

WIND ENERGY SYSTEMS

Weeks 12

- *Theory of horizontal axis wind turbine (HAWT)*
- *Airfoils*
- *Derivation of Blade Element Momentum Theory*
- *Solution procedure*

Objectives of lecture week 12

- To gain an understanding of the aerodynamic limits to wind turbine performance (the *Lanchester-Betz* limit) and basic wind turbine theories: Actuator Disk Theory, Rotor Disk (Galuert) Theory and Blade Element Momentum Theory (BEM);
- To appreciate the importance of the *Lift and Drag* characteristics of the wind turbine blades (aerofoils);
- To understand and predict the aerodynamic forces acting on the blades of a HAWT and hence the power output and rotor thrust load;

WIND TURBINES STANDARDS

- Wind Turbine Generator Systems – Part 1: Safety Requirements. IEC 61400 – 1 (621.312136/13)
- Wind Turbine Generator Systems – Part 2: Safety of Small Wind Turbines CEI IEC 1400-2
- Wind Turbine Generator Systems – Part 11: Acoustic Noise Measurement Techniques. IEC 61400-11 (621.312136/13)
- Wind Turbine Generator Systems – Part 12: Wind Turbine Power Performance Testing. IEC 61400 – 12 (621.312136/13)
- Acoustics – Measurement, Prediction and assessment of noise from wind turbine generators. AS 4959 – 2010
- Wind Turbines Part 21: Measurement and assessment of power quality characteristics of grid connected wind turbines. AS 61400.21 - 2006

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Wind Energy / Wind Devices Websites

- **Australian Wind Energy Institute.** [//www.australianwindenergyinstitute.com](http://www.australianwindenergyinstitute.com)



- **Danish Wind Industry Association.** <http://www.windpower.org/en/>



- **Sandia National Laboratories (US).** <http://www.sandia.gov/wind/>



- **Vestas** <http://www.vestas.dk/>

- **Vestas**

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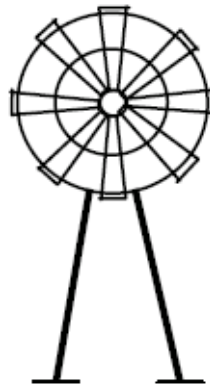
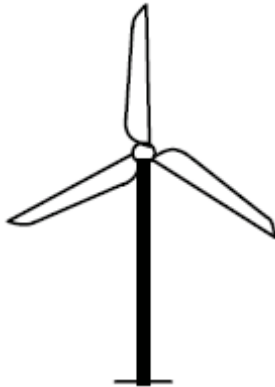


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INTRODUCTION

The most common wind turbines used for electrical power generation are *low solidity* HAWTs with relatively high tip speed ratio, λ .



For application where high torque is required, multi-bladed high solidity turbines are used.



Actuator Disk Theory

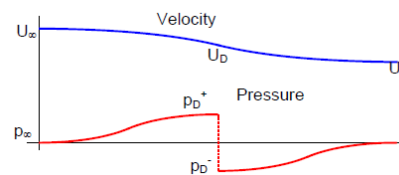
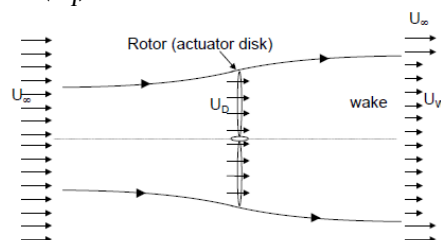
The simplest theory to analyse the performance of HAWT is the *actuator disk theory*. The theory predicts the *optimum (absolute limit) Performance Coefficient (C_p)* and the *Thrust Coefficient (C_T)*.

The turbine is considered to be a **porous disk** (representing the spinning blades) through which the wind flows. The disk extracts kinetic energy from the wind – the air velocity therefore decreases in the wake.

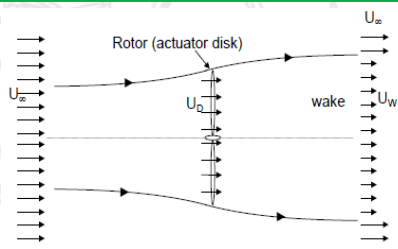
A **streamtube** containing the turbine is shown in the figure.

