{V'_0, V'_2, V'_8, V'_{10}\} &\leftarrow \{W_0^*, W_2^*, W_8^*, W_{10}^*\} \\ &\leftarrow \{X_0^*, X_2^*, X_8^*, X_{10}^*\} \leftarrow \{Y_1, Y_3, Y_9, Y_{11}\} \end{aligned}$$

We replace Bytes $\{Y_1^*, Y_3^*, Y_9^*, Y_{11}^*\}$ with Bytes $\{Y_1, Y_3, Y_9, Y_{11}\}$, and the input and output of MC in Round 5 are

$$\begin{bmatrix} X_0^* & X_4 & X_8^* & X_{12} \\ X_1^* & X_5 & X_9^* & X_{13} \\ X_2^* & X_6 & X_{10}^* & X_{14} \\ X_3^* & X_7 & X_{11}^* & X_{15} \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} Y_0^* & Y_4 & Y_8^* & Y_{12} \\ Y_1 & Y_5 & Y_9 & Y_{13} \\ Y_2^* & Y_6 & Y_{10}^* & Y_{14} \\ Y_3 & Y_7 & Y_{11} & Y_{15} \end{bmatrix}$$

We form two groups of linear functions, marked by (7) and (8). There are two undecided variables X_0^* and X_2^* in (7), and we can solve (7) to get the values of X_0^* and X_2^* . In Equation (8), there are two undecided variables X_8^* and X_{10}^* and we can obtain the values of X_8^* and X_{10}^* by solving (8).

$$\left\{ \begin{aligned} [01 \ 02 \ 03 \ 01] \begin{bmatrix} X_0^* \\ X_1^* \\ X_2^* \\ X_3^* \end{bmatrix} &= Y_1 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} X_0^* \\ X_1^* \\ X_2^* \\ X_3^* \end{bmatrix} &= Y_3 \end{aligned} \right. \quad (7)$$

$$\left\{ \begin{aligned} [01 \ 02 \ 03 \ 01] \begin{bmatrix} X_8^* \\ X_9^* \\ X_{10}^* \\ X_{11}^* \end{bmatrix} &= Y_9 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} X_8^* \\ X_9^* \\ X_{10}^* \\ X_{11}^* \end{bmatrix} &= Y_{11} \end{aligned} \right. \quad (8)$$

After deciding the values of $\{X_0^*, X_2^*, X_8^*, X_{10}^*\}$, we perform SR^{-1} and have four corresponding values $\{W_0^*, W_2^*, W_8^*, W_{10}^*\}$ after SB in Round 5. Bytes V'_0, V'_2, V'_8 and V'_{10} are computed as follows (note that V_0^*, V_2^*, V_8^* and V_{10}^* are obtained from Buffer Three):

$$V'_0 = V_0^* \oplus \text{SB}^{-1}(W_0^*), V'_2 = V_2^* \oplus \text{SB}^{-1}(W_2^*),$$

$$V'_8 = V_8^* \oplus \text{SB}^{-1}(W_8^*), V'_{10} = V_{10}^* \oplus \text{SB}^{-1}(W_{10}^*).$$

At this stage, we have decided the values of $\{G'_i, M'_i, R'_i, V'_i\}$, and Z'_i is not determined and it is equal to the initial value, $i \in \{0, 2, 8, 10\}$.

3.1.4 Deciding Z'_0, Z'_2, Z'_8 and Z'_{10}

Perform Routine Computation Two second time, and the intermediate value after MC in Round 5 is

$$\begin{bmatrix} Y_0^* & Y_4 & Y_8^* & Y_{12} \\ Y_1 & Y_5 & Y_9 & Y_{13} \\ Y_2^* & Y_6 & Y_{10}^* & Y_{14} \\ Y_3 & Y_7 & Y_{11} & Y_{15} \end{bmatrix}$$

Apply ARK to the intermediate value above, we have

$$\begin{bmatrix} Z_0^* & Z_4 & Z_8^* & Z_{12} \\ Z_1 & Z_5 & Z_9 & Z_{13} \\ Z_2^* & Z_6 & Z_{10}^* & Z_{14} \\ Z_3 & Z_7 & Z_{11} & Z_{15} \end{bmatrix}$$

Bytes Z'_0, Z'_2, Z'_8 and Z'_{10} are computed as follows (note that Z_0, Z_2, Z_8 and Z_{10} are obtained from the computation in which the AES algorithm is used to encrypt the plaintext P under the key K (see Round 5 in Figure 2)):

$$Z'_0 = Z_0^* \oplus Z_0, Z'_2 = Z_2^* \oplus Z_2,$$

$$Z'_8 = Z_8^* \oplus Z_8, Z'_{10} = Z_{10}^* \oplus Z_{10}.$$

Finally, we have decided all values of $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}$, $i \in \{0, 2, 8, 10\}$. Now, we carry out a 5-round computation of the AES with extra 20 XOR operations, called Routine Computation Three, by using Bytes $G'_0, G'_2, G'_8, G'_{10}, M'_0, M'_2, M'_8, M'_{10}, R'_0, R'_2, R'_8, R'_{10}, V'_0, V'_2, V'_8, V'_{10}, Z'_0, Z'_2, Z'_8$ and Z'_{10} and we will get the same input to Round 6 as the AES algorithm.

Routine Computation Three

Initial round : ARK

Round 1: SB SR MC ARK $\oplus$

Round 2: SB SR MC ARK $\oplus$

Round 3: SB SR MC ARK $\oplus$

Round 4: SB SR MC.

Remark 2: The most important part of the δ algorithm is solving those eight groups of linear Equations (1)–(8). There is one question needs to be answered. The question is: Are these eight groups of linear equations independent? The answer to this question is choosing different values of Bytes $G'_0, G'_2, G'_8, G'_{10}$ if we face such situations. Among the

20 bytes: $G'_0, G'_2, G'_8, G'_{10}, M'_0, M'_2, M'_8, M'_{10}, R'_0, R'_2, R'_8, R'_{10}, V'_0, V'_2, V'_8, V'_{10}, Z'_0, Z'_2, Z'_8, Z'_{10}$, we can select the values of G'_0, G'_2, G'_8 and G'_{10} freely. As we showed in Remark 1, there are $2^{32}-1$ combinations of these four bytes, and correspondingly, we can have $2^{32}-1$ intermediate values in Figure 3, starting with SB in Round 2 and ending with ARK in Round 10. If we meet any dependent equations, we can overcome this problem by choosing different values of Bytes G'_0, G'_2, G'_8 and G'_{10} . Therefore, this question will not cause any trouble. So far, we have not met any dependent equations in our large-sample experiments.

Remark 3: From Remark 1, we note that there is more than one combination of the 20 output bytes of algorithm δ for a given pair of (P, K) .

Remark 4: For distinct plaintext and cipher key pairs (P, K) , algorithm δ needs to perform individual computations to decide the values of the 20 bytes.

3.2 Variants of algorithm δ

We show that there are other variants of the δ algorithm. In Section 3.1, the locations of the 20 bytes are $\{0, 2, 8, 10\}$, and there are three other combinations, which are $\{4, 6, 12, 14\}$, $\{1, 3, 9, 11\}$ and $\{5, 7, 13, 15\}$. Figure 4 outlines different locations of the 20 bytes. In Figure 3, $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}$ operate in Round $\{1, 2, 3, 4, 5\}$, and they can also operate in Rounds $\{2, 3, 4, 5, 6\}$, $\{3, 4, 5, 6, 7\}$, $\{4, 5, 6, 7, 8\}$ or $\{5, 6, 7, 8, 9\}$. Therefore, there are 4 different combinations for the byte locations, and there are five different combinations for the round numbers in AES-128. In total, there are 20 ($= 4 \times 5$) variants of the δ algorithm for AES-128. The δ algorithm has 28 ($= 4 \times 7$) variants for AES-192, and 36 ($= 4 \times 9$) variants for AES-256.

Figure 4 Different locations of the 20 bytes

0	0	0	0	0	G'_4	0	G'_{12}	0	0	0	0
G'_1	0	G'_9	0	0	0	0	0	0	G'_5	0	G'_{13}
0	0	0	0	0	0	G'_6	0	G'_{14}	0	0	0
G'_3	0	G'_{11}	0	0	0	0	0	0	0	G'_7	0
0	0	0	0	0	0	M'_4	0	M'_{12}	0	0	0
M'_1	0	M'_9	0	0	0	0	0	0	0	M'_5	0
0	0	0	0	0	0	0	M'_6	0	M'_{14}	0	0
M'_3	0	M'_{11}	0	0	0	0	0	0	0	0	M'_7
0	0	0	0	0	0	0	R'_4	0	R'_{12}	0	0
R'_1	0	R'_9	0	0	0	0	0	0	0	R'_5	0
0	0	0	0	0	0	0	R'_6	0	R'_{14}	0	0
R'_3	0	R'_{11}	0	0	0	0	0	0	0	0	R'_7
0	0	0	0	0	0	0	V'_4	0	V'_{12}	0	0
V'_1	0	V'_9	0	0	0	0	0	0	0	0	V'_5
0	0	0	0	0	0	0	V'_6	0	V'_{14}	0	0
V'_3	0	V'_{11}	0	0	0	0	0	0	0	0	V'_7
0	0	0	0	0	0	0	Z'_4	0	Z'_{12}	0	0
Z'_1	0	Z'_9	0	0	0	0	0	0	0	0	Z'_5
0	0	0	0	0	0	0	Z'_6	0	Z'_{14}	0	0
Z'_3	0	Z'_{11}	0	0	0	0	0	0	0	0	Z'_7

4 The modified version of the AES: δ AES

By employing the δ algorithm, we propose a modified version of the AES, which is named δ AES. The major difference between the AES and the δ AES is that the δ AES uses modified AES round keys. In Figure 3 in Section 3, we apply 20 extra XOR operations to the intermediate values after ARK in Rounds 1–5 by using Bytes $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}, i \in \{0, 2, 8, 10\}$. The construction of the δ AES comes from the fact that we can use Bytes $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}$ to XOR with AES round key 1–5 (instead of with the intermediate values after ARK), and we still get the same result, $i \in \{0, 2, 8, 10\}$. There are 20-byte differences between the AES round keys and the δ AES round keys. The δ AES employs the same key scheduling algorithm, constants and round function (i.e. SB, SR, MC and ARK) as the AES.

The construction of the δ AES is adding two procedures, which are calling the δ algorithm and modifying the AES round keys, to the AES algorithm.

- 1 Suppose for a plaintext P and a cipher key K , the AES algorithm produces a ciphertext C , written as $C = \text{AES}_K(P)$.
- 2 By accepting P and K as two inputs, use the δ algorithm to generate 20 output bytes:
$$\{G'_i, M'_i, R'_i, V'_i, Z'_i\}, i \in \{0, 2, 8, 10\}^1$$
- 3 Apply the AES key scheduling algorithm to K and get the round keys.
- 4 Use $\{G'_i, M'_i, R'_i, V'_i, Z'_i\}$ to XOR with the corresponding AES round keys and get the round keys for the δ AES, $i \in \{0, 2, 8, 10\}$. The details of computing the δ AES round keys is described in Section 4.1.
- 5 After carrying out the transformations above, the δ AES uses the same round function (i.e. SB, SR, MC and ARK) to process the plaintext P with modified AES round keys, and finally, the δ AES also generates the same cipher-text C , denoted by $C = \delta\text{AES}(P)$. Appendix provides some examples of the AES and the AES with 20 extra exclusive-or operations.

4.1 AES round keys and δ AES round keys

Suppose K is a 128-bit AES cipher key, and after key expansion, the AES round keys are denoted by

K_0^i	K_4^i	K_8^i	K_{12}^i
K_1^i	K_5^i	K_9^i	K_{13}^i
K_2^i	K_6^i	K_{10}^i	K_{14}^i
K_3^i	K_7^i	K_{11}^i	K_{15}^i

where i is the round number, $i \in \{1, 2, \dots, 10\}$. The round key used in the initial round is the secret key K itself, and the secret key is denoted without the superscript i .

The δ AES round keys come from the following routine (see Figure 5):

- 1 In Initial Round, Rounds 6–10, use the corresponding AES round keys without any changes.
- 2 In Rounds 1–5, use the modified AES round keys. After applying 20 XOR operations to the AES round

keys, the δ AES round key i is calculated by the following formulas:

$$\begin{cases} K_y^i \oplus \beta, & y \in \{0, 2, 8, 10\} \\ K_y^i, & y \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15\} \end{cases}$$

where y is the byte index of the block, $i \in \{1, 2, 3, 4, 5\}$ and β is equal to G'_y, M'_y, R'_y, V'_y or Z'_y when i is equal to 1, 2, 3, 4 or 5, respectively.

