

Phys305 Assignment, Autumn 2006

1 Chapter 1

1.4 At time $t=0$ a particle is represented by the wavefunction

$$\Psi(x, 0) = \begin{cases} Ax/a. & \text{if } 0 \leq x \leq a \\ A(b-x)/(b-a) & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where A , a , and b are constants.

- (a) Normalize Ψ to determine A in terms of a and b .
- (b) Sketch $\Psi(x, 0)$ as a function of x .
- (c) Where is the particle most likely to be found at $t = 0$?
- (d) What is the probability of finding the particle to the left of a . Check your result in the limiting case $b = a$ and $b = 2a$.
- (e) What is the expectation value of x .

1.5 Consider the wavefunction

$$\Psi(x, t) = Ae^{-\lambda|x|}e^{-i\omega t}$$

where A , λ and ω are positive real numbers.

- (a) Normalize Ψ .
- (b) Determine the expectation value of x and x^2 .
- (c) Find the standard deviation of x . Sketch the graph of $|\Psi|^2$ as a function of x and mark the points $(\langle x \rangle - \sigma)$ and $(\langle x \rangle + \sigma)$, to illustrate the sense in which σ represents the 'spread' in x . What is the probability that the particle would be found outside this range.

1.7 Show $d\langle p \rangle/dt = \langle -\partial V/\partial x \rangle$.

1.14 Let $P_{ab}(t)$ be the probability of finding a particle in the range $(a < x < b)$, at time t .

- (a) Show that

$$\frac{dP_{ab}(t)}{dt} = J(a, t) - J(b, t)$$

where

$$J(x, t) \equiv \frac{i\hbar}{2m} \left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right)$$

what are the units of $J(x, t)$. (J is called probability current).

- (b) Find the probability current for the wavefunction $\Psi(x, t) = Ae^{-a[mx^2/\hbar + it]}$

1.18 In general, quantum mechanics is relevant when the de Broglie wavelength of the particle in question h/p is great that the characteristic size of the system, d . In thermal equilibrium at temperature T , the average kinetic energy of a particle is,

$$\frac{p^2}{2m} = \frac{3}{2}k_B T$$

(where k_B is Boltzmann constant), so the typical de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{3mk_B T}}.$$

The purpose of this problem is to anticipate which systems will have to be treated quantum mechanically, and which can be safely described classically.

(a) Solids, The lattice spacing is around $d = 0.3nm$. Find the temperature below which the free electron in a solid are quantum mechanical. Below what temperature are the nuclei in solid quantum mechanical? (use Sodium as a typical case.

(b) Gases. For what temperature are the atoms in an ideal gas at pressure P quantum mechanical. (Hint: use ideal gas law $PV = Nk_B T$ to determine the interatomic spacing).