We use three basic tools of fluid dynamics in the analysis:

- *Conservation of Mass*
- *Impulse-momentum principle*
- *Bernoulli's Equation*



Actuator Disk Theory



Conservation of mass in the streamtube gives:

$$\rho A_{\infty} u_{\infty} = \rho_D A_D u_D = \rho_W A_W u_W \quad (1)$$

We assume incompressible flow and model the fact that the wind must slow down by the time it reaches the rotor by aU_{∞} - i.e. with the "**axial induction factor**", a .

$$u_D = u_{\infty}(1-a) \quad (2)$$

The pressure at the upstream side and downstream of the actuator disk are found from Bernoulli's equation (*neglecting changes in air density and elevation*)

$$p_{\infty} + \frac{1}{2} \rho u_{\infty}^2 = p_D^+ + \frac{1}{2} \rho u_D^2 \quad (3)$$

$$p_D^- + \frac{1}{2} \rho u_D^2 = p_{\infty} + \frac{1}{2} \rho u_W^2 \quad (4)$$

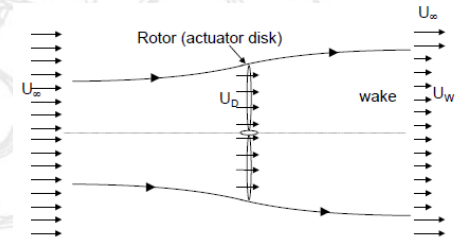
Subtracting the 2nd equation from the 1st we get

$$p_D^+ - p_D^- = \frac{1}{2} \rho (u_{\infty}^2 - u_W^2) \quad (5)$$

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Actuator Disk Theory



The thrust, F , acting on the rotor is given

$$F = A_D (p_D^+ - p_D^-) = \frac{1}{2} \rho A_D (u_{\infty}^2 - u_W^2) \quad (6)$$

Alternatively, the thrust can also be obtained using momentum balance as the rate of change of momentum in a control volume enclosing the rotor.

$$F = \rho A_D u_D (u_{\infty} - u_W) = \rho A_D (1-a) u_{\infty} (u_{\infty} - u_W) \quad (7)$$

Equating (6) and (7) results in

$$\frac{1}{2} (u_{\infty}^2 - u_W^2) = u_{\infty} (1-a) (u_{\infty} - u_W) \quad (8)$$

gives and the power extracted by the rotor is

$$u_W = u_{\infty} (1-2a) \quad (9)$$

$$u_D = \frac{u_{\infty} + u_W}{2} \quad (10)$$

$$\text{Power} = F u_D = 2 \rho A_D u_{\infty}^3 a (1-a)^2 \quad (11)$$

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Actuator Disk Theory

The power coefficient, C_p

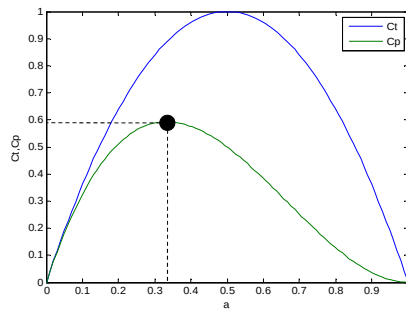
$$C_p = \frac{P}{P_w} = 4a(1-a)^2 \quad (12)$$

We then find the maximum possible power coefficient which occurs when

$$\begin{aligned} dC_p / da &= 0 = 4(1-a)(1-3a) \\ \rightarrow a &= 1/3 \end{aligned}$$

$$\begin{aligned} C_{p,\max} &= 4a(1-a)^2 = 4/3(1-1/3)^2 \\ &= 16/27 = 0.593 \end{aligned}$$

This value is the Lanchester-Betz limit.



We may also define a *Thrust Coefficient*, C_T , to model the axial force on the turbine.

$$C_T = \frac{F}{\rho A_D U_\infty^2 / 2} = 4a(1-a) \quad (13)$$

These coefficients which are derived from simple 1-D momentum model is not valid when $a > 0.5$.



Lanchester – Betz Limit

$$C_p = \frac{P}{P_w} = \frac{P}{\rho A U_\infty^3 / 2}$$

The optimum (theoretical) Power Coefficient, C_p is 59.3% known as Lanchester – Betz Limit

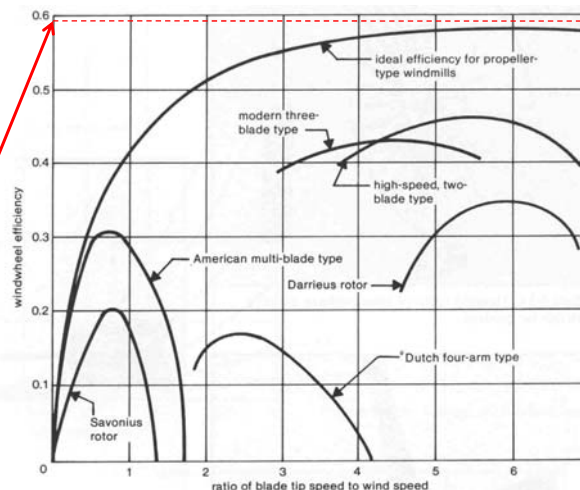


Figure 5-13: Typical performance of several wind machines.