Figure 5 AES round keys and the corresponding δ AES round keys

AES Round Keys					The Corresponding δ AES Round Keys				
cipher Key K	K_0	K_4	K_8	K_{12}	=	K_0	K_4	K_8	K_{12}
	K_1	K_5	K_9	K_{13}		K_1	K_5	K_9	K_{13}
	K_2	K_6	K_{10}	K_{14}		K_2	K_6	K_{10}	K_{14}
	K_3	K_7	K_{11}	K_{15}		K_3	K_7	K_{11}	K_{15}
Initial Round	K_0^1	K_4^1	K_8^1	K_{12}^1	$\oplus$	G_0'	0	G_8'	0
	K_1^1	K_5^1	K_9^1	K_{13}^1		0	0	0	0
	K_2^1	K_6^1	K_{10}^1	K_{14}^1		G_2'	0	G_{10}'	0
	K_3^1	K_7^1	K_{11}^1	K_{15}^1		0	0	0	0
Round Key 1	K_0^2	K_4^2	K_8^2	K_{12}^2	$\oplus$	M_0'	0	M_8'	0
	K_1^2	K_5^2	K_9^2	K_{13}^2		0	0	0	0
	K_2^2	K_6^2	K_{10}^2	K_{14}^2		M_2'	0	M_{10}'	0
	K_3^2	K_7^2	K_{11}^2	K_{15}^2		0	0	0	0
Round Key 2	K_0^3	K_4^3	K_8^3	K_{12}^3	$\oplus$	R_0'	0	R_8'	0
	K_1^3	K_5^3	K_9^3	K_{13}^3		0	0	0	0
	K_2^3	K_6^3	K_{10}^3	K_{14}^3		R_2'	0	R_{10}'	0
	K_3^3	K_7^3	K_{11}^3	K_{15}^3		0	0	0	0
Round Key 3	K_0^4	K_4^4	K_8^4	K_{12}^4	$\oplus$	V_0'	0	V_8'	0
	K_1^4	K_5^4	K_9^4	K_{13}^4		0	0	0	0
	K_2^4	K_6^4	K_{10}^4	K_{14}^4		V_2'	0	V_{10}'	0
	K_3^4	K_7^4	K_{11}^4	K_{15}^4		0	0	0	0
Round Key 4	K_0^5	K_4^5	K_8^5	K_{12}^5	$\oplus$	Z_0'	0	Z_8'	0
	K_1^5	K_5^5	K_9^5	K_{13}^5		0	0	0	0
	K_2^5	K_6^5	K_{10}^5	K_{14}^5		Z_2'	0	Z_{10}'	0
	K_3^5	K_7^5	K_{11}^5	K_{15}^5		0	0	0	0
Round Key 5	K_0^6	K_4^6	K_8^6	K_{12}^6	$\oplus$	K_0^6	K_4^6	K_8^6	K_{12}^6
	K_1^6	K_5^6	K_9^6	K_{13}^6		K_1^6	K_5^6	K_9^6	K_{13}^6
	K_2^6	K_6^6	K_{10}^6	K_{14}^6		K_2^6	K_6^6	K_{10}^6	K_{14}^6
	K_3^6	K_7^6	K_{11}^6	K_{15}^6		K_3^6	K_7^6	K_{11}^6	K_{15}^6
Round Key 6	K_0^7	K_4^7	K_8^7	K_{12}^7	$\oplus$	K_0^7	K_4^7	K_8^7	K_{12}^7
	K_1^7	K_5^7	K_9^7	K_{13}^7		K_1^7	K_5^7	K_9^7	K_{13}^7
	K_2^7	K_6^7	K_{10}^7	K_{14}^7		K_2^7	K_6^7	K_{10}^7	K_{14}^7
	K_3^7	K_7^7	K_{11}^7	K_{15}^7		K_3^7	K_7^7	K_{11}^7	K_{15}^7
Round Key 7	K_0^8	K_4^8	K_8^8	K_{12}^8	$\oplus$	K_0^8	K_4^8	K_8^8	K_{12}^8
	K_1^8	K_5^8	K_9^8	K_{13}^8		K_1^8	K_5^8	K_9^8	K_{13}^8
	K_2^8	K_6^8	K_{10}^8	K_{14}^8		K_2^8	K_6^8	K_{10}^8	K_{14}^8
	K_3^8	K_7^8	K_{11}^8	K_{15}^8		K_3^8	K_7^8	K_{11}^8	K_{15}^8
Round Key 8	K_0^9	K_4^9	K_8^9	K_{12}^9	$\oplus$	K_0^9	K_4^9	K_8^9	K_{12}^9
	K_1^9	K_5^9	K_9^9	K_{13}^9		K_1^9	K_5^9	K_9^9	K_{13}^9
	K_2^9	K_6^9	K_{10}^9	K_{14}^9		K_2^9	K_6^9	K_{10}^9	K_{14}^9
	K_3^9	K_7^9	K_{11}^9	K_{15}^9		K_3^9	K_7^9	K_{11}^9	K_{15}^9
Round Key 9	K_0^{10}	K_4^{10}	K_8^{10}	K_{12}^{10}	$\oplus$	K_0^{10}	K_4^{10}	K_8^{10}	K_{12}^{10}
	K_1^{10}	K_5^{10}	K_9^{10}	K_{13}^{10}		K_1^{10}	K_5^{10}	K_9^{10}	K_{13}^{10}
	K_2^{10}	K_6^{10}	K_{10}^{10}	K_{14}^{10}		K_2^{10}	K_6^{10}	K_{10}^{10}	K_{14}^{10}
	K_3^{10}	K_7^{10}	K_{11}^{10}	K_{15}^{10}		K_3^{10}	K_7^{10}	K_{11}^{10}	K_{15}^{10}
Round Key 10	K_0^{11}	K_4^{11}	K_8^{11}	K_{12}^{11}	$\oplus$	K_0^{11}	K_4^{11}	K_8^{11}	K_{12}^{11}
	K_1^{11}	K_5^{11}	K_9^{11}	K_{13}^{11}		K_1^{11}	K_5^{11}	K_9^{11}	K_{13}^{11}
	K_2^{11}	K_6^{11}	K_{10}^{11}	K_{14}^{11}		K_2^{11}	K_6^{11}	K_{10}^{11}	K_{14}^{11}
	K_3^{11}	K_7^{11}	K_{11}^{11}	K_{15}^{11}		K_3^{11}	K_7^{11}	K_{11}^{11}	K_{15}^{11}

Compared with the AES algorithm, the δ AES needs to do some extra transformations, that is, calling the δ algorithm and modifying the AES round keys. Moreover, for distinct plaintext and cipher key pairs (P, K) , the δ AES needs to carry out individual computations to get Bytes $\{G'_i, M'_i, R'_i, V'_i, Z'_i\} \in \{0, 2, 8, 10\}$.

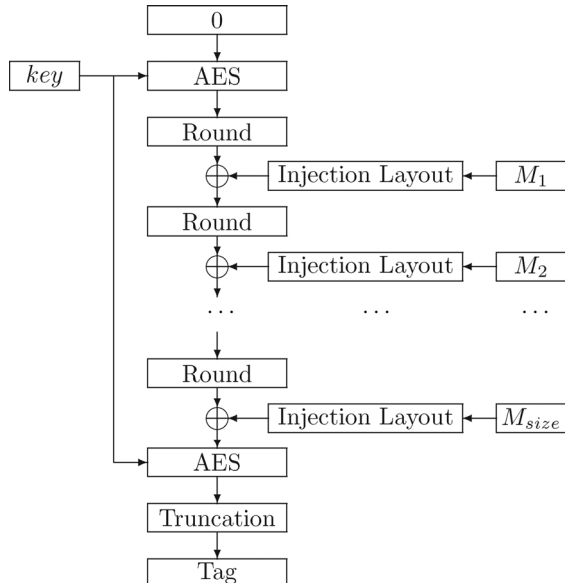
5 Description of the ALPHA-MAC

ALPHA-MAC is a MAC function which uses the building blocks of AES. Similarly to AES, the ALPHA-MAC supports keys of 128, 192 and 256 bits. The word length is 32 bits, and the injection layout places the 4 bytes of each message word $[m_0, m_1, m_2, m_3]$ into a 4×4 array. The format of the injection layout is shown as follows:

$$\begin{bmatrix} m_0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & 0 \\ m_2 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Like AES, the ALPHA-MAC round function contains SB, SR, MC and ARK, and the output of each injection layout acts as the corresponding 128-bit round key. The message padding method appends a single 1 followed by the minimum number of 0 bits such that the length of the result is a multiple of 32. In the initialisation, the state is set to all zeros and AES is applied to the state. For every message word, the chaining method carries out an iteration, and each iteration maps the bits of the message word to an injection input. After that, a sequence of AES round functions are applied to the state, with the round keys replaced by the injection input. In the final transformation, AES is applied to the state. The MAC tag is the first l_m bits of the resulting final state. The length of l_m may have any value less than or equal to 128. The ALPHA-MAC function is depicted in Figure 6.

Figure 6 ALPHA-MAC construction



6 Applying the property to ALPHA-MAC

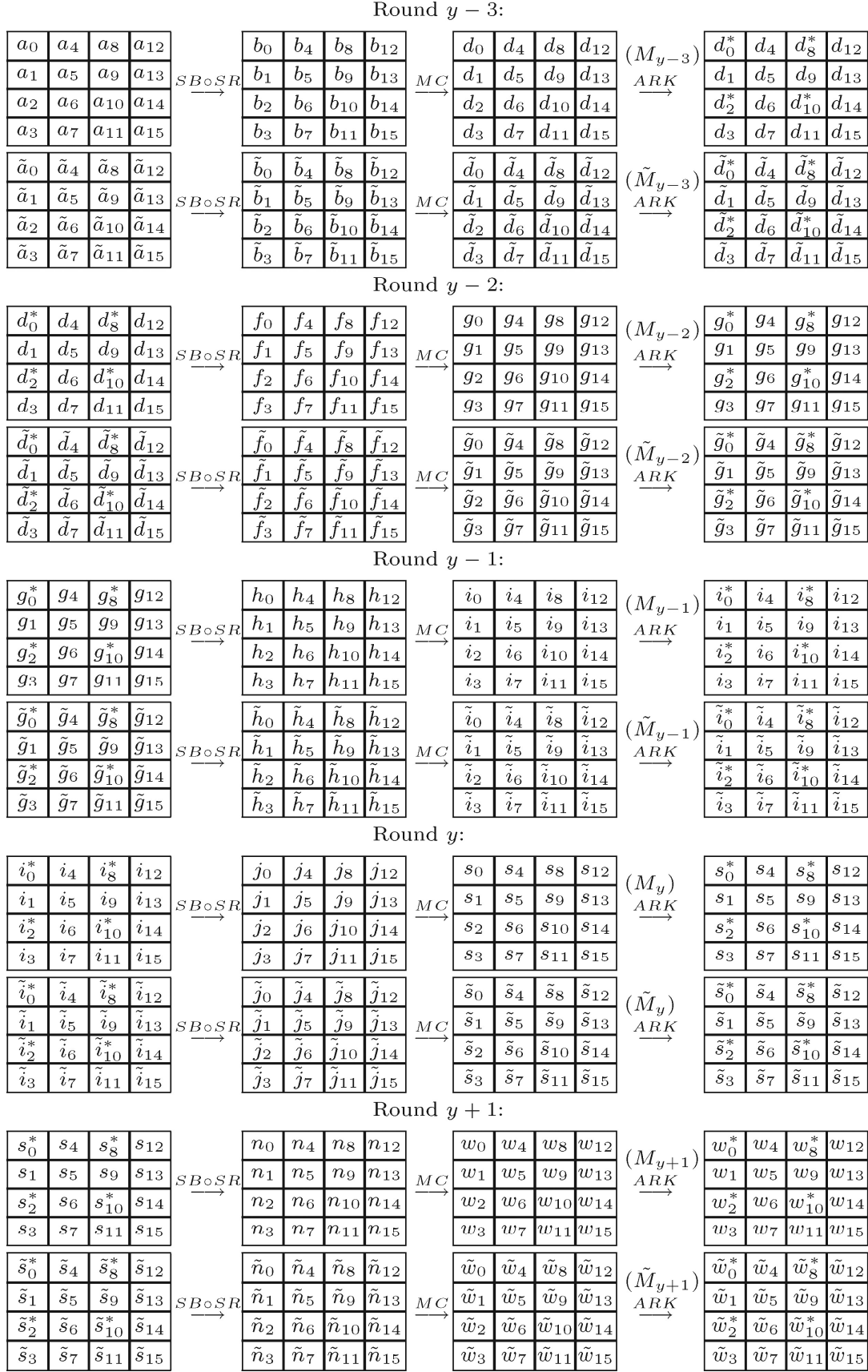
We study the internal structure of the ALPHA-MAC by employing the proposed five-round algebraic property of AES, which is described in Section 3. Firstly, we present a method to find second preimages of the ALPHA-MAC by solving eight groups of linear functions, based on the assumption that an authentication key or an intermediate value of this MAC is known. Each of these eight groups of linear functions contains two equations. We divide the second-preimage search algorithm into two steps: the backwards-and-forwards (BNF) search and the backwards-and-backwards (BNB) search. The BNF search provides an idea for extending 32- to 128-bit collisions² by solving four groups of linear functions. Given a key (or an intermediate value) and one four-block message, the BNB search can generate another four-block message such that these two messages produce 32-bit collisions, which are a prerequisite for the BNF search. To do the BNB search, we need to solve another four groups of linear functions. By combining the BNB search with the BNF search, we can find second preimages of ALPHA-MAC. Secondly, we show that the second-preimage finding method can also be used to generate internal collisions. The proposed collision search method can find two five-block messages such that they produce 128-bit collisions under a selected key (or a selected intermediate value).

6.1 The second-preimage search algorithm

The second-preimage search algorithm aims to find a five-block second-preimage $\tilde{M}$ for a selected five-block message M , under a selected key (or a selected intermediate value). The assumption of this search is that we know two values: a selected key (or a selected intermediate value) and a selected five-block message M . The result of the search is that M and $\tilde{M}$ generate the same 128-bit value after five rounds of ALPHA-MAC iterations, under the selected key (or the selected intermediate value).

We use Figure 7 to illustrate the second-preimage search. Figure 7 depicts five consecutive rounds of the ALPHA-MAC for two different five-block messages M and $\tilde{M}$. We assume that we are able to select an intermediate value of the round functions in some round (e.g. in Round $y-3$), and select five consecutive message blocks $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$. Then we can find another five-block message $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y, \tilde{M}_{y+1})$ such that these two five-block messages collide on 128 bits in Round $y+1$ after ARK. Note that the intermediate value is:

$$\begin{bmatrix} a_0 & a_4 & a_8 & a_{12} \\ a_1 & a_5 & a_9 & a_{13} \\ a_2 & a_6 & a_{10} & a_{14} \\ a_3 & a_7 & a_{11} & a_{15} \end{bmatrix}$$

Figure 7 The five-block collisions

In the case of a selected key, for the sake of simplicity, we assume that $(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$ are the first five blocks of the selected message. Our search algorithm works without assuming that $(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$ are the first five blocks of the selected message.

The second-preimage search algorithm has the following form:

Known:	1 A selected key or a selected intermediate value.
	2 A selected five-block message $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$
Find:	Another five-block message $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y, \tilde{M}_{y+1})$ such that M and $\tilde{M}$ collide on 128 bits after ARK in Round $y + 1$.
Method:	Solve eight groups of linear functions. These eight groups of functions are named as (9)–(16) in this section.

The second-preimage search algorithm consists of two steps: the BNF search and the BNB search. The BNF search can extend 32- to 128-bit collisions, given two messages M and $\tilde{M}$ which collide on 32 bits, namely Bytes s_4, s_{12}, s_6 and s_{14} , after MC in Round y (see Figure 7). Given a key (or an intermediate value) and one four-block message, the BNB search is able to find another four-block message such that these two messages collide on Bytes s_4, s_{12}, s_6 and s_{14} after MC in Round y . The BNB search generates those 32-bit collisions which are required for the BNF search. By merging the BNB search with the BNF search, we can find second preimages of the ALPHA-MAC.

6.1.1 The BNF search

The BNF search has the following form:

Known:	1 A selected key or a selected intermediate value.
	2 Two four-block messages $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$ and $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y, \tilde{M}_{y+1})$ colliding on 32 bits (Bytes s_4, s_{12}, s_6 and s_{14}) after MC in Round y .
Extend:	32-bit collisions to 128-bit collisions in Round $y + 1$.
Method:	Solve four groups of linear functions. These four groups of functions are numbered as (9)–(12) in this section.

The BNF search assumes that we are able to find two messages M and $\tilde{M}$, which collide on Bytes s_4, s_{12}, s_6 and s_{14} after MC in Round y . Based on the algebraic property of the MC transformation and the structure of ALPHA-MAC,

we can extend these 32- to 128-bit collisions within three rounds by solving four groups of linear equations.

6.1.2 Extending 32- to 64-bit collisions

We use the differential XOR property before and after the MC transformation. In Round y before MC, by XORing those two intermediate values, we get the following result:

$$\begin{bmatrix} \tilde{j}_0 \oplus j_0 & \tilde{j}_4 \oplus j_4 & \tilde{j}_8 \oplus j_8 & \tilde{j}_{12} \oplus j_{12} \\ \tilde{j}_1 \oplus j_1 & \tilde{j}_5 \oplus j_5 & \tilde{j}_9 \oplus j_9 & \tilde{j}_{13} \oplus j_{13} \\ \tilde{j}_2 \oplus j_2 & \tilde{j}_6 \oplus j_6 & \tilde{j}_{10} \oplus j_{10} & \tilde{j}_{14} \oplus j_{14} \\ \tilde{j}_3 \oplus j_3 & \tilde{j}_7 \oplus j_7 & \tilde{j}_{11} \oplus j_{11} & \tilde{j}_{15} \oplus j_{15} \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} ? & 0 & ? & 0 \\ 0 & \tilde{s}_5 \oplus s_5 & 0 & \tilde{s}_{13} \oplus s_{13} \\ ? & 0 & ? & 0 \\ 0 & \tilde{s}_7 \oplus s_7 & 0 & \tilde{s}_{15} \oplus s_{15} \end{bmatrix}$$

Here, we use R (to replace $\tilde{j}_0 \oplus j_0$), S (to replace $\tilde{j}_8 \oplus j_8$), T (to replace $\tilde{j}_2 \oplus j_2$) and U (to replace $\tilde{j}_{10} \oplus j_{10}$) so that after the MC transformation in Round y , Bytes $\tilde{s}_1 \oplus s_1, \tilde{s}_3 \oplus s_3, \tilde{s}_9 \oplus s_9$ and $\tilde{s}_{11} \oplus s_{11}$ become zero. Now the question is ‘how to decide R, S, T and U ’. The answer is:

- there exists one and only one pair of (R, T) such that after MC, Bytes $\tilde{s}_1 \oplus s_1$ and $\tilde{s}_3 \oplus s_3$ are both zero
- there exists one and only one pair of (S, U) such that after MC, $\tilde{s}_9 \oplus s_9$ and $\tilde{s}_{11} \oplus s_{11}$ are both zero.

According to the MC transformation, we have the following formula:

$$\begin{bmatrix} ? & 0 & ? & 0 \\ 0 & \tilde{s}_5 \oplus s_5 & 0 & \tilde{s}_{13} \oplus s_{13} \\ ? & 0 & ? & 0 \\ 0 & \tilde{s}_7 \oplus s_7 & 0 & \tilde{s}_{15} \oplus s_{15} \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} R & \tilde{j}_4 \oplus j_4 & S & \tilde{j}_{12} \oplus j_{12} \\ \tilde{j}_1 \oplus j_1 & \tilde{j}_5 \oplus j_5 & \tilde{j}_9 \oplus j_9 & \tilde{j}_{13} \oplus j_{13} \\ T & \tilde{j}_6 \oplus j_6 & U & \tilde{j}_{14} \oplus j_{14} \\ \tilde{j}_3 \oplus j_3 & \tilde{j}_7 \oplus j_7 & \tilde{j}_{11} \oplus j_{11} & \tilde{j}_{15} \oplus j_{15} \end{bmatrix}$$

To find out the values of (R, T) and (S, U) , we need to solve the following two groups of equations.

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} R \\ \tilde{j}_1 \oplus j_1 \\ T \\ \tilde{j}_3 \oplus j_3 \end{bmatrix} = 0 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} R \\ \tilde{j}_1 \oplus j_1 \\ T \\ \tilde{j}_3 \oplus j_3 \end{bmatrix} = 0 \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} S \\ \tilde{j}_9 \oplus j_9 \\ U \\ \tilde{j}_{11} \oplus j_{11} \end{bmatrix} = 0 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} S \\ \tilde{j}_9 \oplus j_9 \\ U \\ \tilde{j}_{11} \oplus j_{11} \end{bmatrix} = 0 \end{array} \right. \quad (10)$$

In the two equations in (9), there are two variables R and T , and therefore there exists one and only one pair of (R, T) to make these two equations hold simultaneously. Similarly, we can decide the values of S and U by solving the two equations in (10).

Once we get the values of R, S, T and U , message block $\tilde{M}_{y-1}$ can be constructed as follows:

- 1 Set the values of $\tilde{j}_0^{\text{new}}, \tilde{j}_8^{\text{new}}, \tilde{j}_2^{\text{new}}, \tilde{j}_{10}^{\text{new}}$, as follows:

$$\tilde{j}_{10}^{\text{new}} = j_0 \oplus R, \tilde{j}_8^{\text{new}} = j_8 \oplus S, \tilde{j}_2^{\text{new}} = \tilde{j}_2 \oplus T \text{ and}$$

$\tilde{j}_{10}^{\text{new}} = j_{10} \oplus U$. Use $\tilde{j}_0^{\text{new}}$ to replace $\tilde{j}_0$, $\tilde{j}_8^{\text{new}}$ to replace $\tilde{j}_8$, $\tilde{j}_2^{\text{new}}$ to replace $\tilde{j}_2$ and $\tilde{j}_{10}^{\text{new}}$ to replace $\tilde{j}_{10}$.

- 2 Perform SR^{-1} (inverse SR) and SB^{-1} (inverse SB). As SR^{-1} and SB^{-1} are permutation and substitution, they do not change the properties we have found. Now we have the outputs of ARK in Round $y - 1$.
- 3 Compute the value of $\tilde{M}_{y-1}^{\text{new}}$ as follows:

$$\tilde{M}_{y-1}^{\text{new}} = (\tilde{j}_0^{\text{new}} \oplus \tilde{i}_0) \parallel (\tilde{j}_8^{\text{new}} \oplus \tilde{i}_8) \parallel (\tilde{j}_{10}^{\text{new}} \oplus \tilde{i}_{10}) \parallel (\tilde{j}_2^{\text{new}} \oplus \tilde{i}_2)$$

Use $\tilde{M}_{y-1}^{\text{new}}$ to replace $\tilde{M}_{y-1}$.

At this stage, two messages $(M_{y-3}, M_{y-2}, M_{y-1})$ and $(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}^{\text{new}})$ collide on 64 bits (Bytes $s_4, s_{12}, s_6, s_{14}, s_1, s_9, s_3$ and s_{11}) in Round y after MC.

6.1.3 Extending 64- to 96-bit collisions

We only need to focus on Rounds y and $y + 1$ to extend 64- to 96-bit collisions. The idea is to choose message block $\tilde{M}_y$ to cancel out the differences between Bytes $(s_5, s_{13}, s_7, s_{15})$ and Bytes $(\tilde{s}_5, \tilde{s}_{13}, \tilde{s}_7, \tilde{s}_{15})$ in Round y . The method of choosing $\tilde{M}_y$ is exactly same as the method for constructing $\tilde{M}_{y-1}$ in Section 6.1.2.

By taking the outputs of ARK in Round y , we perform the SB and SR operations, and then XOR the results after SB and SR:

$$\begin{bmatrix} n_0 & n_4 & n_8 & n_{12} \\ n_1 & n_5 & n_9 & n_{13} \\ n_2 & n_6 & n_{10} & n_{14} \\ n_3 & n_7 & n_{11} & n_{15} \end{bmatrix} \oplus \begin{bmatrix} \tilde{n}_0 & n_4 & \tilde{n}_8 & n_{12} \\ \tilde{n}_1 & n_5 & \tilde{n}_9 & n_{13} \\ \tilde{n}_2 & n_6 & \tilde{n}_{10} & n_{14} \\ \tilde{n}_3 & n_7 & \tilde{n}_{11} & n_{15} \end{bmatrix} = \begin{bmatrix} n_0 \oplus \tilde{n}_0 & 0 & n_8 \oplus \tilde{n}_8 & 0 \\ n_1 \oplus \tilde{n}_1 & 0 & n_9 \oplus \tilde{n}_9 & 0 \\ n_2 \oplus \tilde{n}_2 & 0 & n_{10} \oplus \tilde{n}_{10} & 0 \\ n_3 \oplus \tilde{n}_3 & 0 & n_{11} \oplus \tilde{n}_{11} & 0 \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} ? & 0 & ? & 0 \\ 0 & 0 & 0 & 0 \\ ? & 0 & ? & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, we use π to replace $n_0 \oplus \tilde{n}_0$, ρ to replace $n_8 \oplus \tilde{n}_8$, ϕ to replace $n_2 \oplus \tilde{n}_2$ and ω to replace $n_{10} \oplus \tilde{n}_{10}$ so that after MC in Round $y + 1$, Bytes $w_1 \oplus \tilde{w}_1$, $w_9 \oplus \tilde{w}_9$, $w_3 \oplus \tilde{w}_3$ and $w_{11} \oplus \tilde{w}_{11}$ are zero:

$$\begin{bmatrix} \pi & 0 & \rho & 0 \\ n_1 \oplus \tilde{n}_1 & 0 & n_9 \oplus \tilde{n}_9 & 0 \\ \phi & 0 & \omega & 0 \\ n_3 \oplus \tilde{n}_3 & 0 & n_{11} \oplus \tilde{n}_{11} & 0 \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} ? & 0 & ? & 0 \\ 0 & 0 & 0 & 0 \\ ? & 0 & ? & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now the question is ‘how to decide π, ρ, ϕ and ω ’. The answer is:

- There exists one and only one pair of (π, ϕ) such that after MC, Bytes $w_1 \oplus \tilde{w}_1$ and $w_3 \oplus \tilde{w}_3$ are both zero. The values of (π, ϕ) can be decided by solving (11).
- There exists one and only one pair of (ρ, ω) such that after MC, Bytes $w_9 \oplus \tilde{w}_9$ and $w_{11} \oplus \tilde{w}_{11}$ are both zero. By solving (12), we get the values of (ρ, ω) .