Example

An inventor claims to have developed a water turbine of 1 m diameter that produces 5 kW in a tidal flow of five knots (1 knot = 0.515 m/s). Do you believe the inventor's claim? Why?

$$P_W = \frac{1}{2} \rho A U^3$$

$$= \frac{1}{2} \times 1000 \times \pi \times 0.5^2 \times (5 \times 0.515)^3$$

$$= 6701.49 \text{ W}$$

$$C_{P,\max} = \frac{P}{P_W}$$

$$P_W = 0.593 \times 6701.49$$

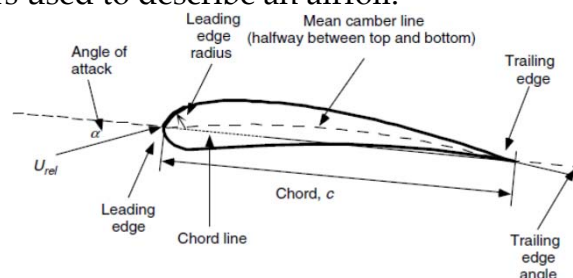
$$= 3.973 \text{ kW}$$

Cautiously believe until he shows the actual unit. It may have other feature such as a diffuser.



Airfoils

Wind turbine blades are designed to extract mechanical forces from the wind flow around the blade. Airfoils are effective devices to do this. And it is important to familiarize ourselves with airfoil characteristics. Figure below shows the important parameters used to describe an airfoil.



The geometric parameters that have an effect on the aerodynamic performance of an aerofoil include: **the leading edge radius, mean camber line, maximum thickness, and thickness distribution of the profile, and the trailing edge angle.**

Airfoils

When an airfoil is placed in real, viscous moving fluid it will experience a force and pitching moment due to the pressure and viscous forces on the surface of the body.

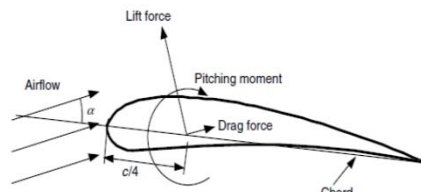
Lift is the component of force on body **normal to the direction** of travel (*i.e.* **normal** to the velocity of fluid *relative* to the body). While **drag** force is **parallel** to the velocity of fluid relative to the body.

These forces and moments for 2D aerofoil are measured in wind tunnel and expressed in terms of coefficients:

$$C_l = \frac{L/l}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\text{Lift force/unit length}}{\text{Dynamic force/unit length}} \quad (25)$$

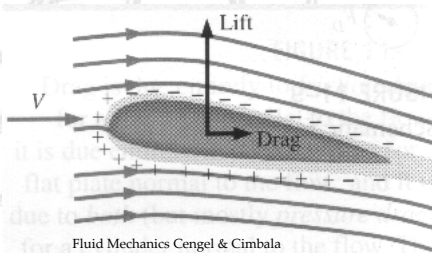
$$C_d = \frac{D/l}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\text{Drag force/unit length}}{\text{Dynamic force/unit length}} \quad (26)$$

$$C_m = \frac{M/l}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\text{Pitching Moment}}{\text{Dynamic Moment}} \quad (27)$$

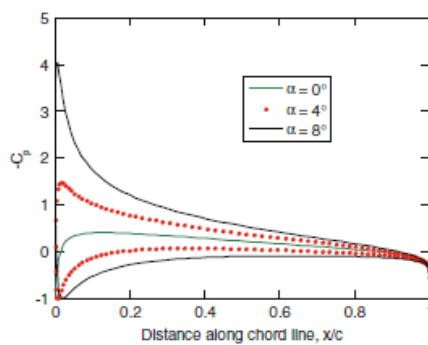


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Aerofoils



Fluid Mechanics Cengel & Cimbala



Small Wind Turbine Analysis, D. Woods

As the fluid flow pass an airfoil at low angle of attack, α , the fluid will need to accelerate along the top surface creating a low pressure (**suction**) region along the top surface.

The surface pressure differences between the top and bottom surface for different α are shown in the figure on the left corner. For NACA 0012 aerofoil.

$$C_p = \frac{P - P_o}{\frac{1}{2}\rho U_\infty^2} \quad (28)$$

As α increases both lift and drag will increase. Initially lift increases more rapid than drag but at some point drag increases rapidly.