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} \pi \\ n_1 \oplus \tilde{n}_1 \\ \phi \\ n_3 \oplus \tilde{n}_3 \end{bmatrix} = 0 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} \pi \\ n_1 \oplus \tilde{n}_1 \\ \phi \\ n_3 \oplus \tilde{n}_3 \end{bmatrix} = 0 \end{array} \right. \quad (11)$$

$$\left\{ \begin{array}{l} [01 \ 02 \ 03 \ 01] \begin{bmatrix} \rho \\ n_9 \oplus \tilde{n}_9 \\ \omega \\ n_{11} \oplus \tilde{n}_{11} \end{bmatrix} = 0 \\ [03 \ 01 \ 01 \ 02] \begin{bmatrix} \rho \\ n_9 \oplus \tilde{n}_9 \\ \omega \\ n_{11} \oplus \tilde{n}_{11} \end{bmatrix} = 0 \end{array} \right. \quad (12)$$

Once we know the values of π , ϕ , ρ and ω , message block $\tilde{M}_y$ can be chosen as follows:

- 1 Set the values of $\tilde{n}_0^{\text{new}}$, $\tilde{n}_8^{\text{new}}$, $\tilde{n}_2^{\text{new}}$ and $\tilde{n}_{10}^{\text{new}}$ as follows:
 $\tilde{n}_0^{\text{new}} = n_0 \oplus \pi$, $\tilde{n}_8^{\text{new}} = n_8 \oplus \rho$, $\tilde{n}_2^{\text{new}} = n_2 \oplus \phi$ and
 $\tilde{n}_{10}^{\text{new}} = n_{10} \oplus \omega$. Use $\tilde{n}_0^{\text{new}}$ to replace $\tilde{n}_0$, $\tilde{n}_8^{\text{new}}$ to replace $\tilde{n}_8$, $\tilde{n}_2^{\text{new}}$ to replace $\tilde{n}_2$ and $\tilde{n}_{10}^{\text{new}}$ to replace $\tilde{n}_{10}$.
- 2 Perform SR^{-1} and SB^{-1} . Since SR^{-1} and SB^{-1} are permutation and substitution, they do not affect the properties we have found. Now we have the outputs of ARK in Round y .
- 3 Compute the value of $\tilde{M}_y$ as follows:

$$\tilde{M}_y = \left(\tilde{n}_0^{\text{new}} \oplus \tilde{s}_0 \right) \left\| \left(\tilde{n}_8^{\text{new}} \oplus \tilde{s}_8 \right) \right\| \left(\tilde{n}_{10}^{\text{new}} \oplus \tilde{s}_2 \right) \left\| \left(\tilde{n}_2^{\text{new}} \oplus \tilde{s}_{10} \right) \right\|$$

So far, two messages $(M_{y-3}, M_{y-2}, M_{y-1}, M_y)$ and $(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y)$ collide on 96 bits (i.e. Bytes $w_1, w_3, w_4, w_5, w_6, w_7, w_9, w_{11}, w_{12}, w_{13}, w_{14}$ and w_{15}) in Round $y+1$ after MC transformation.

6.1.4 Extending 96- to 128-bit collisions

This step is straightforward as we can select message M_{y+1} arbitrarily, and construct message $\tilde{M}_{y+1}$ to cancel the differences between Bytes w_0, w_8, w_2 and w_{10} . The construction is provided as follows:

$$\tilde{M}_{y+1} = \left((w_0 \oplus \tilde{w}_0) \right\| (w_8 \oplus \tilde{w}_8) \left\| (w_2 \oplus \tilde{w}_2) \right\| (w_{10} \oplus \tilde{w}_{10}) \right) \oplus M_{y+1}$$

6.1.5 The BNB search

The BNB search has the following form:

Known:	1 A selected key or a selected intermediate value. 2 One selected four-block message $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y)$
Find:	Another four-block message $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y)$ such that these two messages collide on 32 bits (Bytes s_4, s_{12}, s_6 and s_{14}) after MC in Round y
Method:	Solve four groups of linear functions. These four groups of functions are named as (13)–(16).

We propose a method to find 32-bit collisions on Bytes s_4, s_{12}, s_6 and s_{14} (see Figure 7) by solving four groups of linear

functions. This search assumes that for a selected key (or a selected intermediate value) and a selected four-block message $(M_{y-3}, M_{y-2}, M_{y-1}, M_y)$, we can generate another four-block message $(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y)$ such that these two messages collide on Bytes s_4, s_{12}, s_6 and s_{14} after MC in Round y . The method used by the BNB search is similar to the idea employed by the BNF search, but works in only one direction (i.e. only backwards).

6.1.6 Deciding four values $(\tilde{j}_5, \tilde{j}_7, \tilde{j}_{13}$ and $\tilde{j}_{15})$

In the beginning, we choose $(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y)$ randomly. Assume that the input and the output of MC in Round y are listed as follows:

$$\begin{bmatrix} \tilde{j}_0 & \tilde{j}_4 & \tilde{j}_8 & \tilde{j}_{12} \\ \tilde{j}_1 & \tilde{j}_5^{\text{old}} & \tilde{j}_9 & \tilde{j}_{13}^{\text{old}} \\ \tilde{j}_2 & \tilde{j}_6 & \tilde{j}_{10} & \tilde{j}_{14} \\ \tilde{j}_3 & \tilde{j}_7^{\text{old}} & \tilde{j}_{11} & \tilde{j}_{15}^{\text{old}} \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} \tilde{s}_0 & \tilde{s}_4 & \tilde{s}_8 & \tilde{s}_{12} \\ \tilde{s}_1 & \tilde{s}_5 & \tilde{s}_9 & \tilde{s}_{13} \\ \tilde{s}_2 & \tilde{s}_6 & \tilde{s}_{10} & \tilde{s}_{14} \\ \tilde{s}_3 & \tilde{s}_7 & \tilde{s}_{11} & \tilde{s}_{15} \end{bmatrix}$$

Now we do not use the values of $\tilde{j}_5^{\text{old}}, \tilde{j}_7^{\text{old}}, \tilde{j}_{13}^{\text{old}}$ or $\tilde{j}_{15}^{\text{old}}$. Instead, we use $\tilde{j}_5$ (to replace $\tilde{j}_5^{\text{old}}$), $\tilde{j}_7$ (to replace $\tilde{j}_7^{\text{old}}$), $\tilde{j}_{13}$ (to replace $\tilde{j}_{13}^{\text{old}}$), and $\tilde{j}_{15}$ (to replace $\tilde{j}_{15}^{\text{old}}$) such that we get values $\tilde{s}_4, \tilde{s}_{12}, \tilde{s}_6$ and $\tilde{s}_{14}$, respectively (illustrated as follows):

$$\begin{bmatrix} \tilde{j}_0 & \tilde{j}_4 & \tilde{j}_8 & \tilde{j}_{12} \\ \tilde{j}_1 & \tilde{j}_5 & \tilde{j}_9 & \tilde{j}_{13} \\ \tilde{j}_2 & \tilde{j}_6 & \tilde{j}_{10} & \tilde{j}_{14} \\ \tilde{j}_3 & \tilde{j}_7 & \tilde{j}_{11} & \tilde{j}_{15} \end{bmatrix} \xrightarrow{\text{MC}} \begin{bmatrix} \tilde{s}_0 & \tilde{s}_4 & \tilde{s}_8 & \tilde{s}_{12} \\ \tilde{s}_1 & \tilde{s}_5 & \tilde{s}_9 & \tilde{s}_{13} \\ \tilde{s}_2 & \tilde{s}_6 & \tilde{s}_{10} & \tilde{s}_{14} \\ \tilde{s}_3 & \tilde{s}_7 & \tilde{s}_{11} & \tilde{s}_{15} \end{bmatrix}$$

Now the question is ‘how can we make this happen’. Our answer is to solve two groups of linear functions. For the values of s_4 and s_6 , we have two linear equations in (13) with only two unknown variables ($\tilde{j}_5$ and $\tilde{j}_7$). Therefore, we can solve (13) to obtain the values of $\tilde{j}_5$ and $\tilde{j}_7$

$$\left\{ \begin{array}{l} [02 \ 03 \ 01 \ 01] \begin{bmatrix} \tilde{j}_4 \\ \tilde{j}_5 \\ \tilde{j}_6 \\ \tilde{j}_7 \end{bmatrix} = s_4 \\ [01 \ 01 \ 02 \ 03] \begin{bmatrix} \tilde{j}_4 \\ \tilde{j}_5 \\ \tilde{j}_6 \\ \tilde{j}_7 \end{bmatrix} = s_6 \end{array} \right. \quad (13)$$

$$\left\{ \begin{array}{l} \begin{bmatrix} 02 & 03 & 01 & 01 \end{bmatrix} \begin{bmatrix} \tilde{j}_{12} \\ \tilde{j}_{13} \\ \tilde{j}_{14} \\ \tilde{j}_{15} \end{bmatrix} = s_{12} \\ \begin{bmatrix} 01 & 01 & 02 & 03 \end{bmatrix} \begin{bmatrix} \tilde{j}_{12} \\ \tilde{j}_{13} \\ \tilde{j}_{14} \\ \tilde{j}_{15} \end{bmatrix} = s_{14} \end{array} \right. \quad (14)$$

Similarly, for the values of s_{12} and s_{14} , we have two linear functions in (14) with two unknown variable ($\tilde{j}_{13}$ and $\tilde{j}_{15}$). We can solve (14) to decide the values of $\tilde{j}_{13}$ and $\tilde{j}_{15}$. After getting four vehicles ($\tilde{j}_5, \tilde{j}_7, \tilde{j}_{13}$ and $\tilde{j}_{15}$) decided, we perform the SR^{-1} and SB^{-1} transformations. As SR^{-1} transformation. As SR^{-1} is permutation and SB^{-1} is substitution $\tilde{j}_5, \tilde{j}_7, \tilde{j}_{13}$ and $\tilde{j}_{15}$ are first relocated then substituted by another four values $\tilde{i}_9, \tilde{i}_3, \tilde{i}_1$ and $\tilde{i}_{11}$, respectively. As the message injection layout does not change the values of $\tilde{i}_9, \tilde{i}_3, \tilde{i}_1$ and $\tilde{i}_{11}$ these four values are not changed after we do ARK. So, we get four known values ($\tilde{i}_9, \tilde{i}_3, \tilde{i}_1$ and $\tilde{i}_{11}$) after MC in Round $y-1$. Our next target is to modify message block $\tilde{M}_{y-2}$ so that we get those four values $\tilde{i}_9, \tilde{i}_3, \tilde{i}_1$ and $\tilde{i}_{11}$ after MC in Round $y-1$.

6.1.7 Modifying message block $\tilde{M}_{y-2}$

Suppose by using the original message block $\tilde{M}_{y-2}$, we have the following states in Round $y-1$.

$$\begin{bmatrix} \tilde{g}_0^{*old} & \tilde{g}_4 & \tilde{g}_8^{*old} & \tilde{g}_{12} \\ \tilde{g}_1 & \tilde{g}_5 & \tilde{g}_9 & \tilde{g}_{13} \\ \tilde{g}_2^{*old} & \tilde{g}_6 & \tilde{g}_{10}^{*old} & \tilde{g}_{14} \\ \tilde{g}_3 & \tilde{g}_7 & \tilde{g}_{11} & \tilde{g}_{15} \end{bmatrix} \xrightarrow{\text{SB} \circ \text{SR}} \begin{bmatrix} \tilde{h}_0^{old} & \tilde{h}_4 & \tilde{h}_8^{old} & \tilde{h}_{12} \\ \tilde{h}_1 & \tilde{h}_5 & \tilde{h}_9 & \tilde{h}_{13} \\ \tilde{h}_2^{old} & \tilde{h}_6 & \tilde{h}_{10}^{old} & \tilde{h}_{14} \\ \tilde{h}_3 & \tilde{h}_7 & \tilde{h}_{11} & \tilde{h}_{15} \end{bmatrix}$$

$$\underline{\text{MC}} \begin{bmatrix} ? & \tilde{i}_4 & ? & \tilde{i}_{12} \\ ? & \tilde{i}_5 & ? & \tilde{i}_{13} \\ ? & \tilde{i}_6 & ? & \tilde{i}_{14} \\ ? & \tilde{i}_7 & ? & \tilde{i}_{15} \end{bmatrix}$$

Now we replace values $(\tilde{h}_0^{old}, \tilde{h}_2^{old}, \tilde{h}_8^{old}, \tilde{h}_{10}^{old})$ with $(\tilde{h}_0, \tilde{h}_2, \tilde{h}_8, \tilde{h}_{10})$ and then we get those four values ($\tilde{i}_9, \tilde{i}_3, \tilde{i}_1$ and $\tilde{i}_{11}$) located as follows:

$$\begin{bmatrix} \tilde{g}_0^* & \tilde{g}_4 & \tilde{g}_8^* & \tilde{g}_{12} \\ \tilde{g}_1 & \tilde{g}_5 & \tilde{g}_9 & \tilde{g}_{13} \\ \tilde{g}_2^* & \tilde{g}_6 & \tilde{g}_{10}^* & \tilde{g}_{14} \\ \tilde{g}_3 & \tilde{g}_7 & \tilde{g}_{11} & \tilde{g}_{15} \end{bmatrix} \xrightarrow{\text{SB} \circ \text{SR}} \begin{bmatrix} \tilde{h}_0 & \tilde{h}_4 & \tilde{h}_8 & \tilde{h}_{12} \\ \tilde{h}_1 & \tilde{h}_5 & \tilde{h}_9 & \tilde{h}_{13} \\ \tilde{h}_2 & \tilde{h}_6 & \tilde{h}_{10} & \tilde{h}_{14} \\ \tilde{h}_3 & \tilde{h}_7 & \tilde{h}_{11} & \tilde{h}_{15} \end{bmatrix}$$

$$\underline{\text{MC}} \begin{bmatrix} ? & \tilde{i}_4 & ? & \tilde{i}_{12} \\ \tilde{i}_1 & \tilde{i}_5 & \tilde{i}_9 & \tilde{i}_{13} \\ ? & \tilde{i}_6 & ? & \tilde{i}_{14} \\ \tilde{i}_3 & \tilde{i}_7 & \tilde{i}_{11} & \tilde{i}_{15} \end{bmatrix}$$

Based on the property of MC transformation, we can form the following two groups of linear functions:

$$\left\{ \begin{array}{l} \begin{bmatrix} 01 & 02 & 03 & 01 \end{bmatrix} \begin{bmatrix} \tilde{h}_0 \\ \tilde{h}_1 \\ \tilde{h}_2 \\ \tilde{h}_3 \end{bmatrix} = \tilde{i}_1 \\ \begin{bmatrix} 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} \tilde{h}_0 \\ \tilde{h}_1 \\ \tilde{h}_2 \\ \tilde{h}_3 \end{bmatrix} = \tilde{i}_3 \end{array} \right. \quad (15)$$

$$\left\{ \begin{array}{l} \begin{bmatrix} 01 & 02 & 03 & 01 \end{bmatrix} \begin{bmatrix} \tilde{h}_8 \\ \tilde{h}_9 \\ \tilde{h}_{10} \\ \tilde{h}_{11} \end{bmatrix} = \tilde{i}_9 \\ \begin{bmatrix} 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} \tilde{h}_8 \\ \tilde{h}_9 \\ \tilde{h}_{10} \\ \tilde{h}_{11} \end{bmatrix} = \tilde{i}_{11} \end{array} \right. \quad (16)$$

We know the values of $\tilde{h}_1, \tilde{h}_3, \tilde{h}_9$ and $\tilde{h}_{11}$ from the original message block $\tilde{M}_{y-2}$. We can get the values of $(\tilde{h}_0, \tilde{h}_2)$ by solving (15), and get the values of $(\tilde{h}_8, \tilde{h}_{10})$ by solving (16). After finding the values of $(\tilde{h}_0, \tilde{h}_2, \tilde{h}_8, \tilde{h}_{10})$, we perform SR^{-1} and SB^{-1} , and obtain the corresponding four values $(\tilde{g}_0^*, \tilde{g}_2^*, \tilde{g}_8^*, \tilde{g}_{10}^*)$. Once we know the values of $(\tilde{g}_0^*, \tilde{g}_2^*, \tilde{g}_8^*, \tilde{g}_{10}^*)$, we replace $\tilde{M}_{y-2}$ with $\tilde{M}_{y-2}^{\text{new}}$. $\tilde{M}_{y-2}^{\text{new}}$ is constructed as follows (note that $\tilde{g}_0, \tilde{g}_8, \tilde{g}_2$ and $\tilde{g}_{10}$ are known from the message block $\tilde{M}_{y-3}$ in Round $y-3$):

$$\tilde{M}_{y-3}^{\text{new}} = (\tilde{g}_0^* \oplus \tilde{g}_0) \| (\tilde{g}_8^* \oplus \tilde{g}_8) \| (\tilde{g}_2^* \oplus \tilde{g}_2) \| (\tilde{g}_{10}^* \oplus \tilde{g}_{10})$$

6.1.8 Combining the BNB search with the BNF search

The second-preimage search algorithm combines the BNB search with the BNF search. To search for a second preimage of the ALPHA-MAC, we perform the following steps:

- 1 Select a key or an intermediate value.
- 2 Select a five-block message
 $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$.
- 3 Generate the second preimage
 $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y, \tilde{M}_{y+1})$ randomly. We need to guarantee that $\tilde{M}_{y-3}$ is not equal to M_{y-3} .
- 4 Perform the BNB search to generate 32-bit collisions. The BNB search is done by modifying message block $\tilde{M}_{y-2}$.
- 5 Use the BNF search to extend those 32- to 128-bit collisions. The BNF search is carried out by modifying the values of $\tilde{M}_{y-1}, \tilde{M}_y$ and $\tilde{M}_{y+1}$. Message $\tilde{M}(\tilde{M}_{y-3}, \tilde{M}_{y-2}, \tilde{M}_{y-1}, \tilde{M}_y, \tilde{M}_{y+1})$ is a second preimage of message $M(M_{y-3}, M_{y-2}, M_{y-1}, M_y, M_{y+1})$ under the selected key (or the selected intermediate value).

The routine of finding second preimages is shown in Table 1, and Figure 8 depicts this finding. The name of the BNB search comes from the fact that searching for $\tilde{M}_{y-2}$ is carried out by moving backwards and then backwards, and

the name of the BNF search comes from the fact that searching for $\tilde{M}_{y-1}, \tilde{M}_y$ and $\tilde{M}_{y+1}$ is performed by moving backwards and then forwards (see Table 1). A personal computer takes about 1 sec to find a second preimage of the ALPHA-MAC. A found second preimage of a selected key K (see Table 2) and a selected five-block message M (see Table 3) is $\tilde{M}$ (shown in Table 3). The 128-bit colliding value is listed in Table 4 (note that these two messages are listed after injection layout).

Figure 8 The second-preimage search

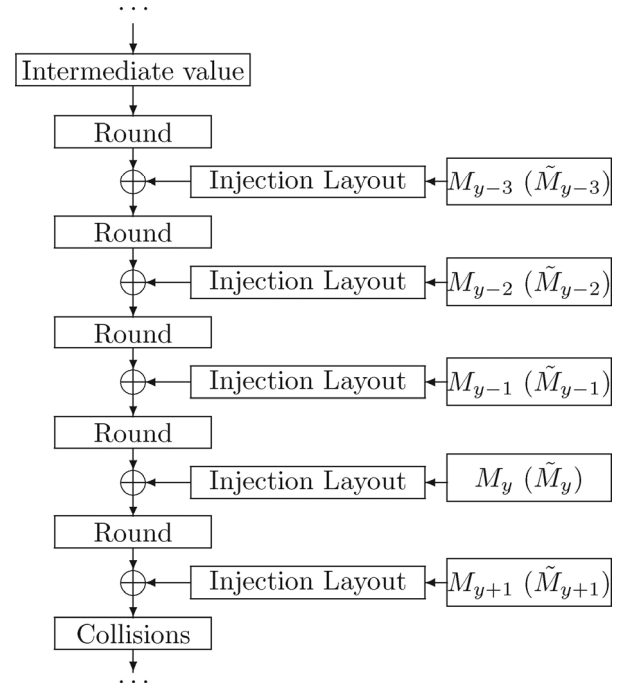


Table 1 Second-preimage search = BNB search + BNF search

Search	R	Round $y - 2$	Di	Round $y - 1$	Di	Round y
BNB	1					$\tilde{s}_4 \rightarrow s_4, \tilde{s}_{12} \rightarrow s_{12},$ $\tilde{s}_6 \rightarrow s_6, \tilde{s}_{14} \rightarrow s_{14}$
	2		$\Leftarrow$	$h_0^{\text{old}} \rightarrow h_0, h_2^{\text{old}} \rightarrow h_2,$ $\tilde{h}_8^{\text{old}} \rightarrow \tilde{h}_8, \tilde{h}_{10}^{\text{old}} \rightarrow \tilde{h}_{10}$		
	3	$M_{y-2} \rightarrow M_{y-2}^{\text{new}}$				
BNF		Round $y - 1$	Di	Round y	Di	Round $y + 1$
	4	Modify M_{y-1}	$\Leftarrow$	collisions on s_4, s_{12}, s_6 and s_{14}		
	5		$\Rightarrow$	collisions on $s_4, s_{12}, s_6, s_{14}, s_1,$ s_9, s_3 and s_{11}		
	6			modify $\tilde{M}_y$	$\Rightarrow$	96-bit collisions
	7					modify $M_{y+1} \rightarrow 128\text{-bit collisions}$

Note: Di – Direction; R – Routine.

Table 2 The selected key K

83	55	2d	81
88	2c	05	67
c1	63	be	c2
2a	a2	52	a4

Table 3 Two five-block messages

M (the selected message)																			
M_{y-3}					M_{y-2}					M_{y-1}					M_y				
c4 0	8c 0	e6 0	2a 0	77 0	fd 0	ef 0	a1 0	81 0	9f 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
94 0	f3 0	95 0	04 0	4c 0	37 0	68 0	09 0	25 0	2c 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
$\tilde{M}$ (the found second preimage)																			
$\tilde{M}_{y-3}$					$\tilde{M}_{y-2}$					$\tilde{M}_{y-1}$					$\tilde{M}_y$				
1d 0	43 0	22 0	04 0	e4 0	83 0	2f 0	e5 0	69 0	06 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1c 0	0d 0	2f 0	30 0	2f 0	9b 0	d4 0	30 0	f4 0	3a 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0

Table 4 The 128-bit collisions

7d	69	88	d7
02	cb	1f	af
b9	d8	7b	5e
0e	10	79	21

6.2 The collision search algorithm

Known: A selected key or a selected intermediate value.

Find: Two five-block messages M and $\tilde{M}$ such that they collide under the selected key or the intermediate value.

Method: Employ the second-preimage search.

In the second-preimage search, we choose the first five-block message arbitrarily, and once it is decided, we do not modify it. All we need to do is modify the second five-block message so that 128-bit collisions happen. Therefore, the second-preimage search can also be used to find two colliding five-block messages under a selected key (or a selected intermediate value).

7 Conclusion

We described a five-round algebraic property of the AES algorithm. In the presented property, we change 20 bytes from 5 intermediate values at some fixed locations in 5 consecutive rounds by carrying out 20 extra XOR

operations, and we show that after 5 rounds of processing, such modifications do not change the intermediate result and finally, still produce the same ciphertext. We defined an algorithm named δ , and by employing the δ algorithm, we constructed a modified version of the AES, the δ AES. For a plaintext and a key, the AES and the δ AES produce the same ciphertext.

We then showed that the five-round algebraic property of the AES can be used to analyse the internal structure of the ALPHA-MAC, a MAC function whose underlying block cipher is AES. We provided a second-preimage search algorithm and a collision search algorithm.

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Notes

¹For simplicity, we use only one variant of the δ algorithm here. Other variants of the δ algorithm also work.

²Here and in the rest of this section ‘collisions’ stands for ‘internal collisions’.

Appendix*Examples of AES with 20 XOR operations*

We provide seven examples of the outputs of the five algorithm and their corresponding plaintexts, secret keys and ciphertexts in Figure A1(a)–(g).