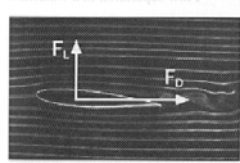
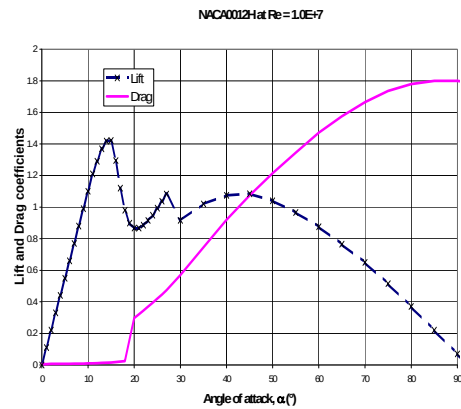
Aerofoils

Lift and Drag Coefficients are functions of:

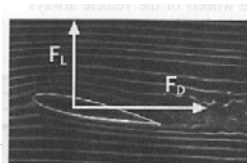
- *Angle of attack,*
- *Shape of aerofoil*
- *Reynolds number ($Re = Uc/\nu$).*

Data generally given for *sections* (infinite span).

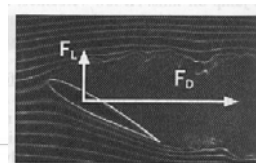
Note: it is the ratio C_L/C_D that is usually the most important parameter in regard to aerofoil performance in wind turbines.



(a) 5°



(b) 15°

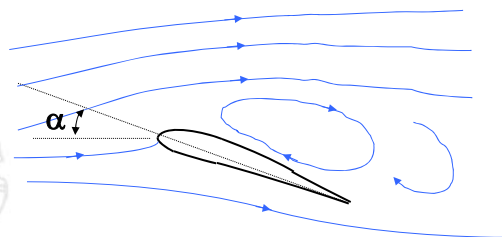


(c) 30°

Aerofoils : Stall

As the angle of attack is increased so the lift increases almost linearly until the "boundary layer" cannot remain attached.

At this point stall occurs and the aerofoil is no longer streamlined and lift drops dramatically.

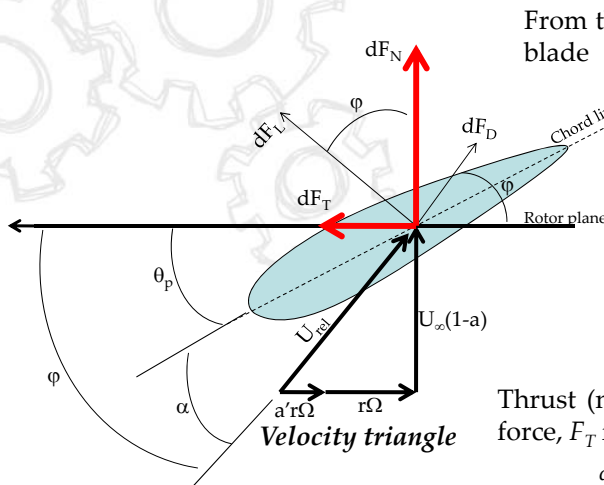


Aerofoil data for aeroplane applications often present only for small values of α - but wind turbines can operate with very wide range of α .

Difference in aerofoils for flying object and wind turbines, in wind turbines:

- *High angle of attacks*
- *Low Reynolds number*

Turbine Blades: Lift & Drag



From the diagram, looking down blade

$$\tan \phi = \frac{(1-a)u_\infty}{(1+a')\Omega r} = \frac{(1-a)}{(1+a')\lambda_r}$$

$$U_{rel} = u_\infty(1-a) / \sin \phi \quad (29)$$

Lift and drag for a blade element

$$dF_L = C_L \frac{1}{2} \rho U_{rel}^2 c dr \quad (30)$$

$$dF_D = C_D \frac{1}{2} \rho U_{rel}^2 c dr \quad (31)$$

Thrust (normal), F_N and tangential force, F_T for a blade element

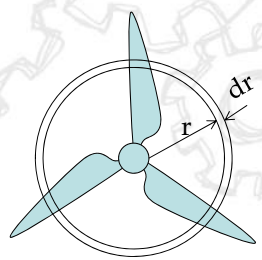
$$dF_N = dF_L \cos \phi + dF_D \sin \phi \quad (32)$$

$$dF_T = dF_L \sin \phi - dF_D \cos \phi \quad (33)$$

Velocity triangle

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Blade Element Momentum Theory (BEM)