Figure A1 The values of P , K , AES round keys and the 20 bytes

Plaintext P	cipher Key K	Key Expansion →	AES Round Keys																																																	
<table><tr><td>3b</td><td>d1</td><td>bd</td><td>7c</td></tr><tr><td>87</td><td>ad</td><td>29</td><td>25</td></tr><tr><td>4d</td><td>1d</td><td>cf</td><td>41</td></tr><tr><td>d2</td><td>d7</td><td>07</td><td>ae</td></tr></table>	3b	d1	bd	7c	87	ad	29	25	4d	1d	cf	41	d2	d7	07	ae	<table><tr><td>03</td><td>43</td><td>9c</td><td>c9</td></tr><tr><td>a6</td><td>f4</td><td>57</td><td>9f</td></tr><tr><td>26</td><td>c5</td><td>90</td><td>29</td></tr><tr><td>11</td><td>30</td><td>39</td><td>e2</td></tr></table>	03	43	9c	c9	a6	f4	57	9f	26	c5	90	29	11	30	39	e2		<table><tr><td>03</td><td>43</td><td>9c</td><td>c9</td></tr><tr><td>a6</td><td>f4</td><td>57</td><td>9f</td></tr><tr><td>26</td><td>c5</td><td>90</td><td>29</td></tr><tr><td>11</td><td>30</td><td>39</td><td>e2</td></tr></table>	03	43	9c	c9	a6	f4	57	9f	26	c5	90	29	11	30	39	e2	
3b	d1	bd	7c																																																	
87	ad	29	25																																																	
4d	1d	cf	41																																																	
d2	d7	07	ae																																																	
03	43	9c	c9																																																	
a6	f4	57	9f																																																	
26	c5	90	29																																																	
11	30	39	e2																																																	
03	43	9c	c9																																																	
a6	f4	57	9f																																																	
26	c5	90	29																																																	
11	30	39	e2																																																	
	Initial Round Key																																																			
		Round Key 1	<table><tr><td>d9</td><td>9a</td><td>06</td><td>cf</td></tr><tr><td>03</td><td>f7</td><td>a0</td><td>3f</td></tr><tr><td>be</td><td>7b</td><td>eb</td><td>c2</td></tr><tr><td>cc</td><td>fc</td><td>c5</td><td>27</td></tr></table>	d9	9a	06	cf	03	f7	a0	3f	be	7b	eb	c2	cc	fc	c5	27	<table><tr><td>ed</td><td>0</td><td>dc</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>9b</td><td>0</td><td>04</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	ed	0	dc	0	0	0	0	0	9b	0	04	0	0	0	0	0																
d9	9a	06	cf																																																	
03	f7	a0	3f																																																	
be	7b	eb	c2																																																	
cc	fc	c5	27																																																	
ed	0	dc	0																																																	
0	0	0	0																																																	
9b	0	04	0																																																	
0	0	0	0																																																	
		Round Key 2	<table><tr><td>ae</td><td>34</td><td>32</td><td>fd</td></tr><tr><td>26</td><td>d1</td><td>71</td><td>4e</td></tr><tr><td>72</td><td>09</td><td>e2</td><td>20</td></tr><tr><td>46</td><td>ba</td><td>7f</td><td>58</td></tr></table>	ae	34	32	fd	26	d1	71	4e	72	09	e2	20	46	ba	7f	58	<table><tr><td>58</td><td>0</td><td>81</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>fd</td><td>0</td><td>81</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	58	0	81	0	0	0	0	0	fd	0	81	0	0	0	0	0																
ae	34	32	fd																																																	
26	d1	71	4e																																																	
72	09	e2	20																																																	
46	ba	7f	58																																																	
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0	0	0	0																																																	
fd	0	81	0																																																	
0	0	0	0																																																	
		Round Key 3	<table><tr><td>85</td><td>b1</td><td>83</td><td>7e</td></tr><tr><td>91</td><td>40</td><td>31</td><td>7f</td></tr><tr><td>18</td><td>11</td><td>f3</td><td>d3</td></tr><tr><td>12</td><td>a8</td><td>d7</td><td>8f</td></tr></table>	85	b1	83	7e	91	40	31	7f	18	11	f3	d3	12	a8	d7	8f	<table><tr><td>72</td><td>0</td><td>ed</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>5d</td><td>0</td><td>26</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	72	0	ed	0	0	0	0	0	5d	0	26	0	0	0	0	0																
85	b1	83	7e																																																	
91	40	31	7f																																																	
18	11	f3	d3																																																	
12	a8	d7	8f																																																	
72	0	ed	0																																																	
0	0	0	0																																																	
5d	0	26	0																																																	
0	0	0	0																																																	
		Round Key 4	<table><tr><td>5f</td><td>ee</td><td>6d</td><td>13</td></tr><tr><td>f7</td><td>b7</td><td>86</td><td>f9</td></tr><tr><td>6b</td><td>7a</td><td>89</td><td>5a</td></tr><tr><td>e1</td><td>49</td><td>9e</td><td>11</td></tr></table>	5f	ee	6d	13	f7	b7	86	f9	6b	7a	89	5a	e1	49	9e	11	<table><tr><td>cd</td><td>0</td><td>3e</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>c6</td><td>0</td><td>9f</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	cd	0	3e	0	0	0	0	0	c6	0	9f	0	0	0	0	0																
5f	ee	6d	13																																																	
f7	b7	86	f9																																																	
6b	7a	89	5a																																																	
e1	49	9e	11																																																	
cd	0	3e	0																																																	
0	0	0	0																																																	
c6	0	9f	0																																																	
0	0	0	0																																																	
		Round Key 5	<table><tr><td>d6</td><td>38</td><td>55</td><td>46</td></tr><tr><td>49</td><td>fe</td><td>78</td><td>81</td></tr><tr><td>e9</td><td>93</td><td>1a</td><td>40</td></tr><tr><td>9c</td><td>d5</td><td>4b</td><td>5a</td></tr></table>	d6	38	55	46	49	fe	78	81	e9	93	1a	40	9c	d5	4b	5a	<table><tr><td>69</td><td>0</td><td>19</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>4f</td><td>0</td><td>04</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	69	0	19	0	0	0	0	0	4f	0	04	0	0	0	0	0																
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49	fe	78	81																																																	
e9	93	1a	40																																																	
9c	d5	4b	5a																																																	
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4f	0	04	0																																																	
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		Round Key 6	<table><tr><td>fa</td><td>c2</td><td>97</td><td>d1</td></tr><tr><td>40</td><td>be</td><td>c6</td><td>47</td></tr><tr><td>57</td><td>c4</td><td>de</td><td>9e</td></tr><tr><td>c6</td><td>13</td><td>58</td><td>02</td></tr></table>	fa	c2	97	d1	40	be	c6	47	57	c4	de	9e	c6	13	58	02																																	
fa	c2	97	d1																																																	
40	be	c6	47																																																	
57	c4	de	9e																																																	
c6	13	58	02																																																	
		Round Key 7	<table><tr><td>1a</td><td>d8</td><td>4f</td><td>9e</td></tr><tr><td>4b</td><td>f5</td><td>33</td><td>74</td></tr><tr><td>20</td><td>e4</td><td>3a</td><td>a4</td></tr><tr><td>f8</td><td>eb</td><td>b3</td><td>b1</td></tr></table>	1a	d8	4f	9e	4b	f5	33	74	20	e4	3a	a4	f8	eb	b3	b1																																	
1a	d8	4f	9e																																																	
4b	f5	33	74																																																	
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f8	eb	b3	b1																																																	
		Round Key 8	<table><tr><td>08</td><td>d0</td><td>9f</td><td>01</td></tr><tr><td>02</td><td>f7</td><td>c4</td><td>b0</td></tr><tr><td>e8</td><td>0c</td><td>36</td><td>92</td></tr><tr><td>f3</td><td>18</td><td>ab</td><td>1a</td></tr></table>	08	d0	9f	01	02	f7	c4	b0	e8	0c	36	92	f3	18	ab	1a																																	
08	d0	9f	01																																																	
02	f7	c4	b0																																																	
e8	0c	36	92																																																	
f3	18	ab	1a																																																	
		Round Key 9	<table><tr><td>f4</td><td>24</td><td>bb</td><td>ba</td></tr><tr><td>4d</td><td>ba</td><td>7e</td><td>ce</td></tr><tr><td>4a</td><td>46</td><td>70</td><td>e2</td></tr><tr><td>8f</td><td>97</td><td>3c</td><td>26</td></tr></table>	f4	24	bb	ba	4d	ba	7e	ce	4a	46	70	e2	8f	97	3c	26																																	
f4	24	bb	ba																																																	
4d	ba	7e	ce																																																	
4a	46	70	e2																																																	
8f	97	3c	26																																																	
Ciphertext		Round Key 10	<table><tr><td>49</td><td>6d</td><td>d6</td><td>6c</td></tr><tr><td>d5</td><td>6f</td><td>11</td><td>df</td></tr><tr><td>bd</td><td>fb</td><td>8b</td><td>69</td></tr><tr><td>7b</td><td>ec</td><td>d0</td><td>f6</td></tr></table>	49	6d	d6	6c	d5	6f	11	df	bd	fb	8b	69	7b	ec	d0	f6																																	
49	6d	d6	6c																																																	
d5	6f	11	df																																																	
bd	fb	8b	69																																																	
7b	ec	d0	f6																																																	
<table><tr><td>da</td><td>68</td><td>03</td><td>a0</td></tr><tr><td>c9</td><td>7e</td><td>cc</td><td>09</td></tr><tr><td>4d</td><td>b6</td><td>93</td><td>8a</td></tr><tr><td>38</td><td>ea</td><td>62</td><td>48</td></tr></table>	da	68	03	a0	c9	7e	cc	09	4d	b6	93	8a	38	ea	62	48																																				
da	68	03	a0																																																	
c9	7e	cc	09																																																	
4d	b6	93	8a																																																	
38	ea	62	48																																																	

(a)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P	cipher Key K	Key Expansion →	AES Round Keys																																																	
<table><tr><td>b7</td><td>4d</td><td>6b</td><td>86</td></tr><tr><td>9b</td><td>8a</td><td>36</td><td>53</td></tr><tr><td>4e</td><td>f8</td><td>ca</td><td>1c</td></tr><tr><td>5d</td><td>63</td><td>08</td><td>2b</td></tr></table>	b7	4d	6b	86	9b	8a	36	53	4e	f8	ca	1c	5d	63	08	2b	<table><tr><td>6d</td><td>4e</td><td>e7</td><td>cf</td></tr><tr><td>db</td><td>c1</td><td>c7</td><td>0c</td></tr><tr><td>27</td><td>86</td><td>33</td><td>23</td></tr><tr><td>b4</td><td>70</td><td>4a</td><td>5c</td></tr></table>	6d	4e	e7	cf	db	c1	c7	0c	27	86	33	23	b4	70	4a	5c	Initial Round Key	<table><tr><td>6b</td><td>4e</td><td>e7</td><td>cf</td></tr><tr><td>db</td><td>c1</td><td>c7</td><td>0c</td></tr><tr><td>27</td><td>86</td><td>33</td><td>23</td></tr><tr><td>b4</td><td>70</td><td>4a</td><td>5c</td></tr></table>	6b	4e	e7	cf	db	c1	c7	0c	27	86	33	23	b4	70	4a	5c	
b7	4d	6b	86																																																	
9b	8a	36	53																																																	
4e	f8	ca	1c																																																	
5d	63	08	2b																																																	
6d	4e	e7	cf																																																	
db	c1	c7	0c																																																	
27	86	33	23																																																	
b4	70	4a	5c																																																	
6b	4e	e7	cf																																																	
db	c1	c7	0c																																																	
27	86	33	23																																																	
b4	70	4a	5c																																																	
		Round Key 1	<table><tr><td>92</td><td>dc</td><td>3b</td><td>f4</td></tr><tr><td>fd</td><td>3c</td><td>fb</td><td>f7</td></tr><tr><td>6d</td><td>eb</td><td>d8</td><td>fb</td></tr><tr><td>3e</td><td>4e</td><td>04</td><td>58</td></tr></table>	92	dc	3b	f4	fd	3c	fb	f7	6d	eb	d8	fb	3e	4e	04	58	The twenty bytes																																
92	dc	3b	f4																																																	
fd	3c	fb	f7																																																	
6d	eb	d8	fb																																																	
3e	4e	04	58																																																	
		Round Key 2	<table><tr><td>f8</td><td>24</td><td>1f</td><td>eb</td></tr><tr><td>f2</td><td>ce</td><td>35</td><td>c2</td></tr><tr><td>07</td><td>ec</td><td>34</td><td>cf</td></tr><tr><td>81</td><td>cf</td><td>cb</td><td>93</td></tr></table>	f8	24	1f	eb	f2	ce	35	c2	07	ec	34	cf	81	cf	cb	93	<table><tr><td>fb</td><td>0</td><td>bb</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>07</td><td>0</td><td>11</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	fb	0	bb	0	0	0	0	0	07	0	11	0	0	0	0	0																
f8	24	1f	eb																																																	
f2	ce	35	c2																																																	
07	ec	34	cf																																																	
81	cf	cb	93																																																	
fb	0	bb	0																																																	
0	0	0	0																																																	
07	0	11	0																																																	
0	0	0	0																																																	
		Round Key 3	<table><tr><td>d9</td><td>fd</td><td>e2</td><td>ff</td></tr><tr><td>78</td><td>b6</td><td>83</td><td>cc</td></tr><tr><td>db</td><td>37</td><td>03</td><td>41</td></tr><tr><td>68</td><td>a7</td><td>6c</td><td>09</td></tr></table>	d9	fd	e2	ff	78	b6	83	cc	db	37	03	41	68	a7	6c	09	<table><tr><td>74</td><td>0</td><td>64</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>67</td><td>0</td><td>af</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	74	0	64	0	0	0	0	0	67	0	af	0	0	0	0	0																
d9	fd	e2	ff																																																	
78	b6	83	cc																																																	
db	37	03	41																																																	
68	a7	6c	09																																																	
74	0	64	0																																																	
0	0	0	0																																																	
67	0	af	0																																																	
0	0	0	0																																																	
		Round Key 4	<table><tr><td>52</td><td>af</td><td>4d</td><td>44</td></tr><tr><td>33</td><td>85</td><td>06</td><td>47</td></tr><tr><td>cd</td><td>fa</td><td>f9</td><td>35</td></tr><tr><td>69</td><td>ce</td><td>a2</td><td>5d</td></tr></table>	52	af	4d	44	33	85	06	47	cd	fa	f9	35	69	ce	a2	5d	<table><tr><td>85</td><td>0</td><td>e3</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>39</td><td>0</td><td>1b</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	85	0	e3	0	0	0	0	0	39	0	1b	0	0	0	0	0																
52	af	4d	44																																																	
33	85	06	47																																																	
cd	fa	f9	35																																																	
69	ce	a2	5d																																																	
85	0	e3	0																																																	
0	0	0	0																																																	
39	0	1b	0																																																	
0	0	0	0																																																	
		Round Key 5	<table><tr><td>e2</td><td>4d</td><td>00</td><td>44</td></tr><tr><td>a5</td><td>20</td><td>26</td><td>61</td></tr><tr><td>81</td><td>7b</td><td>82</td><td>b7</td></tr><tr><td>72</td><td>bc</td><td>1e</td><td>43</td></tr></table>	e2	4d	00	44	a5	20	26	61	81	7b	82	b7	72	bc	1e	43	<table><tr><td>9b</td><td>0</td><td>ec</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>10</td><td>0</td><td>bb</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	9b	0	ec	0	0	0	0	0	10	0	bb	0	0	0	0	0																
e2	4d	00	44																																																	
a5	20	26	61																																																	
81	7b	82	b7																																																	
72	bc	1e	43																																																	
9b	0	ec	0																																																	
0	0	0	0																																																	
10	0	bb	0																																																	
0	0	0	0																																																	
		Round Key 6	<table><tr><td>2d</td><td>60</td><td>60</td><td>24</td></tr><tr><td>0c</td><td>2c</td><td>0a</td><td>6b</td></tr><tr><td>9b</td><td>e0</td><td>62</td><td>d5</td></tr><tr><td>69</td><td>d5</td><td>cb</td><td>88</td></tr></table>	2d	60	60	24	0c	2c	0a	6b	9b	e0	62	d5	69	d5	cb	88	<table><tr><td>ca</td><td>0</td><td>05</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>90</td><td>0</td><td>0d</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	ca	0	05	0	0	0	0	0	90	0	0d	0	0	0	0	0																
2d	60	60	24																																																	
0c	2c	0a	6b																																																	
9b	e0	62	d5																																																	
69	d5	cb	88																																																	
ca	0	05	0																																																	
0	0	0	0																																																	
90	0	0d	0																																																	
0	0	0	0																																																	
		Round Key 7	<table><tr><td>12</td><td>72</td><td>12</td><td>36</td></tr><tr><td>0f</td><td>23</td><td>29</td><td>42</td></tr><tr><td>5f</td><td>bf</td><td>dd</td><td>08</td></tr><tr><td>5f</td><td>8a</td><td>41</td><td>c9</td></tr></table>	12	72	12	36	0f	23	29	42	5f	bf	dd	08	5f	8a	41	c9																																	
12	72	12	36																																																	
0f	23	29	42																																																	
5f	bf	dd	08																																																	
5f	8a	41	c9																																																	
		Round Key 8	<table><tr><td>be</td><td>cc</td><td>de</td><td>e8</td></tr><tr><td>3f</td><td>1c</td><td>35</td><td>77</td></tr><tr><td>82</td><td>3d</td><td>e0</td><td>e8</td></tr><tr><td>5a</td><td>d0</td><td>91</td><td>58</td></tr></table>	be	cc	de	e8	3f	1c	35	77	82	3d	e0	e8	5a	d0	91	58																																	
be	cc	de	e8																																																	
3f	1c	35	77																																																	
82	3d	e0	e8																																																	
5a	d0	91	58																																																	
		Round Key 9	<table><tr><td>50</td><td>9c</td><td>42</td><td>aa</td></tr><tr><td>a4</td><td>b8</td><td>8b</td><td>fa</td></tr><tr><td>e8</td><td>d5</td><td>35</td><td>dd</td></tr><tr><td>c1</td><td>11</td><td>80</td><td>d8</td></tr></table>	50	9c	42	aa	a4	b8	8b	fa	e8	d5	35	dd	c1	11	80	d8																																	
50	9c	42	aa																																																	
a4	b8	8b	fa																																																	
e8	d5	35	dd																																																	
c1	11	80	d8																																																	
Ciphertext		Round Key 10	<table><tr><td>4b</td><td>d7</td><td>95</td><td>3f</td></tr><tr><td>65</td><td>dd</td><td>50</td><td>aa</td></tr><tr><td>89</td><td>5c</td><td>69</td><td>b4</td></tr><tr><td>6d</td><td>7c</td><td>fc</td><td>24</td></tr></table>	4b	d7	95	3f	65	dd	50	aa	89	5c	69	b4	6d	7c	fc	24																																	
4b	d7	95	3f																																																	
65	dd	50	aa																																																	
89	5c	69	b4																																																	
6d	7c	fc	24																																																	
<table><tr><td>f8</td><td>05</td><td>b3</td><td>09</td></tr><tr><td>e6</td><td>f8</td><td>42</td><td>45</td></tr><tr><td>36</td><td>b8</td><td>4d</td><td>e3</td></tr><tr><td>bb</td><td>44</td><td>a0</td><td>21</td></tr></table>	f8	05	b3	09	e6	f8	42	45	36	b8	4d	e3	bb	44	a0	21																																				
f8	05	b3	09																																																	
e6	f8	42	45																																																	
36	b8	4d	e3																																																	
bb	44	a0	21																																																	

(b)

(b)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P				cipher Key K				Key Expansion →	AES Round Keys																							
16	2f	90	60	48	f1	43	a0	Initial Round Key	48	f1	43	a0	The twenty bytes <table><tr><td>bd</td><td>0</td><td>26</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>3e</td><td>0</td><td>ec</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				bd	0	26	0	0	0	0	0	3e	0	ec	0	0	0	0	0
bd	0	26	0																													
0	0	0	0																													
3e	0	ec	0																													
0	0	0	0																													
1c	c5	84	f8	c5	59	7b	bd	c5	59	7b	bd																					
a8	c6	c1	a4	68	d5	21	32	68	d5	21	32																					
0f	00	c2	ae	88	ca	43	fb		88	ca	43	fb																				
								Round Key 1	33	c2	81	21	<table><tr><td>bd</td><td>0</td><td>26</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>3e</td><td>0</td><td>ec</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				bd	0	26	0	0	0	0	0	3e	0	ec	0	0	0	0	0
bd	0	26	0																													
0	0	0	0																													
3e	0	ec	0																													
0	0	0	0																													
								e6	bf	c4	79	0	0	0	0																	
								67	b2	93	a1	3e	0	ec	0																	
								68	a2	e1	1a	0	0	0	0																	
								Round Key 2	87	45	c4	e5	<table><tr><td>68</td><td>0</td><td>86</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>c6</td><td>0</td><td>46</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				68	0	86	0	0	0	0	0	c6	0	46	0	0	0	0	0
68	0	86	0																													
0	0	0	0																													
c6	0	46	0																													
0	0	0	0																													
								d4	6b	af	d6	0	0	0	0																	
								c5	77	e4	45	c6	0	46	0																	
								95	37	d6	cc	0	0	0	0																	
								Round Key 3	75	30	f4	11	<table><tr><td>f0</td><td>0</td><td>4e</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0d</td><td>0</td><td>ee</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				f0	0	4e	0	0	0	0	0	0d	0	ee	0	0	0	0	0
f0	0	4e	0																													
0	0	0	0																													
0d	0	ee	0																													
0	0	0	0																													
								ba	d1	7e	a8	f0	0	4e	0																	
								8e	f9	1d	58	0	0	0	0																	
								4c	7b	ad	61	0d	0	ee	0																	
								Round Key 4	bf	8f	7b	6a	<table><tr><td>22</td><td>0</td><td>79</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>a9</td><td>0</td><td>bc</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				22	0	79	0	0	0	0	0	a9	0	bc	0	0	0	0	0
22	0	79	0																													
0	0	0	0																													
a9	0	bc	0																													
0	0	0	0																													
								d0	01	7f	d7	0	0	0	0																	
								61	98	85	dd	a9	0	bc	0																	
								ce	b5	18	79	0	0	0	0																	
								Round Key 5	a1	2e	55	3f	<table><tr><td>1e</td><td>0</td><td>7c</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>12</td><td>0</td><td>82</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				1e	0	7c	0	0	0	0	0	12	0	82	0	0	0	0	0
1e	0	7c	0																													
0	0	0	0																													
12	0	82	0																													
0	0	0	0																													
								11	10	6f	b8	1e	0	7c	0																	
								d7	4f	ca	17	0	0	0	0																	
								cc	79	61	18	12	0	82	0																	
								Round Key 6	ed	c3	96	a9																				
									e1	f1	9e	26					0	0	0	0												
									7a	35	ff	e8					12	0	82	0												
									b9	c0	a1	b9					0	0	0	0												
								Round Key 7	5a	99	0f	a6																				
									7a	8b	15	33					1e	0	7c	0												
									2c	19	e6	0e					0	0	0	0												
									6a	aa	0b	b2					12	0	82	0												
								Round Key 8	4e	80	8f	29																				
									1b	5a	4f	7c					1e	0	7c	0												
									d1	02	e4	ea					0	0	0	0												
									19	e4	ef	5d					12	0	82	0												
								Round Key 9	12	92	1d	34																				
									56	0c	43	3f					56	0c	43	3f												
									57	55	b1	5b					57	55	b1	5b												
									eb	0f	e0	bd					eb	0f	e0	bd												
								Round Key 10	51	c3	de	ea																				
									6f	63	20	1f					51	c3	de	ea												
									2d	78	c9	92					6f	63	20	1f												
									f3	fc	1c	a1					2d	78	c9	92												
Ciphertext																																
69	2d	52	ad																													
27	7b	50	8d																													
69	2c	d3	8c																													
42	72	a6	ac																													

(c)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P				cipher Key K				Key Expansion →				AES Round Keys											
87	d8	b0	e1	2c	03	f5	58	Initial Round Key	2c	03	f5	c7	Round Key 1	3c	3f	ca	92	The twenty bytes	26	0	8e	0	
04	bd	51	ef	17	35	f9	e3		17	35	f9	5d		5b	6e	97	74		0	0	0	0	
a6	8d	77	b0	2a	a7	82	5d		2a	a7	82	c3		ec	4b	c9	94		b5	0	bc	0	
67	3f	a2	8b	46	64	91	c7		46	64	91	58		2c	48	d9	1e		0	0	0	0	
								Round Key 2	ac	93	59	ec	Round Key 3	17	84	dd	16		db	0	de	0	
									79	17	80	88		bd	aa	2a	de		0	0	0	0	
									9e	d5	1c	f4		50	85	99	11		49	0	ad	0	
									63	2b	f2	cb		7c	57	a5	49		0	0	0	0	
								Round Key 4	02	86	5b	4d	Round Key 5	fd	7b	20	6d		4c	0	79	0	
									3f	95	bf	61		c7	5e	78	3f		0	0	0	0	
									6b	ee	77	66		f2	ba	85	dc		00	0	de	0	
									3b	6c	c9	80		e4	50	2d	d0		0	0	0	0	
								Round Key 6	7d	06	26	4b	Round Key 7	48	4e	68	23		57	0	e9	0	
									c7	5e	78	3f		0c	99	26	47		0	0	0	0	
									f2	ba	85	dc		a6	48	3f	59		3f	0	1b	0	
									e4	50	2d	d0		d8	b4	7d	fd		0	0	0	0	
								Round Key 8	a2	ec	84	a7	Round Key 9	a0	4c	c8	6f						
									ae	b1	d6	8e		1d	ac	7a	f4						
									af	97	2a	4b		8b	1c	36	7d						
									71	76	5c	a6		2d	5b	07	a1						
								Round Key 10	29	65	ad	c2											
									e2	4e	34	c0											
									b9	a5	93	ee											
									85	de	d9	78											
Ciphertext																							
79	9c	7e	58																				
bd	b7	2b	11																				
5b	eb	b0	48																				
56	b0	37	b7																				

(d)

(d)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P				cipher Key K				Key Expansion →	AES Round Keys																							
b9	22	bd	ea	10	db	32	b9	Initial Round Key	10	db	32	b9	The twenty bytes																			
ca	e9	4b	ac	db	d6	e0	43		db	d6	e0	43																				
66	b4	67	96	4c	c3	8c	10		4c	c3	8c	10																				
de	3b	64	fc	68	f8	63	00		68	f8	63	00																				
								Round Key 1	0b	d0	e2	5b	<table><tr><td>b4</td><td>0</td><td>bb</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>c9</td><td>0</td><td>17</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				b4	0	bb	0	0	0	0	0	c9	0	17	0	0	0	0	0
b4	0	bb	0																													
0	0	0	0																													
c9	0	17	0																													
0	0	0	0																													
								11	c7	27	64																					
								2d	ce	42	52																					
								3e	c6	a5	a5																					
								Round Key 2	4a	9a	78	23	<table><tr><td>81</td><td>0</td><td>2a</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>e4</td><td>0</td><td>9c</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				81	0	2a	0	0	0	0	0	e4	0	9c	0	0	0	0	0
81	0	2a	0																													
0	0	0	0																													
e4	0	9c	0																													
0	0	0	0																													
								11	d6	f1	95																					
								2b	e5	a7	f5																					
								07	c1	64	c1																					
								Round Key 3	64	fe	86	a5	<table><tr><td>36</td><td>0</td><td>e4</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>bc</td><td>0</td><td>3d</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				36	0	e4	0	0	0	0	0	bc	0	3d	0	0	0	0	0
36	0	e4	0																													
0	0	0	0																													
bc	0	3d	0																													
0	0	0	0																													
								f7	21	d0	45																					
								53	b6	11	e4																					
								21	e0	84	45																					
								Round Key 4	02	fc	7a	df	<table><tr><td>9b</td><td>0</td><td>1f</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>ae</td><td>0</td><td>f8</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				9b	0	1f	0	0	0	0	0	ae	0	f8	0	0	0	0	0
9b	0	1f	0																													
0	0	0	0																													
ae	0	f8	0																													
0	0	0	0																													
								9e	bf	6f	2a																					
								3d	8b	9a	7e																					
								27	c7	43	06																					
								Round Key 5	f7	0b	71	ae	<table><tr><td>0</td><td>0</td><td>73</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>45</td><td>0</td><td>75</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>				0	0	73	0	0	0	0	0	45	0	75	0	0	0	0	0
0	0	73	0																													
0	0	0	0																													
45	0	75	0																													
0	0	0	0																													
								6d	d2	bd	97																					
								52	d9	43	3d																					
								b9	7e	3d	3b																					
								Round Key 6	5f	54	25	8b																				
									4a	98	25	b2																				
									b0	69	2a	17																				
									5d	23	1e	25																				
								Round Key 7	28	7c	59	d2																				
									ba	22	07	b5																				
									8f	e6	cc	db																				
									60	43	5d	78																				
								Round Key 8	7d	01	58	8a																				
									03	21	26	93																				
									33	d5	19	c2																				
									d5	96	cb	b3																				
								Round Key 9	ba	bb	e3	69																				
									26	07	21	b2																				
									5e	8b	92	50																				
									ab	3d	f6	45																				
Ciphertext								Round Key 10	bb	00	e3	8a																				
									75	72	53	e1																				
									30	bb	29	79																				
									52	6f	99	dc																				
<table><tr><td>ad</td><td>75</td><td>0e</td><td>98</td></tr><tr><td>d1</td><td>b6</td><td>06</td><td>f7</td></tr><tr><td>9f</td><td>92</td><td>00</td><td>d4</td></tr><tr><td>a2</td><td>cb</td><td>6f</td><td>e2</td></tr></table>								ad	75	0e	98	d1	b6	06	f7	9f	92	00	d4	a2	cb	6f	e2									
ad	75	0e	98																													
d1	b6	06	f7																													
9f	92	00	d4																													
a2	cb	6f	e2																													