If the rotor has N blades, (combining (30) & (31) in (32) & (33)) the total normal force on the section at a distance, r from the center is:

$$dF_N = N \frac{1}{2} \rho U_{rel}^2 C_a c dr \quad (34)$$

where

The total tangential force is $C_a = C_L \cos \phi + C_D \sin \phi$

$$dF_T = N \frac{1}{2} \rho U_{rel}^2 C_{a'} c dr \quad (35)$$

$C_{a'} = C_L \sin \phi - C_D \cos \phi$

The differential torque is

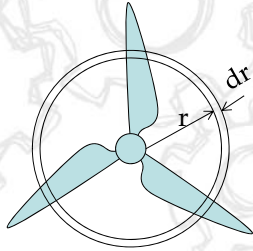
$$dQ = N \frac{1}{2} \rho U_{rel}^2 C_{a'} c r dr \quad (36)$$

Defining local solidity, σ'

$$\sigma' = N c / 2\pi r \quad (37)$$

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BEM



The elemental thrust from axial momentum (see Eq. 13)

$$dF_N = 4\pi \rho u_\infty^2 a(1-a)r dr \quad (38)$$

Equating (34) and (38),

$$dF_N = N \frac{1}{2} \rho U_{rel}^2 C_a c dr \quad (34)$$

we can obtain relation between a and C_l and C_d .

$$a(1-a) = \frac{U_{rel}^2}{4U_\infty^2} \sigma' C_a \quad (39)$$

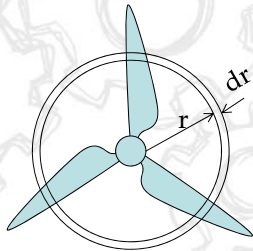
Eq. (39) provides an iterative way to find a and C_l and C_d . From assumed a , relative velocity is obtained hence lift and drag coefficients. Using Eq. (39) new a is found and iteration is continued until a is converged. So it is common to write Eq. (39) as

$$a_n(1-a_n) = \frac{U_{rel}^2}{4U_\infty^2} \sigma' C_a \quad (40) \quad \text{where } a_n \text{ is new } a \text{ value.}$$

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BEM



The elemental torque from rotating disk theory (see Eq. 16)

$$dQ = 4\pi \rho \Omega U_\infty a'(1-a)r^3 dr \quad (41)$$

Equating (41) and (36)

$$dQ = N \frac{1}{2} \rho U_{rel}^2 C_{a'} c r dr \quad (36)$$

We obtain relation between a and a' .

$$a'(1-a) = \frac{\sigma' U_{rel}^2}{4 U_\infty^2} \frac{C_{a'}}{\lambda_r} \quad (42)$$

Combining (40) and (42), we can get

$$a' = \frac{a C_{a'}}{C_a \lambda_r} \quad (43)$$

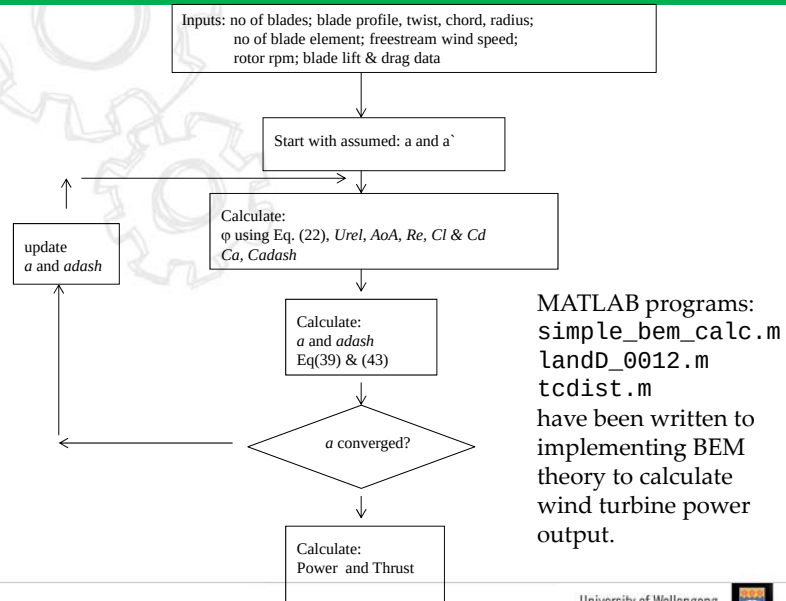
After a and a' have been obtained, blade element thrust and torque can be calculated using (34) and (36). Total torque and thrust can be obtained by integration over the whole blade span.

The iterative procedure is shown in the next slide.

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Solution Procedure



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