(e)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P	cipher Key K	Key Expansion →	AES Round Keys																																																	
<table><tr><td>eb</td><td>6b</td><td>cb</td><td>f3</td></tr><tr><td>90</td><td>15</td><td>45</td><td>68</td></tr><tr><td>25</td><td>db</td><td>57</td><td>7c</td></tr><tr><td>55</td><td>37</td><td>26</td><td>6c</td></tr></table>	eb	6b	cb	f3	90	15	45	68	25	db	57	7c	55	37	26	6c	<table><tr><td>f4</td><td>b2</td><td>70</td><td>19</td></tr><tr><td>9e</td><td>76</td><td>c7</td><td>a4</td></tr><tr><td>73</td><td>1f</td><td>96</td><td>c2</td></tr><tr><td>8b</td><td>8c</td><td>36</td><td>39</td></tr></table>	f4	b2	70	19	9e	76	c7	a4	73	1f	96	c2	8b	8c	36	39	Initial Round Key	<table><tr><td>f4</td><td>b2</td><td>70</td><td>19</td></tr><tr><td>9e</td><td>76</td><td>c7</td><td>a4</td></tr><tr><td>73</td><td>1f</td><td>96</td><td>c2</td></tr><tr><td>8b</td><td>8c</td><td>36</td><td>39</td></tr></table>	f4	b2	70	19	9e	76	c7	a4	73	1f	96	c2	8b	8c	36	39	
eb	6b	cb	f3																																																	
90	15	45	68																																																	
25	db	57	7c																																																	
55	37	26	6c																																																	
f4	b2	70	19																																																	
9e	76	c7	a4																																																	
73	1f	96	c2																																																	
8b	8c	36	39																																																	
f4	b2	70	19																																																	
9e	76	c7	a4																																																	
73	1f	96	c2																																																	
8b	8c	36	39																																																	
		Round Key 1	<table><tr><td>bc</td><td>0e</td><td>7e</td><td>67</td></tr><tr><td>bb</td><td>cd</td><td>0a</td><td>ae</td></tr><tr><td>61</td><td>7e</td><td>e8</td><td>2a</td></tr><tr><td>5f</td><td>d3</td><td>e5</td><td>dc</td></tr></table>	bc	0e	7e	67	bb	cd	0a	ae	61	7e	e8	2a	5f	d3	e5	dc	The twenty bytes <table><tr><td>42</td><td>0</td><td>e9</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>de</td><td>0</td><td>72</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	42	0	e9	0	0	0	0	0	de	0	72	0	0	0	0	0																
bc	0e	7e	67																																																	
bb	cd	0a	ae																																																	
61	7e	e8	2a																																																	
5f	d3	e5	dc																																																	
42	0	e9	0																																																	
0	0	0	0																																																	
de	0	72	0																																																	
0	0	0	0																																																	
		Round Key 2	<table><tr><td>5a</td><td>54</td><td>2a</td><td>4d</td></tr><tr><td>5e</td><td>93</td><td>99</td><td>37</td></tr><tr><td>e7</td><td>99</td><td>71</td><td>5b</td></tr><tr><td>da</td><td>09</td><td>ec</td><td>30</td></tr></table>	5a	54	2a	4d	5e	93	99	37	e7	99	71	5b	da	09	ec	30	<table><tr><td>6c</td><td>0</td><td>40</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>f5</td><td>0</td><td>f5</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	6c	0	40	0	0	0	0	0	f5	0	f5	0	0	0	0	0																
5a	54	2a	4d																																																	
5e	93	99	37																																																	
e7	99	71	5b																																																	
da	09	ec	30																																																	
6c	0	40	0																																																	
0	0	0	0																																																	
f5	0	f5	0																																																	
0	0	0	0																																																	
		Round Key 3	<table><tr><td>c4</td><td>90</td><td>ba</td><td>f7</td></tr><tr><td>67</td><td>f4</td><td>6d</td><td>5a</td></tr><tr><td>e3</td><td>7a</td><td>0b</td><td>50</td></tr><tr><td>39</td><td>30</td><td>dc</td><td>ec</td></tr></table>	c4	90	ba	f7	67	f4	6d	5a	e3	7a	0b	50	39	30	dc	ec	<table><tr><td>94</td><td>0</td><td>25</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>1b</td><td>0</td><td>95</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	94	0	25	0	0	0	0	0	1b	0	95	0	0	0	0	0																
c4	90	ba	f7																																																	
67	f4	6d	5a																																																	
e3	7a	0b	50																																																	
39	30	dc	ec																																																	
94	0	25	0																																																	
0	0	0	0																																																	
1b	0	95	0																																																	
0	0	0	0																																																	
		Round Key 4	<table><tr><td>72</td><td>e2</td><td>58</td><td>af</td></tr><tr><td>34</td><td>c0</td><td>ad</td><td>f7</td></tr><tr><td>2d</td><td>57</td><td>5c</td><td>0c</td></tr><tr><td>51</td><td>61</td><td>bd</td><td>51</td></tr></table>	72	e2	58	af	34	c0	ad	f7	2d	57	5c	0c	51	61	bd	51	<table><tr><td>35</td><td>0</td><td>4a</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>67</td><td>0</td><td>28</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	35	0	4a	0	0	0	0	0	67	0	28	0	0	0	0	0																
72	e2	58	af																																																	
34	c0	ad	f7																																																	
2d	57	5c	0c																																																	
51	61	bd	51																																																	
35	0	4a	0																																																	
0	0	0	0																																																	
67	0	28	0																																																	
0	0	0	0																																																	
		Round Key 5	<table><tr><td>0a</td><td>e8</td><td>b0</td><td>1f</td></tr><tr><td>ca</td><td>0a</td><td>a7</td><td>50</td></tr><tr><td>fc</td><td>ab</td><td>f7</td><td>fb</td></tr><tr><td>28</td><td>49</td><td>f4</td><td>a5</td></tr></table>	0a	e8	b0	1f	ca	0a	a7	50	fc	ab	f7	fb	28	49	f4	a5	<table><tr><td>f3</td><td>0</td><td>21</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>b1</td><td>0</td><td>f4</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	f3	0	21	0	0	0	0	0	b1	0	f4	0	0	0	0	0																
0a	e8	b0	1f																																																	
ca	0a	a7	50																																																	
fc	ab	f7	fb																																																	
28	49	f4	a5																																																	
f3	0	21	0																																																	
0	0	0	0																																																	
b1	0	f4	0																																																	
0	0	0	0																																																	
		Round Key 6	<table><tr><td>79</td><td>91</td><td>21</td><td>3e</td></tr><tr><td>c5</td><td>cf</td><td>68</td><td>38</td></tr><tr><td>fa</td><td>51</td><td>a6</td><td>5d</td></tr><tr><td>e8</td><td>a1</td><td>55</td><td>f0</td></tr></table>	79	91	21	3e	c5	cf	68	38	fa	51	a6	5d	e8	a1	55	f0																																	
79	91	21	3e																																																	
c5	cf	68	38																																																	
fa	51	a6	5d																																																	
e8	a1	55	f0																																																	
		Round Key 7	<table><tr><td>3e</td><td>af</td><td>8e</td><td>b0</td></tr><tr><td>89</td><td>46</td><td>2e</td><td>16</td></tr><tr><td>76</td><td>27</td><td>81</td><td>dc</td></tr><tr><td>5a</td><td>fb</td><td>ae</td><td>5e</td></tr></table>	3e	af	8e	b0	89	46	2e	16	76	27	81	dc	5a	fb	ae	5e																																	
3e	af	8e	b0																																																	
89	46	2e	16																																																	
76	27	81	dc																																																	
5a	fb	ae	5e																																																	
		Round Key 8	<table><tr><td>f9</td><td>56</td><td>d8</td><td>68</td></tr><tr><td>0f</td><td>49</td><td>67</td><td>71</td></tr><tr><td>2e</td><td>09</td><td>88</td><td>54</td></tr><tr><td>bd</td><td>46</td><td>e8</td><td>b6</td></tr></table>	f9	56	d8	68	0f	49	67	71	2e	09	88	54	bd	46	e8	b6																																	
f9	56	d8	68																																																	
0f	49	67	71																																																	
2e	09	88	54																																																	
bd	46	e8	b6																																																	
		Round Key 9	<table><tr><td>41</td><td>17</td><td>cf</td><td>a7</td></tr><tr><td>2f</td><td>66</td><td>01</td><td>70</td></tr><tr><td>60</td><td>69</td><td>e1</td><td>b5</td></tr><tr><td>f8</td><td>be</td><td>56</td><td>e0</td></tr></table>	41	17	cf	a7	2f	66	01	70	60	69	e1	b5	f8	be	56	e0																																	
41	17	cf	a7																																																	
2f	66	01	70																																																	
60	69	e1	b5																																																	
f8	be	56	e0																																																	
Ciphertext		Round Key 10	<table><tr><td>26</td><td>31</td><td>fe</td><td>59</td></tr><tr><td>fa</td><td>9c</td><td>9d</td><td>ed</td></tr><tr><td>81</td><td>e8</td><td>09</td><td>bc</td></tr><tr><td>a4</td><td>1a</td><td>4c</td><td>ac</td></tr></table>	26	31	fe	59	fa	9c	9d	ed	81	e8	09	bc	a4	1a	4c	ac																																	
26	31	fe	59																																																	
fa	9c	9d	ed																																																	
81	e8	09	bc																																																	
a4	1a	4c	ac																																																	

(f)

Figure A1 The values of P , K , AES round keys and the 20 bytes (continued)

Plaintext P				cipher Key K				Key Expansion →	AES Round Keys											
99	30	d8	1f	11	b2	60	3c	Initial Round Key	11	b2	60	3c	The twenty bytes							
7e	99	08	84	2f	30	c2	6b		2f	30	c2	6b								
c4	24	91	d1	3a	c5	8c	c4		3a	c5	8c	c4								
b0	21	92	1e	d0	82	1e	5b		d0	82	1e	5b								
													6f	dd	bd	81				
													33	03	c1	aa				
													03	c6	48	8c				
													3b	b9	a7	fc				
													c1	1c	a1	20				
													57	54	95	3f				
													b3	75	3d	b1				
													37	8e	29	d5				
													b0	ac	0d	2d				
													9f	cb	5e	61				
													b0	c5	f8	49				
													80	0e	27	f2				
													57	fb	f6	db				
													a4	6f	31	50				
													39	fc	04	4d				
													58	56	71	83				
													14	ef	19	c2				
													47	28	19	49				
													d5	29	2d	60				
													e1	b7	c6	45				
													0f	e0	f9	3b				
													97	bf	a6	ef				
													bb	92	bf	df				
													c4	73	b5	f0				
													90	70	89	b2				
													09	b6	10	ff				
													37	a5	1a	c5				
													26	55	e0	10				
													06	76	ff	4d				
													af	19	09	f6				
													fd	58	42	87				
													11	44	a4	b4				
													5f	29	d6	9b				
													b8	a1	a8	5e				
													70	28	6a	ed				
													f2	b6	12	a6				
													31	18	ce	55				
													ed	4c	e4	ba				
													54	7c	16	fb				
													e6	50	42	e4				
Ciphertext																				
f2	c4	0a	90																	
5b	8f	0a	41																	
05	ae	83	56																	
26	df	06	0d																	

(g)